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ABSTRACT
We draw parallels between the pandemic and foreclosure in network industries by applying "Susceptible, Infected, Recovered" (SIR) modelling to an antitrust setting. We consider a digital service seeking to grow into an addressable market occupied by an incumbent platform. The entrant can grow organically, but amassing more users allows it to spread faster as users invite friends or generate content increasing its attractiveness. We consider the impact of the incumbent taking steps (e.g. reducing interoperability) to make the entrant "less infectious" with three main implications for antitrust policy: conduct may have large effects even if the targeted service continues to grow; conduct is most effective when applied against nascent services before they can harness netwrok effects; and conduct can have non-linear effects, with the most "viral" services continuing to grow while others are eliminated. Each result has parallels with the experience of the pandemic and implications for innovation incentives.

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KEYWORDS Platforms; exclusionary abuses; foreclosure; network effects; SIR

1. Introduction

For most antitrust practitioners the two most common conversation topics over the last 18 months have surely been "COVID-19" and "Big Tech". This paper argues these topics are more closely related than they might seem. As well as the obvious observation that the pandemic has benefited tech firms by shifting a greater share of communication, shopping, and entertainment online, there is a further connection: the same mathematical concepts that have become familiar in the context of the pandemic: "R-rates", "flattening the curve", and "herd immunity" can also tell us something about antitrust theories of harm and foreclosure in industries subject to network effects.

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We discuss how the "Susceptible, Infected, Recovered" (SIR) models used to understand epidemiological spread can be adapted to consider the case of a dominant "gatekeeper" platform seeking to prevent the emergence of a new entrant in a market with strong network effects. This mirrors the increasing volume of research applying these models to economic issues.

We build a model in which a new entrant app is seeking to penetrate an addressable market of consumers who already make use of the services of an incumbent platform. We assume that this new entrant can grow to some extent organically, but that amassing more users also allows it to spread more effectively (e.g. because its current users can send invites to their friends or will generate content which makes the service more attractive). We assume also that the incumbent platform can take steps (e.g. reducing interoperability or reducing users' ability to share content or invite their friends) which make the rival service "less infectious".

This model set-up endogenously creates an "S-curve" dynamic in which growth of the service first accelerates (more users means more scope for invites/content sharing, drawing in more users more quickly) and then slows (as the service grows it starts to saturate the addressable market meaning less scope to acquire new users). This S-curve is analogous to the growth of a viral pandemic where growth accelerates and then slows as the population achieves "herd immunity".

The model has three main takeaways:

- First, it shows how conduct can have large growth effects *relative to a counterfactual* even while a service continues to grow *in absolute terms*. There are parallels here with the control of a virus where a public health intervention can "flatten the curve" and significantly slow the spread of an infection while still failing to stop its spread entirely. This implies that empirical analysis of effects should look for growth effects as well as level effects and should be "on the lookout" for conduct which prevents services from moving onto the "steep" part of the S-curve.
- Second, it shows how foreclosing conduct is likely to be particularly effective when applied early in a service's life before it has amassed a user base and moved onto the steep part of the S-curve. This is important as it means that conduct might be effective at preventing rivals' spread into new geographies or product lines even if it is unable to remove the service entirely from those markets where it is already

established. It also implies that conduct which is only partially effective when applied to already established entrants may be effective at reducing entry and funding elsewhere as potential entrants will recognize that they will be unsuccessful if subjected to said conduct before they "get off the ground".

- Third, it illustrates how conduct can have non-linear effects: a particularly "viral" service might be able to resist foreclosing conduct and continue to grow (albeit at a lower rate relative to a counterfactual without the conduct) while a less inherently fast-growing service may find the conduct is sufficient to stop it in its tracks even if it would have otherwise grown successfully. There are parallels here with epidemiology where a virus will continue to grow exponentially as long as its R rate is above 1, but will be suppressed if it is reduced below this level. There is an analogy also with the emergence of more infectious variants of COVID-19 that have been able to spread notwithstanding restrictions which had proven sufficient to suppress the original virus.

The rest of the paper is structured as follows. Section 2 provides background on relevant antitrust cases and the attention given to the absolute level of targeted firms' growth as a measure of anticompetitive effects. Section 3 provides background on SIR models and their growing use in the economic literature. Section 4 presents our SIR model applied to foreclosure theories of harm in the technology space. Section 5 presents some key implications of our model. Section 6 discusses the implications for antitrust policy. Section 7 concludes.

2. Background on relevant antitrust cases

Almost all of the concerns around exclusionary abuses in the technology space share two characteristics: (i) some kind of strategy that impedes rivals' ability to acquire customers in absolute or relative terms, whether that be removing them from search results, refusing to preinstall their apps, restricting access to APIs and/or data or otherwise reducing inter-operability; and (ii) an important role for scale and/or network effects which means that this reduction in customer acquisition can be anticipated to materially hinder the development of rivals or even "tip" the market to monopoly. These properties are shared in, for example, the original Microsoft cases; the Google Search, Android and Ad Tech cases; the FTC's case against Facebook; as well as some of the cases brought against Amazon and Apple.[1] Typically, but not always, the accusation is that the platform is engaged in a "dynamic leveraging" strategy that is either defensive in nature (forestalling a potential competitive threat) or offensive ("swinging" market power into a new market).[2]

These sorts of theories of harm are most blatant when there is evidence of wholesale market exit by targeted competitors and there are "bodies on the floor" as a result of the conduct.[3] When this is not the case, and the impacted firms continue to grow in absolute terms, it is natural for accused firms to argue this is proof against foreclosing effects.[4]

In turn, the natural counter to this is that we do not necessarily care only about *absolute* reductions in an impacted firm's volume of business, but rather about the impact *relative to the counterfactual*. While in mature industries the two are likely to go hand in hand, in new, fast-growing industries it may be that impacted rivals continue to grow in absolute terms, at least for a time, while ceding market share and expanding less than they would have done in the counterfactual. In a world of network effects and dynamic competition this could result in rivals falling behind over time, ultimately resulting in anticompetitive effects.

The question though is how to operationalize these concerns. This paper argues that "Susceptible, Infected, Recovered" (SIR) models similar to those used in modelling epidemiological spread can be used to estimate counterfactual growth paths and shed light on potential foreclosing effects over the long run.

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[1]See, for example, USA v. Microsoft Corporation, 253 F.3d 34 (D.C. Cir. 2001); EC cases 39740 'Google Search (Shopping)'; 40099 'Google Android'; FTC v. Facebook, Inc FTC MATTER/FILE NUMBER: 191 0134. See also the EC's press releases in respect of investigations into Apple and Amazon.

[2]The classic references for such dynamic foreclosure theories are DW Carlton and M Waldman, 'The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries' (2002) 33(2) The RAND Journal of Economics 194 and JP Choi and C Stefanadis, 'Tying, Investment, and the Dynamic Leverage Theory' (2001) 32(1) The RAND Journal of Economics 52. For a discussion in the context of tech platforms see S Athey and FS Morton, 'Platform Annexation' (2021) SIEPR Working Paper 21-015. These and many other papers show why foreclosure theories of harm in dynamic technology markets need not fall-foul of the "One Monopoly Profit" critique which argues that a monopolist will not want to use its market power to colonize adjacent/complementary markets. For a short summary, see J Baker, 'Note on "Single Monopoly Profit" Theory' (2014).

[3]For example, the European Commission's Google Shopping decision pointed to falls in traffic for price comparison sites of circa 80 to 92% while Google's own service saw traffic increase 14–45 fold. https://ec.europa.eu/commission/presscorner/detail/en/IP_17_1784.

[4]For an example of such commentary see: https://chillingcompetition.com/2021/05/03/the-commission-sends-an-so-to-apple-common-carrier-antitrust-picks-up-speed/. Similarly, Google's reaction to the shopping decision argues that the continued growth of eBay and Amazon provides evidence against anticompetitive foreclosure https://blog.google/around-the-globe/google-europe/european-commission-decision-shopping-google-story/.

3. Background on SIR models and their use in epidemiology and economics

It is not a coincidence that the term "going viral" has been used to describe the growth of digital apps, services and content. Just as an infectious disease spreads through a population as long as each newly infected person infects at least one more person, digital services can rely on similar dynamics by encouraging users to share content or invite new users or by relying on the fact that new users generate network effects (e.g. by generating the content of interest to others or encouraging the development of a surrounding ecosystem) that draw more users in. Both infectious diseases and digital services can take advantage of exponential growth to spread at dramatic rates (Figure 1).[5]

The same mathematical processes can therefore be used to understand these quite different applications. Just as the spread of a virus is determined by the level of infectiousness as measured by the "basic reproductive number", R, the growth of a service will be governed by the degree to which users share content or invite new users, and the extent to which users who try the service "convert" into long term users. Similarly, just as the growth of a virus is ultimately limited by the size of the susceptible population, and growth will tend to slow as the number of uninfected individuals declines and the population achieves "herd immunity", the growth of a new digital service is bound by its "addressable market" and will see its growth slow as this becomes saturated and the number of potential new users to try the service declines.[6]

Epidemiologists have used the SIR approach to model viral spread. These models split the population between those who are uninfected hence "Susceptible", those who are "Infected" and potentially infectious, and those who are "Recovered" or otherwise immune. They then make assumptions (e.g. about the extent to which individuals interact, the extent interactions result in infection, and so on) to model the spread of an infection through a population. These models have traditionally been used to study the spread of viruses

[5]We note that the term "exponential growth" is often used imprecisely as a synonym for "fast growth". This is incorrect: exponential growth refers to a process which grows in a compound fashion (e.g. by increasing at a constant percentage rate over time). Technically we refer here to exponential growth with saturation (e.g. a virus or digital service spreading over a fixed population), which is described by the logistic growth function exhibiting S-curve dynamics where an exponential stage of increasing growth is followed by an inflection point followed by decreasing growth.

[6]In light of this, strategy writers have emphasized the importance of "jumping the S-curve" (i.e. the importance of firms ensuring that they enter new product lines before their existing ones reach saturation point). See, for example, PF Nunes and T Breene, 'Jumping the S-Curve-How to Beat the Growth Cycle, Get on Top and Stay There' *Harvard Business Review Press* (2011).

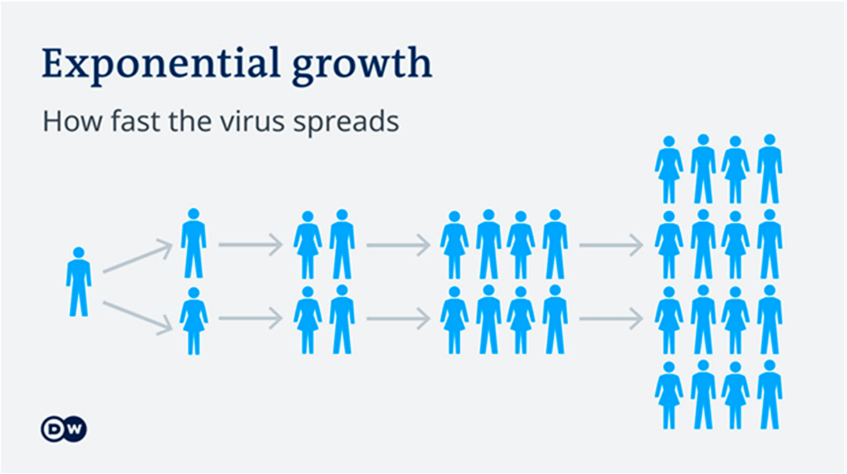

Figure 1. The power of exponential growth. Source: https://www.dw.com/en/corona-confusion-how-to-make-sense-of-the-numbers-and-terminology/a-52825433.

(e.g. influenza,[7] dengue fever,[8] SARS[9] and COVID-19[10]). However, they are increasingly being used in non-epidemiological applications to study the spread of computer viruses[11] or financial contagion[12] as well as the diffusion of marketing, ideas/information and influence in social networks.[13] More recently, the COVID-19 epidemic has led to a further resurgence of the SIR model in seminal work on virus network diffusion where economists have expanded on classical epidemiology work to improve our understanding of health policy decisions and

[7]See e.g. SAA Karim and R Razali, 'A Proposed Mathematical Model of Influenza A, H1N1 for Malaysia' (2011) 11 Journal of Applied Sciences 1457.

[8]For example, HS Rodrigues, MTT Monteiro and DFM Torres, 'Dengue in Cape Verde: Vector Control and Vaccination' in JP Bourguignon and others (eds), *Dynamics, Games and Science – International Conference and Advanced School Planet Earth DGS II* (Springer International Publishing Switzerland 2015), 593–605.

[9]See e.g. T Mkhatshwa and A Mummert, 'Modeling Super-Spreading Events for Infectious Diseases: Case Study SARS' (2011) 41 IAENG International Journal of Applied Mathematics 2.

[10]C Pizzuti and others, 'Network-Based Prediction of COVID-19 Epidemic Spreading in Italy' (2020) 5 Applied Network Science 91.

[11]For a review of the literature see HS Rodrigues, 'Application of SIR Epidemiological Model: New Trends' (2016) 10 International Journal of Applied Mathematics and Informatics 92.

[12]Daron Acemoglu, Asuman Ozdaglar and Alireza Tahbaz-Salehi, 'Systemic Risk and Stability in Financial Networks' (2015) 105(2) American Economic Review, American Economic Association 564.

[13]For specific examples of recent academic publications see e.g. S Ma, L Feng and CH Lai, 'Mechanistic Modelling of Viral Spreading on Empirical Social Network and Popularity Prediction' (2018) 8 Scientific Reports 13126, W Wang and WN Street, 'Modeling and Maximizing Influence Diffusion in Social Networks for Viral Marketing' (2018) 3 Applied Network Science 6, and P Kumar and A Sinha, 'Information Diffusion Modeling and Analysis for Socially Interacting Networks' (2021) 11 Social Network Analysis and Mining 11.

individual strategic decision making during an epidemic.[14] For example, Toxvaerd studies equilibrium social distancing using an SIR-based model,[15] McAdams develops a Nash-equilibrium extension of the SIR model to study strategic behaviour,[16] and Acemoglu et al. study optimal targeted lockdowns in a multi-group SIR setup.[17]

To our knowledge, no one to date has used an SIR model in an antitrust context to model foreclosing effects and this is the contribution of this paper.

4. Using an SIR model to analyze foreclosure

Consider a simple model of "diffusion" of a new app or service "A" that relies on an incumbent dominant platform "P" for distribution. We assume a fixed population of agents $\boldsymbol{W}$[18] that represent A's addressable market and, as is standard in an SIR model, we distinguish the total addressable market into three groups: those who are already users of A (corresponding to the infected population); those who are yet to use A (the susceptible population) and those who have used, and subsequently abandoned it (the recovered population). As is intuitive in the case of an app or other digital service, we assume that users who abandon it (i.e. become inactive) can be induced to reactivate their account and are hence not immune to "re-infection". In order to capture the typical scenario in an exclusionary abuse case, we assume that the entrant relies on an incumbent platforms for distribution. Formally, we assume that a large share of A's addressable market are already users of P.

User growth dynamics without intervention by *P*. To model the dynamics of A's growth we allow for some susceptible users to discover the service spontaneously at a fixed rate $\bar{p}$; and for some proportion of infected users to spontaneously stop using the service. We further allow for current users of A to induce their connections to use the service. We capture this by allowing current users of A to recommend or "share" the app with their connections which, with some probability,

[14]For more detail, see Christopher Avery and others 'An Economist's Guide to Epidemiology Models of Infectious Disease' (2020) 34(4) Journal of Economic Perspectives 79.

[15]F Toxvaerd, 'Equilibrium Social Distancing' (2020) Cambridge-INET Working Paper Series No: 2020/08, Cambridge Working Papers in Economics 2021, Faculty of Economics, University of Cambridge.

[16]David McAdams, 'Nash SIR: An Economic-Epidemiological Model of Strategic Behavior During a Viral Epidemic' (2020) Covid Economics (forthcoming).

[17]Daron Acemoglu and others, 'Optimal Targeted Lockdowns in a Multi-Group SIR Model' (2020) NBER Working Papers 27102, National Bureau of Economic Research, Inc.

[18]Bold capital letters will signify sets whereas the respective non-bold capital letter will be the size of the set.

induces them to become users of A. This could reflect direct network effects (e.g. sharing of content or sending of messages to one's friends or family) or indirect network effects (e.g. if users of A develop user-generated content which makes the value of the app higher for developers who invest in additional features that then in turn make A more attractive to susceptible users).

The probability of a new user joining equals the probability that they spontaneously join plus the probability that they "accept" an invite sent by a current user to them. The higher the number of users, the higher the total number of invites that each susceptible user receives hence the lower the probability that they reject all of them and do not join the app.

Therefore, in each period t each susceptible agent has a, symmetric across agents, total probability of joining the app equal to:

$$p_t = \bar{p} + (1 - \bar{p})q(zW_t^A) = \bar{p} + (1 - \bar{p})q(zW_t^A)$$

where $q(zW_t^A)$ is the probability with which a susceptible agent becomes a new user of the app, and z is the (assumed fixed) probability with which each susceptible agent receives one invite from each current user of the app, hence zW_t^A is the average number of invitations received by each susceptible agent.

We define τ as the probability that a susceptible agent accepts an invite they receive. For simplicity, we assume that this probability remains fixed in time (and independent of how many invites have been rejected). The probability that a susceptible individual who has not joined the app spontaneously accepts an invite among the ones received is hence given by:

$$q(zW_t^A) = (1 - (1 - \tau)^{zW_t^A})$$

We next assume that the app also faces an exogenous churn rate[19] i.e. that a positive share c of users are removed from the infected/infectious population every period. This could refer to users who do not really like the app so stop signing in and become "ghost" users with zero content viewing, creating and sharing activity.

[19]If we assume that there is zero churn in the app i.e. once a susceptible user becomes a user of the app they never quit, this amounts to zero removal rate from the infected population and, in the absence of foreclosure conduct (e.g. blocking), zero removal rate from the infectious population as well. Under this assumption, it will be easy to check that the conduct can harm the app by reducing its growth only down to the minimum growth rate that is solely due to the spontaneous probability of a new user joining. If this exogenous probability is positive then the app remains an "epidemic" besides P's conduct.

We assume churned users are removed from the infected/infectious population but that they re-enter the susceptible population so that they can re-join with probability p_{t+1} in the next time period. We assume that users who have newly joined stay active for at least one period before considering de-activating,[20] and that no one can be removed from the population or become immune to re-joining.

At any time period t, we then have a fully connected network with the agent population divided into two compartments, a population of W_t^A infected/infectious agents:[21]

$$\begin{aligned} W_t^A &= (1-c)W_{t-1}^A + p_{t-1}W_{t-1}^N \\ &= (1-c)W_{t-1}^A + [\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^A})]W_{t-1}^N \end{aligned}$$

and a population of W_t^N susceptible agents:

$$\begin{aligned} W_t^N &= W_{t-1}^N - p_{t-1}W_{t-1}^N + cW_{t-1}^A \\ &= (1-\bar{p}-(1-\bar{p})(1-(1-\tau)^{zW_{t-1}^A}))W_{t-1}^N + cW_{t-1}^A \end{aligned}$$

with

$$W_t^A + W_t^N = W$$

This set up endogenously introduces an "S-curve" dynamic: when the population of A users is very small, growth will come only from "spontaneous" discovery of the service. As more users are added, they will find that there is a large user base of susceptible users and each user of A will interact with many susceptible users, resulting in accelerating growth.[22] However, as the user base of A increases, the volume of users will grow relative to the size of the remaining addressable market and each A user will interact with fewer individuals who are not already users of the service and hence growth through this channel will slow. Thus, growth will initially accelerate, but then slow: the classic "S-curve".

Allowing for intervention by *P*. To model potential exclusionary conduct we allow for P to engage in activities which weaken the "spread" of A between current users and susceptible users (i.e. we assume that, as a result of P's conduct, in each time period a share (e.g.

[20]This is a simplifying but reasonable assumption. According to this, newly infected or spontaneously joining users will be infected for the full next period but then may leave with probability c.

[21]Our assumption that new infection cases only become infectious to others from the next time period onwards is consistent with the incubation period in epidemiology.

[22]In principle, there are even more feedback network effects of the user base growth: for example, one could allow for a larger user base to contribute to more user-generated content or reputational effects which increased the spontaneous rate of growth.

50%) of the infected population is not infectious). This could capture a range of factors such as reducing the ability of users of A to send invites or content shares through *P's* platform or otherwise reducing interoperability of the product (e.g. as Facebook is alleged to have done by the FTC) or by reducing the visibility of A's service (as is the accusation against Google).

We thereby assume that a share s of the app's users are no longer able to share content/invites with non-users or, in a more general context, prevented from contributing to new user acquisition through P's conduct.[23] This captures the fact that, while all app users may be affected by P's conduct, the frictions imposed are not perfect or there may be some workarounds to overcome obstacles imposed by the conduct. The set of "Blocked users" who are prevented from contributing to user acquisition remain part of the infected population but will not be able to recruit new app users through their activity. Blocked users can find/use workarounds in the next period but overall a constant share of the infected population is blocked and not infectious in each time period.

In addition, we assume that a share of the app's users do not quit "spontaneously" but their quitting is induced by P's conduct. This allows us to model the scenario when a share k of the blocked users W_{t-1}^{AB} quit the app (in the broad sense of inactivity described previously) due to the fact that they could no longer interact with non-app users in P. We will assume that users need to be blocked for at least one period before considering leaving due to blocking.[24] This could reflect the fact that P is for these users their primary social network so they do not care for any content sharing/creation activity if it cannot be integrated in their P activity, and/or that these users respond to blocking by replacing the app with another app, such as one owned/favoured by P, which is not subject to the conduct.

The remaining agents still remain part of the app i.e. the fact they cannot share content or the reduced interoperability and interaction with users of P does not induce them to quit the app. They will, therefore, be part of the infected population but will be blocked instead of infectious since they will not be able to recruit new app users through their activity.

[23]The corollary in an epidemiological model is that some app users are still "infected" but are no longer "infectious".

[24]This is a simplifying but reasonable assumption. According to this, newly blocked users will be infected for the full period but then may leave with probability k as a result of being blocked and hence dissatisfied with the app.

At any time period t, we will then have a fully connected network of agents in three groups: W_t^{AB} blocked app users; W_t^{AI} infectious agents; and W_t^{N} susceptible agents:

$$W_t^{AI} = (1-s)(1-c)(1-k)W_{t-1}^{AB} + (1-s)(1-c)W_{t-1}^{AI} + (1-s)[\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^{AI}})]W_{t-1}^{N}$$

$$W_t^{AB} = s(1-c)(1-k)W_{t-1}^{AB} + s(1-c)W_{t-1}^{AI} + s[\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^{AI}})]W_{t-1}^{N}$$

$$W_t^{N} = [1-\bar{p}-(1-\bar{p})(1-(1-\tau)^{zW_{t-1}^{AI}})]W_{t-1}^{N} + cW_{t-1}^{AI} + (1-k)cW_{t-1}^{AB} + kW_{t-1}^{AB}$$

$$W_t^{AI} + W_t^{AB} + W_t^{N} = W$$

This system of equations determines the growth path and performance of A. We now consider some implications of the model and the lessons for analysis of potential anticompetitive conduct.

5. Implications of the model

While deliberately stylized, the SIR model set out above[25] presents a number of important qualitative implications for the assessment of potential exclusionary conduct.

First, SIR models underline how conduct can have large effects *relative to a counterfactual* even if the entrant continued to grow in absolute terms. This is illustrated in Figure 2 which simulates the model to generate a path for A's userbase under illustrative parameter values allowing for P to begin its conduct after 18 months.[26] One can see that the entrant app in the counterfactual without any intervention from P achieves a user base of 75 m over an example addressable market of P users of 100 m. However, with P's conduct (which, as above, could reflect degradation of interoperability or some other strategy to suppress the spread of A) growth of A is limited to a more linear, instead of exponential trajectory, where it achieves a user base of less than 30 m. Therefore, the observation of "healthy" growth for the entrant service can mask

[25]Further detail on the model is presented in the mathematical Appendix.

[26]The parameters used for the paper's simulation illustrations are: $W = 100{,}000{,}000$, $\bar{p} = 0.0001$, $\tau = 0.005$, $z = 0.000001$, $s = 0.45$, $c = 0.1$, $k = 0.12$, a life-span of 36 months and various times of introducing the conduct as indicated.

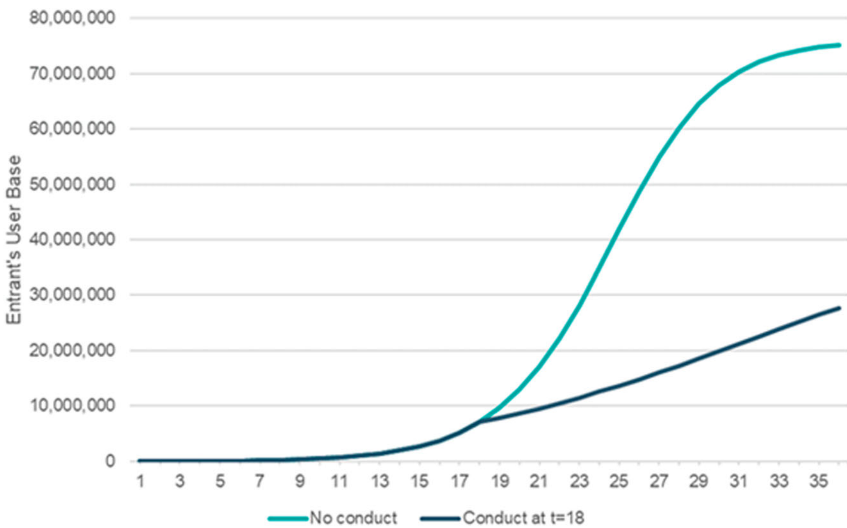

Figure 2. The conduct leads to a significantly lower user base compared to the counterfactual.

significant effects of *P*'s conduct without an appropriate counterfactual analysis.

There are parallels here with the experience during the pandemic: suppression strategies (e.g. mask wearing or vaccinations) will reduce the reproductive number but may not do so sufficiently to prevent continued growth. In these circumstances, the impact will be to "flatten the curve", slowing growth without necessarily suppressing it entirely.

As well as slowing its trajectory, the conduct can also cause *A* to "max out" its userbase at a lower level. In our model, the entrant will continue to grow as long as the volume of new users due to organic growth and growth via network interaction exceeds the loss of users due to both exogenous churn and churn due to the conduct (this is analogous to a virus always growing as long as the R value is greater than 1). Because the conduct reduces the rate of growth and increases the rate of churn it not only slows the entrants' growth but can also lead to the steady state level of userbase being lower than it would be without the conduct. Indeed, for some parameter values, the conduct can cause an app that would otherwise have penetrated the entire addressable market to top out at a substantially lower level.[27]

[27]In a richer model one would expect further mechanisms by which this slower growth could result in a permanent failure to reach the addressable market. For example, our model abstracts away from competition dynamics, while in a richer model one could allow for the dominant platform having its own competing service which could be expected to use the conduct to grow faster than the entrant and

Second, the illustration above shows how suppressive conduct can prevent the emergence of an "S-curve" dynamic (in the Figure above, the entrant would have grown in an exponential fashion with accelerating growth without the conduct, but grows more linearly under the conduct). This is important because it means a simple "before and after" analysis of growth rates will understate the impact of the conduct: one might see what looks like a modest slowing in growth, but fail to recognize that the targeted firm was on the cusp of *accelerating* growth due to S-curve dynamics.

Third, SIR models introduce important non-linearities in the effect of the conduct. More "viral" and intrinsically fast growing-services continue to grow (albeit at a slower rate) while services which are less naturally fast growing may be fatally harmed.

This is illustrated in Figure 3 where we look at the same conduct (which we assume results in approximately 50% of users being unable to contribute to new user acquisition and a 10% increase in churn), introduced at the same time (6 months into the life of the app), but applied to apps with different propensities for growth absent the conduct (which we capture by taking three different values for the "organic" churn rate). For each of these three values the solid line indicates outcomes with the conduct and the dotted line without the conduct.

We observe that, absent the conduct, all three apps would have grown effectively (the dotted lines all show growth and an S-curve dynamic). However, only the "most viral" app (the blue line) is able to survive the conduct (albeit still growing at a slower rate) with the two "less viral" apps flatlining. Again there are parallels with experiences within the pandemic: by some estimates, the transmissibility of Covid-19 has dramatically increased over the course of the pandemic with the original virus identified in Wuhan having an R of 2.5 and the Delta variant that emerged in India having an R as high as 8.[28] Thus, interventions which reduced transmission by just over 60% would be sufficient to reduce R below 1 in the first case and result in the infection "fizzling out", but this same intervention would only achieve slower exponential growth in the second case.

Fourth, the model illustrates the importance of early intervention: the network effects in our model mean that a service which has just started out will be "further down the S-curve" and growing at a slower rate

ultimately prevail in the long run. Alternatively, slower growth could result in reduced funding and investment, and hence less ability to expand.

[28]https://www.bbc.co.uk/news/health-57431420.

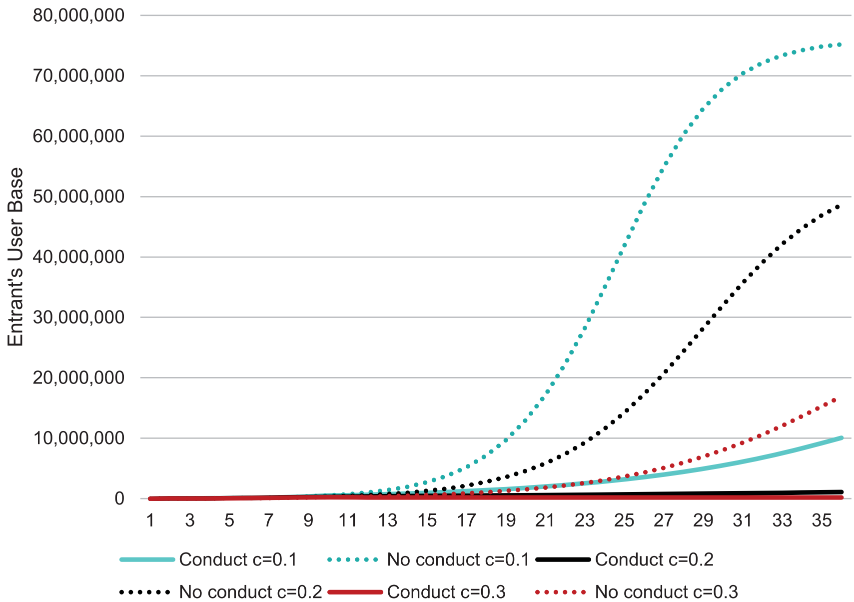

Figure 3. Early blocking can kill an innovator whose "viral" or other quality features lead to less user loyalty.

than one which has amassed a user base. As such, a service which is still in its infancy is inherently easier to suppress than one which has been operating for some time. This is illustrated in Figure 4 which calibrates the model for exactly the same parameter values, changing only the assumed date that *P* begins its conduct to either 2, 10, 14, or 18 months after the launch of the app. One sees that, while the conduct always reduces the growth of the targeted firm, the impact is more severe the earlier the conduct is initiated. We observe that, if the conduct is introduced soon after the app's entry, it is particularly detrimental with its userbase growth flatlining. In the competitive innovative context of technology markets, achieving early traction is key for acquiring funding and hence determining an app's success or failure. Crucially, even an app that would otherwise achieve exponential growth, could be potentially killed due to early exclusionary conduct preventing user acquisition.[29]

[29] This is consistent with allegations against Facebook that it used acquired company Onavo to collect users' app usage and mobile browsing data that would allow it to identify very early upticks in the growth of rivals, much earlier than these signs would be visible to investors or other third party. This issue is raised in the FTC's amended complaint against Facebook. See: https://www.ftc.gov/system/files/documents/cases/ecf_75-1_ftc_v_facebook_public_redacted_fac.pdf and https://

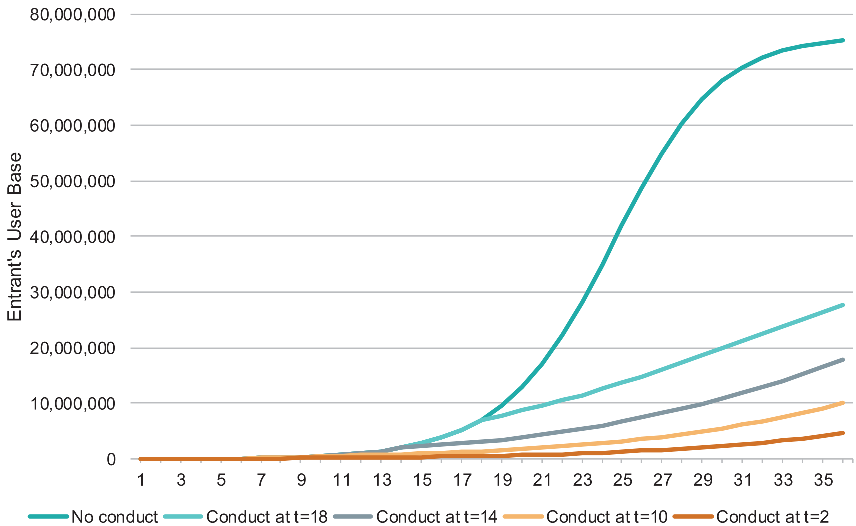

Figure 4. The earlier the conduct the larger the impact with nascent firm's flatlining.

6. Takeaways for antitrust policy

In light of the analysis above, we see three main takeaways:

First, the model provides a formal underpinning for why, at least in industries with strong network effects, the emphasis should be on assessing performance vs. a counterfactual rather than just considering absolute effects. This should not be a license for speculation, but means that empirical work should be looking not just at "level effects" (i.e. evidence that the conduct caused targeted firms' volumes to decline in absolute terms), but also "first derivative" effects (whether the conduct resulted in noticeable declines in *growth*). Further, when looking at growth effects one should be on the lookout for evidence of S-curve effects which mean that growth might have been expected to accelerate in the counterfactual.

Other elements of an empirical "roadmap" would be to consider calibrating an SIR model like the one presented above to determine whether the impugned conduct could plausibly suppress growth to a significant enough extent to result in competitive harm. Of course, any such analysis should consider also alternative explanations: a targeted firm might slow because of deficiencies in its product or entry by other players. Ideally,

techcrunch.com/2018/12/05/seized-cache-of-facebook-docs-raise-competition-and-consent-questions/.

one would hope to use econometric techniques to unpack the effect of the impugned conduct from these other considerations.

Second, the fact that exclusionary conduct is likely to be most effective in its early stages and when targeted at less "viral" services implies that conduct by dominant firms might prove effective at preventing the expansion of targeted services into new markets while allowing them to survive in places where they had already established a user base.[30] This indicates that a potential test for the impact of the conduct is to compare the performance of a service across sub-markets where it was already established vs. those it was not.

Third, when it comes to conducts which are applied by a platform across a wide range of applications, the model illustrates why one should be careful in interpreting the presence of some "winners" in particular applications. It may be that these winners have features which make them particularly "infectious" and more likely to only see their curve flattened by the conduct rather than experiencing outright foreclosure. As such, for each such success story there may be other entrants who failed or did not receive funding to enter in the first place. The model, therefore, has parallels with a growing literature and policy debate exploring whether there is a "kill zone" around major platforms.[31]

7. Conclusions

The same mathematical processes that govern the spread of infectious diseases have parallels with the emergence and growth of products exhibiting network effects. Such products and services similarly lend themselves to rapid exponential growth and to S-curve dynamics whereby growth first accelerates as the service gains users and can leverage network effects, and then slows once the addressable market is penetrated and the population achieves something akin to "herd immunity". As such, the same "SIR models" used in epidemiology can be usefully applied in an antitrust context.

Taking this analogy further, a dominant firm seeking to stop the spread of potentially competing products faces a similar problem to a government seeking to suppress a new virus and can be expected to try

[30] A candidate example would be Yelp (a long-standing complainant against Google who the authors have worked with). While Yelp continues to operate a viable business in those parts of the US where it first launched it has ceased attempts to expand internationally: https://techcrunch.com/2016/11/03/yelp-lays-off-175-in-sales-and-marketing-as-it-retrenches-internationally/.

[31] See for example, Sai Krishna Kamepalli and others, 'Kill Zone' (2021) NBER Working Paper No. w27146.

and reduce transmissibility and to adopt policies to, at the very least, “flatten the curve”, if it cannot achieve outright suppression. The key contrast, of course, is that, unlike a pandemic, social welfare is likely to benefit from allowing this spread to occur.

Applying SIR models in this context delivers some important takeaways for the assessment of potentially anticompetitive conduct. First, they show how, in fast-growing industries, it is all the more important to consider impacts vs. a counterfactual rather than absolute effects on output or revenues. Second, they illustrate how conduct can be particularly effective when applied to firms who are in their infancy and have not yet reached the point at which S-curve dynamics “kick in”. Third, they provide a further rationale to focus on long-run innovation and entry incentives.

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Disclosure statement

The authors are consultants at Charles River Associates and have worked for complainants in antitrust cases involving alleged foreclosure by digital platforms including Google and Tencent. They have advised multiple operators in the technology space including Amazon, Apple, Grab, Kelkoo, Microsoft, PayPal, Uber, Visa, Yandex, Yelp and others. The views expressed here are their own.

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Appendix. Mathematical derivations

We assume a fixed population of agents $\boldsymbol{W}$[32] that represent A's addressable market. For simplicity, we assume that the addressable market coincides with P's user base and that this network is fully connected i.e. that all agents are connected and can hence communicate with everyone else.[33] At any given point in time, A's addressable market can be divided into two compartments:

- $\boldsymbol{W}_t^{A}$: the agents who are already using A. This compartment can be further broken down into two sub-compartments:
  - $\boldsymbol{W}_t^{AP}$ those who are users of both A and P, and
  - $\boldsymbol{W}_t^{AN}$ those who are users of A but not of P.
- $\boldsymbol{W}_t^{N}$: the agents who are not users of A. This compartment can be further broken down into two sub-compartments:
  - $\boldsymbol{W}_t^{NP}$ those who are users of P but not of A, and
  - $\boldsymbol{W}_t^{NN}$ those who are users neither of A nor P.

In antitrust cases where P has a strong market position, we would expect the $\boldsymbol{W}_t^{NN}$ and $\boldsymbol{W}_t^{AN}$ groups to be small and most of the addressable market of A to be existing users of P. To account for this and simplify the derivations we assume that $\boldsymbol{W}_t^{NN} = \boldsymbol{W}_t^{AN} = \emptyset$ hence $\boldsymbol{W}_t^{AP} \equiv \boldsymbol{W}_t^{A}$ and $\boldsymbol{W}_t^{NP} \equiv \boldsymbol{W}_t^{N}$.

We further assume that, in each discrete time period, an exogenously determined share of A users is "removed" i.e. they stop using it and/or

[32]Bold capital letters will signify sets whereas the respective non-bold capital letter will be the size of the set.

[33]This is in line with the baseline SIR epidemiology model which assumes a well-mixed population meaning that any infected agent has a probability of contacting any susceptible agent that is well approximated by the average.

deactivate their account so move from the infected back to the susceptible population. This amounts to assuming that although the users "recover" they never become immune i.e. they can re-join the infected population in the next period.

All agents who are already users of the app can use P to interact with their connections (e.g. by sharing content or invites) both in $\boldsymbol{W}_t^A$ and in $\boldsymbol{W}_t^N$. Interacting with someone in $\boldsymbol{W}_t^A$ is innocuous as the recipient is already "infected".[34] We will, therefore, limit our attention to the interaction between app users and those who are, at the beginning of that period, non-users of the app. We assume that, within each time period, each agent in $\boldsymbol{W}_t^A$ sends to each of the agents in $\boldsymbol{W}_t^N$ one invite with some probability r. We assume that this probability is symmetric across all app users. The receipt of an invite incentivizes non-users to join the app.

We will assume that an agent in $\boldsymbol{W}_t^N$ also has a probability $\bar{p} \geq 0$ to join the app spontaneously, without receiving any invites. This is assumed to be exogenous and symmetric across agents. In contrast with baseline epidemiological models, we will assume that this probability of spontaneous app sign-up, corresponding to infection from the virus via environmental channels other than human transmission such as food/water/contaminated surfaces, is positive (and exogenous).[35]

Therefore, the additional probability of joining the app due to the receipt of an invite will be bounded from above by $1 - \bar{p}$ and we will assume it is increasing in the amount of total invites v_t that the agent receives. This will in turn be assumed a positive function of the content stock of the app, which, in general, depends on both the app's number of users and their average content creation productivity $q > 0$ (assumed constant over time). We also assume that content depreciates based on a parameter $0 < d < 1$.

Therefore, in each period t each agent $i \in \boldsymbol{W}_t^N$ has a, symmetric across agents, total probability of joining the app equal to:

$$p_t = \bar{p} + (1 - \bar{p})q(v(V_t))$$

with

$$V_t = \sum_{j=0}^{t} d^{t-j} q W_j^A$$

which is increasing in the number of users in the platform. In particular, $v(V_t)$ will be increasing in V_t and will be symmetric for all agents in $\boldsymbol{W}_t^N$ i.e. all non-users will have the same invite receipt level for any given content stock V_t.[36] Since each agent in $\boldsymbol{W}_t^N$ receives one invite from each agent in $\boldsymbol{W}_t^A$ with probability r, we

[34]Although engagement in the platform is important and more sharing on the platform means more engagement, we will focus on interaction between infectious and susceptible P users which can lead to app usage diffusion (e.g. content sharing or friend invitations). User engagement will be captured through users' contribution to the stock of the app V_t.

[35]A positive probability of "infection" through other means than contact with the "infected" population of app users will allow us to capture in the model the fact that only some proportion of new app sign-ups within a time period are due to content shares/invites via P.

[36]Here and everywhere agents that are in the addressable market are assumed to have the same features as non-users.

have that the expected average number of invites received is:

$$v(V_t) = r(V_t)W_t^A$$

hence

$$p_t = \bar{p} + (1 - \bar{p})q(v(V_t)) = \bar{p} + (1 - \bar{p})q(r(V_t)W_t^A)$$

Therefore, a higher number of app users can have both a **direct network effect** in the spread of the app's user base, as higher W_t^A means higher $v(V_t)$, but also an **indirect network effect** through the probability $r(V_t)$ as higher W_t^A means more content V_t hence higher probability for each agent to share content.

We next look in detail at the probability $r(V_t)$ with which each infectious agent will share an invite with each of the susceptible agents. In principle, the probability $r(V_t)$ with which an infected individual in W_t^A will make contact with each of the susceptible agents in W_t^N can be **increasing, decreasing or fixed in** W_t^A.

On one hand, as the infection spreads and the app gets more users, the content stock of the platform V_t increases and so does the propensity of all users to share content/invites with other users. As all agents face a time constraint though it is intuitive that the rate of this increase will be bounded. This captures the **indirect positive network effect**. However, at the same time, the susceptible agents set shrinks and so the proportion of content/invites reaching susceptible agents falls. Therefore, the overall probability $r(V_t)$ that an infectious agent will interact with each susceptible agent may increase, fall or remain constant as time passes.

To simplify the analysis we will assume that indirect network effects are exactly strong enough to keep the probability $\boldsymbol{r(V_t)}$ fixed.

$$r(V_t) = z, \ \ 0 < z < 1$$

We note that, even when the probability $r(V_t)$ is constant, the direct network effect would still cause the total number of interactions experienced by each susceptible individual $v(V_t) = r(V_t)W_t^A$ to increase.

Therefore, $v(V_t) = r(V_t)W_t^A = zW_t^A$ captures both the indirect and the direct network effects at play.

We next assume that the app also faces an exogenous churn rate[37] i.e. that a positive share c of users are removed from the infected/infectious population every period. This could refer to users who do not really like the app so stop signing in and become "ghost" users with zero content viewing, creating and sharing activity.

We can, therefore, capture not only users who delete their app account but also those that are effectively removed due to inactivity. Users who delete their accounts can easily register again, similarly to "ghost" users who can simply resume activity in the future. We hence simplify this by treating these two groups in the same way and assuming they are removed from the infected/infectious population but that they re-

[37]If we assume that there is zero churn in the app i.e. once a susceptible user becomes a user of the app they never quit, this amounts to zero removal rate from the infected population and, in the absence of the conduct, zero removal rate from the infectious population as well. Under this assumption, it will be easy to check that the conduct can harm the app by reducing its growth only down to the minimum growth rate that is solely due to the spontaneous probability of a new user joining. If this exogenous probability is positive then the app remains an "epidemic" besides *P*'s conduct.

enter the susceptible population so that they can re-join with probability p_{t+1} in the next time period. We assume that users who have newly joined stay active for at least one period before considering de-activating,[38] and that no one can be removed from the population due to death or with immunity to re-joining.

At any time period t, we then have a fully connected network with the agent population divided into two compartments, an infected/infectious compartment

$$W_t^A = W_{t-1}^A + p_{t-1} W_{t-1}^N - cW_{t-1}^A = (1-c)W_{t-1}^A + p_{t-1} W_{t-1}^N$$

and W_t^N susceptible agents

$$W_t^N = W_{t-1}^N - p_{t-1} W_{t-1}^N + cW_{t-1}^A = (1-p_{t-1})W_{t-1}^N + cW_{t-1}^A$$

with

$$W_t^A + W_t^N = W$$

For $p_0 > 0, \quad W_0^A > 0$, as time passes and W_t^A increases, V_t and p_t further grow but the set of susceptible agents $\boldsymbol{W_t^N}$ shrinks.

Replacing the total probability of a new infection in the infected population formula we get:

$$\begin{aligned} W_t^A &= (1-c)W_{t-1}^A + p_{t-1} W_{t-1}^N \\ &= (1-c)W_{t-1}^A + (\bar{p} + (1-\bar{p})q(r(V_{t-1})W_{t-1}^A))W_{t-1}^N \end{aligned}$$

Define τ as the probability that a susceptible agent accepts an invite received and assume that this remains fixed in time (and independent of how many invites have been rejected). The probability $q(r(V_t)W_t^A)$ that a susceptible individual who has not joined the app spontaneously accepts an invite among the ones received and hence becomes an app user in period t will be equal to the probability of not joining spontaneously times one minus the probability of rejecting **all** $r(V_t)W_t^A$ invites received in period t:

$$q(r(V_t)W_t^A) = (1-(1-\tau)^{r(V_t)W_t^A})$$

Therefore, the **total number of new infections** within the time period will be given by:

$$[\bar{p} + (1-\bar{p})(1-(1-\tau)^{r(V_t)W_{t-1}^A})]W_{t-1}^N$$

and the total number of the infected will be:[39]

$$W_t^A = (1-c)W_{t-1}^A + [\bar{p} + (1-\bar{p})(1-(1-\tau)^{r(V_t)W_{t-1}^A})]W_{t-1}^N$$

[38] This is a simplifying but reasonable assumption. According to this, newly infected or spontaneously joining users will be infected for the full next period but then may leave with probability c.

[39] Our assumption that new infection cases only become infectious to others from the next time period onwards is consistent with the incubation period in epidemiology.

Growth rate and reproduction number in the counterfactual

One of the most popular metrics[40] used by the literature to describe the spreading potential of a virus and whether it can invade a population and become an epidemic is the **Reproduction number** R_0, defined as the **number** of secondary app new users produced by a single app user in an, otherwise, completely susceptible population of non-app users.[41] Note that this defines a dimensionless number and not a rate, giving the number of susceptible individuals that will become infected at time zero by the single infected agent.

In the recent epidemiological literature, the reproduction number R_0 has often been used as the threshold quantity that determines whether a disease can invade a population.

The probability that each susceptible agent, who does not join spontaneously, becomes infected due to an interaction is, by definition, equal to $q(r(V_0)W_0^A)$. Therefore, the number of secondary app users produced by a single app user in an, otherwise, completely susceptible population is:

$$R_0 = (1 - \bar{p})W_0^N q(r(V_0)W_0^A)$$

where $W_0^A = 1$.

Given the fully connected network, the infected agent sends $r(V_0)W_0^N$ invites. Since at the beginning only one agent is infectious, all susceptible agents receive $r(V_0)$ invites, therefore, the reproduction number will be:

$$\begin{aligned} R_0 &= (1 - \bar{p})W_0^N q(r(V_t)) = W_0^N(1 - \bar{p})(1 - (1 - \tau)^{r(V_0)W_0^A}) \\ &= W_0^N(1 - \bar{p})(1 - (1 - \tau)^{r(V_0)}) \end{aligned}$$

with $[(1 - \tau)^{r(V_0)}]$ giving the probability that a susceptible agent rejects all invites they receive and hence remain uninfected at the end of period 0.

In a standard model, the infectious individual would either die or recover i.e. cease to be infectious but also become immune to new infection and in any case be removed from the population by the next time period. Therefore, for the infection to be an epidemic (with no environmental spread) we would need that $R_0 > 1$. In our context, however, this condition is no longer necessary: since, in the absence of blocking/foreclosure conduct from P, each infected individual remains infected forever i.e. once they become an app user they can send content/invites to non-app users forever afterwards, we only need that the number of total new infections is larger than the churn:

$$\Delta W_t^A = [\bar{p} + (1 - \bar{p})(1 - (1 - \tau)^{r(V_t)W_t^A})]W_{t-1}^N - cW_{t-1}^A > 0$$

[40]Others include the Contact number and the Replacement number which are defined at any time t.

[41]At time zero, we assume that there is a single app user so that the susceptible population is almost equal to the entire population $W_0^N = W - 1$.

For the first time period this gives:

$$\Delta W_1^A = W_1^A - W_0^A > 0 \rightarrow W_0^N[\bar{p} + (1-\bar{p})(1-(1-\tau)^{r(V_0)})] > cW_0^A$$
$$\rightarrow W_0^N\bar{p} + R_0 > cW_0^A$$

Therefore, for low enough churn i.e. high enough quality of services, the app "virus" will always invade the susceptible population of agents that are not app users and become an epidemic.

Allowing P to engage in conduct to suppress A's "spread"

We will continue to assume that the sets $\boldsymbol{W_t^{NN}}$ and $\boldsymbol{W_t^{AN}}$ are empty (or negligible in size). Then $\boldsymbol{W_t^A \equiv W_t^{AP}}$ and $\boldsymbol{W_t^N \equiv W_t^{NP}}$**.** In other words, we will limit attention to the effects of conduct within the P network.

We assume that a share s of the app's users are no longer able to interact with non-users or otherwise prevented from contributing to new user acquisition through P's conduct.[42] This captures the fact that, while all app users may be affected by P's conduct, frictions imposed are not perfect or there may be some workarounds to overcome obstacles imposed by the conduct in the interaction between users and non-users of the app such that e.g. invites could still be shared to some degree. The set of "Blocked users" who are prevented from contributing to user acquisition remain part of the infected population but will not be able to recruit new app users through their activity. Blocked users can find/use workarounds in the next period but overall a constant share of the infected population is blocked and not infectious in each time period.

In addition, we assume that a share of the app's users do not quit "spontaneously" but their quitting is induced by P's conduct. This allows us to model the scenario when a share k of the blocked users W_{t-1}^{AB} quit the app (in the broad sense of inactivity described previously) due to the fact that they could no longer interact with non-app users in P. We will assume that users need to be blocked for at least one period before considering leaving due to blocking.[43] This could reflect the fact that e.g., P is for these users their primary social network so they do not care for any content sharing/creation activity if it cannot be integrated in their P activity, and/or that these users respond to blocking by replacing the app with another app, such as one owned/favoured by P, which is not subject to the conduct.

The remaining agents still remain part of the app i.e. the fact they cannot share content or the reduced interoperability and interaction with users of P do not

[42]The corollary in an epidemiological model is that some app users are still "infected" but are no longer "infectious".

[43]This is a simplifying but reasonable assumption. According to this, newly blocked users will be infected for the full period but then may leave with probability k as a result of being blocked hence dissatisfied with the app.

induce them to quit the app. They will, therefore, be part of the infected population but will not be infectious since they will not be able to recruit new app users through their activity.

At any time period t, we will then have a fully connected network of agents in three groups: W_t^{AB} blocked app users who are hence not infectious: W_t^{AI} infectious agents and W_t^{N} susceptible agents:

$$\begin{aligned} W_t^{AI} &= (1-s)(1-c)(1-k)W_{t-1}^{AB} + (1-s)(1-c)W_{t-1}^{AI} \\ &\quad + (1-s)[\bar{p} + (1-\bar{p})(1-(1-\tau)^{r(V_{t-1})W_{t-1}^{AI}})]W_{t-1}^{N} \end{aligned}$$

$$\begin{aligned} W_t^{AB} &= s(1-c)(1-k)W_{t-1}^{AB} + s(1-c)W_{t-1}^{AI} \\ &\quad + s[\bar{p} + (1-\bar{p})(1-(1-\tau)^{r(V_{t-1})W_{t-1}^{AI}})]W_{t-1}^{N} \end{aligned}$$

$$\begin{aligned} W_t^{N} &= [1-\bar{p}-(1-\bar{p})(1-(1-\tau)^{r(V_{t-1})W_{t-1}^{AI}})]W_{t-1}^{N} + cW_{t-1}^{AI} + (1-k)cW_{t-1}^{AB} \\ &\quad + kW_{t-1}^{AB} \end{aligned}$$

with $W_t^{AI} + W_t^{AB} + W_t^{N} = W$.

Now the infected population equals the blocked plus the infectious users:

$$\begin{aligned} W_t^{A} &= W_t^{AI} + W_t^{AB} \\ &= (1-s)(1-c)(1-k)W_{t-1}^{AB} + (1-s)(1-c)W_{t-1}^{AI} \\ &+ [\bar{p} + (1-\bar{p})(1-(1-\tau)^{r(V_{t-1})W_{t-1}^{DI}})]W_{t-1}^{N} \\ &+ s(1-c)(1-k)W_{t-1}^{AB} + s(1-c)W_{t-1}^{AI} = (1-c)(1-k)W_{t-1}^{AB} + (1-c)W_{t-1}^{AI} \\ &+ [\bar{p} + (1-\bar{p})(1-(1-\tau)^{r(V_{t-1})W_{t-1}^{AI}})]W_{t-1}^{N} \\ &= W_{t-1}^{AB} + W_{t-1}^{AI} - (k+c-ck)W_{t-1}^{AB} - cW_{t-1}^{AI} \\ &+ [\bar{p} + (1-\bar{p})(1-(1-\tau)^{r(V_{t-1})W_{t-1}^{AI}})]W_{t-1}^{N} \end{aligned}$$

Therefore,

$$\begin{aligned} \Delta W_t^{A} &= W_t^{A} - W_{t-1}^{A} = W_t^{AI} + W_t^{AB} - W_{t-1}^{AB} - W_{t-1}^{AI} \\ &= [\bar{p} + (1-\bar{p})(1-(1-\tau)^{r(V_{t-1})W_{t-1}^{AI}})]W_{t-1}^{N} - (k+c-ck)W_{t-1}^{AB} - cW_{t-1}^{AI} \end{aligned}$$

and we see that now the growth rate of the app's population is positive only for:

$$[\bar{p} + (1-\bar{p})(1-(1-\tau)^{r(V_{t-1})W_{t-1}^{AI}})]W_{t-1}^{N} > (1-(1-k)(1-c))W_{t-1}^{AB} + cW_{t-1}^{AI}$$

$$p_{t-1}W_{t-1}^{N} > (k+c-ck)W_{t-1}^{AB} + cW_{t-1}^{AI}$$

i.e. if the newly infected exceed those removed due to spontaneous churn or conduct-induced churn.

The app can hence shrink and blocking, which both reduces the probability p_t and causes the additional churn rate k can lead to the elimination of the app in the P network.

Dynamic evolution of the app's user base after the conduct is introduced

Using our assumption that $r(V_t) = z$ at all t the system becomes:

$$\begin{aligned} W_t^{AI} = {} & (1-s)(1-c)(1-k)W_{t-1}^{AB} + (1-s)(1-c)W_{t-1}^{AI} \\ & + (1-s)[\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^{AI}})]W_{t-1}^{N} \end{aligned}$$

$$\begin{aligned} W_t^{AB} = {} & s(1-c)(1-k)W_{t-1}^{AB} + s(1-c)W_{t-1}^{AI} \\ & + s[\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^{AI}})]W_{t-1}^{N} \end{aligned}$$

$$W_t^{N} = [1-\bar{p}-(1-\bar{p})(1-(1-\tau)^{zW_{t-1}^{AI}})]W_{t-1}^{N} + cW_{t-1}^{AI} + (1-k)cW_{t-1}^{AB} + kW_{t-1}^{AB}$$

$$W_t^{AI} + W_t^{AB} + W_t^{N} = W$$

or in summary form, using the last equation to drop the third one:

$$\begin{aligned} W_t^{AI} = {} & (1-s)(1-c)(1-k)W_{t-1}^{AB} + (1-s)(1-c)W_{t-1}^{AI} \\ & + (1-s)[\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^{AI}})](W - W_{t-1}^{AI} - W_{t-1}^{AB}) \end{aligned}$$

$$\begin{aligned} W_t^{AB} = {} & s(1-c)(1-k)W_{t-1}^{AB} + s(1-c)W_{t-1}^{AI} \\ & + s[\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^{AI}})](W - W_{t-1}^{AI} - W_{t-1}^{AB}) \end{aligned}$$

In growth rate form we have:

$$\begin{aligned} \Delta W_t^{AI} = {} & (1-s)[\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^{AI}})]W_{t-1}^{N} + (1-s)(1-c)(1-k)W_{t-1}^{AB} \\ & - (1-(1-s)(1-c))W_{t-1}^{AI} \end{aligned}$$

$$\begin{aligned} \Delta W_t^{AB} = {} & W_t^{AB} - W_{t-1}^{AB} = s(1-c)W_{t-1}^{AI} - (1-s(1-c)(1-k))W_{t-1}^{AB} \\ & + s[\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^{AI}})]W_{t-1}^{N} \end{aligned}$$

$$\Delta W_t^{AI} + \Delta W_t^{AB} + \Delta W_t^{N} = 0$$

This is a two-dimensional non-linear system of difference equations with no analytical solution, which can however be simulated using calibrated or estimated parameter values and initial conditions.[44]

[44] See e.g., Chapter 2 of MRS Kulenovic and O Merino, 'Discrete Dynamical Systems and Difference Equations with Mathematica'.

Dynamic evolution of the app's user base with complete effective blocking

We notice that even if the entire population of infected agents were effectively blocked starting at period t such that $s = 1$, we would have:

$$W_{\tilde{t}}^{AI} = 0$$

$$W_{\tilde{t}}^{AB} = (1-c)(1-k)W_{\tilde{t}-1}^{AB} + (1-c)W_{\tilde{t}-1}^{AI} + \left[\bar{p} + (1-\bar{p})\left(1-(1-\tau)^{zW_{\tilde{t}-1}^{AI}}\right)\right]W_{\tilde{t}-1}^{N}$$

$$W_{\tilde{t}}^{N} = \left[1-\bar{p}-(1-\bar{p})\left(1-(1-\tau)^{zW_{\tilde{t}-1}^{AI}}\right)\right]W_{\tilde{t}-1}^{N} + cW_{\tilde{t}-1}^{AI} + (1-k)cW_{\tilde{t}-1}^{AB} + kW_{\tilde{t}-1}^{AB}$$

hence the infected population would be equal to the blocked population:

$$W_{\tilde{t}}^{A} = W_{\tilde{t}}^{AB} = (1-c)(1-k)W_{\tilde{t}-1}^{AB} + (1-c)W_{\tilde{t}-1}^{AI} + \left[\bar{p} + (1-\bar{p})\left(1-(1-\tau)^{zW_{\tilde{t}-1}^{AI}}\right)\right]W_{\tilde{t}-1}^{N}$$

In the following period, we would then have:

$$W_{\tilde{t}+1}^{AI} = 0$$

$$W_{\tilde{t}+1}^{AB} = (1-c)(1-k)W_{\tilde{t}}^{AB} + \bar{p}W_{\tilde{t}}^{N}$$

$$W_{\tilde{t}+1}^{N} = (1-\bar{p})W_{\tilde{t}}^{N} + (c+k-ck)W_{\tilde{t}}^{AB}$$

with

$$W_{\tilde{t}+1}^{A} = W_{\tilde{t}+1}^{AB} = (1-c)(1-k)W_{\tilde{t}}^{AB} + \bar{p}W_{\tilde{t}}^{N}$$

and growth rate

$$\Delta W_{\tilde{t}+1}^{A} = \Delta W_{\tilde{t}+1}^{AB} = W_{\tilde{t}+1}^{AB} - W_{\tilde{t}}^{AB} = \bar{p}W_{\tilde{t}}^{N} - [1-(1-c)(1-k)]W_{\tilde{t}}^{AB}$$

For high enough $\bar{p}$ and low enough spontaneous churn and churn due to blocking, this growth rate can still be positive. Therefore, this exogenous probability of joining the app spontaneously limits the maximum harm of the conduct.

Dynamic evolution of the app's user base in the counterfactual and pre-conduct period

The above system captures the post-conduct scenario. During the time periods pre-conduct, as well as in the counterfactual with no blocking, the infectious population coincides with the infected as the blocked population is zero. This corresponds to $s = 0$ hence $k = 0$ i.e. churn is only spontaneous. Therefore, we have:

$$W_t^A = W_t^{AI} = (1-c)W_{t-1}^A + [\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^A})]W_{t-1}^N$$

$$W_t^{AB} = 0$$

$$W_t^N = W - W_t^A$$

or in summary form:

$$W_t^A = (1-c)W_{t-1}^A + [\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^A})](W - W_{t-1}^A)$$

or in growth rate form:

$$\Delta W_t^A = \Delta W_t^{AI} = [\bar{p} + (1-\bar{p})(1-(1-\tau)^{zW_{t-1}^A})]W_{t-1}^N - cW_{t-1}^A$$

$$\Delta W_t^N = -\Delta W_t^A$$

This system can again be simulated using calibrated or estimated parameter values and initial conditions.