A NOTE ON VERTICAL MERGERS WITH AN UPSTREAM MONOPOLIST: FORECLOSURE AND CONSUMER WELFARE EFFECTS

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1. Introduction and Summary

This note develops a simple model in which an upstream monopolist sells an input used by two downstream competitors.[1] Those downstream firms in turn compete to sell their products to consumers. The note addresses two related issues. First, does a vertical merger between the upstream monopolist and one of the downstream firms create an incentive for the merged firm to raise price to its unintegrated downstream rival, or at the extreme, completely cut off sales to that rival? Second, does that vertical merger increase or decrease downstream prices, that is, does it reduce or raise the welfare of consumers. Both these questions need to be analyzed because

1 It is straightforward to extend the analysis to the case with *n* downstream firms.

downstream prices could fall even if the upstream monopolist raised price to its unintegrated downstream rival.[2]

In the simplest vertical merger model, in which the monopolist's input is used in fixed proportions and the downstream firms sell homogeneous products in perfect (Bertrand price) competition, the "single monopoly profit" result obtains and the merger has no effect on downstream prices or consumer welfare. When the downstream products are differentiated, the model analyzed here shows that, for a broad range of plausible values of the model parameters, the vertically integrated company has no incentive to raise price to the unintegrated downstream firm, much less cut off all sales. In fact, the model shows that the merger leads to *lower* downstream prices, hence increased consumer welfare.[3]

While the model is highly stylized, it remains a useful tool for illustrating the different incentives created by the vertical merger. In particular, the model shows how the incentive to lower price to the integrated downstream firm (because of the "elimination of double marginalization" effect) can be quite powerful, and that this pro-competitive effect can offset any anti-competitive incentive to "raise rivals' costs." The model is also useful for illustrating a third effect, not often discussed in the context of vertical mergers, which can be called the "reduced demand" effect. The "reduced demand" effect arises in this model from the elimination of double marginalization, although more generally, it can also result from merger-related efficiencies. As discussed in more detail below, this reduced demand effect is pro-competitive as it creates an incentive to lower price to the unintegrated downstream rival. Thus, assessing the competitive impact of the vertical merger requires determining the net effect of balancing these three effects.

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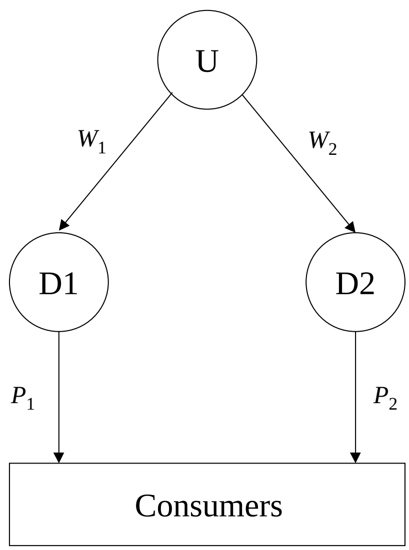
2 The analysis ignores any production cost savings stemming from the merger; any such savings make downstream price reductions more likely.

3 If the downstream firms do not sell to final consumers directly, reductions in downstream prices are assumed to lead to reductions in retail prices.

The existing literature on vertical mergers has paid little attention to the reduced demand effect. For example, in Ordover et al. (1990), there is no elimination of double marginalization or merger-related efficiencies, and thus no reduced demand effect. This lack of pro-competitive incentives follows from their assumption that two upstream firms produce a homogeneous product and price at competitive levels. Absent those two pro-competitive effects, there is no balancing necessary in their model: the only remaining incentive effect is the raising rivals' costs incentive which results in higher prices.[4] In a world where there is less than perfect competition between upstream firms or with merger-related efficiency benefits, though, the merger's impact on price cannot be so easily predicted; instead, pro-competitive and anti-competitive effects must be balanced.[5]

2. Overview of the Results

The model considered in this note is a simple one, as illustrated in the following figure:



4 See also Reiffen (1992) and Ordover et al. (1992).

5 See Michael Riordan et al. (1995) for a discussion on balancing pro-competitive and anti-competitive effects.

Specifically, there is an upstream monopolist ("U") selling to two downstream firms ("D1" and "D2"). Those downstream firms in turn compete to sell their products to consumers. The first task, therefore, is to solve for pre-merger input prices (W_1 and W_2) and downstream prices (P_1 and P_2). Then, the second task is to assume the upstream firm merges with D1 and determine the new profit-maximizing input prices and downstream prices, including whether the upstream monopolist wants to set its input price W_2 so high that D2 (the unintegrated downstream firm) is effectively cut off.

In the pre-merger world, U sets input prices W_1 and W_2 to maximize its own profits. In doing so, it recognizes that D1 and D2 will treat those input prices as costs, and based on how vigorous the downstream competition between D1 and D2, they will choose output prices P_1 and P_2. It is well known that, because D1 and D2 in effect add a "markup" to the input price set by U, the downstream output prices faced by consumers (P_1 and P_2) will be higher than the prices that a vertically integrated monopolist would charge. The extent to which these output prices exceed the prices that an integrated monopolist would charge depends on the extent of competition between D1 and D2. The greater that downstream competition, the smaller the markup they add to the input prices, thus the closer the final prices will be to the integrated monopoly prices.

If D1 and D2 produce homogeneous products an engage in Bertrand price competition, the premerger downstream price equals the marginal costs (including the input cost) of these competitors. Under these circumstances, a vertical merger between U and D1 would have no effect on input or downstream prices. This is the classic "single monopoly profit" result.

When downstream products are differentiated or are not perfectly competitive, the single monopoly profit result does not hold. Instead, a merger of U and D1 has three effects. The first two are well-known: the merger creates an incentive to lower the price charged by the integrated downstream firm D1 (the "elimination of double marginalization" effect) and an incentive to raise price to the unintegrated rival (the "raising rivals' costs" effect). But there is also a third, less well-known effect that also can create an incentive to lower the input price to the unintegrated downstream firm. This effect is called the "reduced demand" effect. The next paragraphs discuss each of these effects in turn.

One important pro-competitive effect of the vertical merger results from the "elimination of double marginalization" between the upstream monopolist and the integrated downstream firm. This point is best explained if one initially assumes that there are only two firms, i.e., an upstream monopolist and a downstream monopolist. In this case, the downstream firm has a post-merger incentive to lower the downstream price because this increases its volume of sales and hence the profits of the upstream monopolist (which accrue to the downstream firm after the acquisition). In fact, the downstream firm will lower its post-merger price to the level that maximizes the sum of the upstream and downstream profits.

To see that the "vertically integrated optimal downstream price" is lower than the pre-merger price, notice that the upstream monopolist could induce the downstream firm to charge the vertically integrated optimal price by selling the input to the downstream firm at a price equal to marginal cost. Since pre-merger the upstream monopolist sells the input at a price greater than marginal cost, the downstream firm sells the output at a price greater than the vertically integrated optimal price. Therefore, in the case with a single upstream and single downstream firm and a double marginalization, a vertical merger leads to a reduction in the price of the output.

Suppose now that there are two downstream firms that purchase their inputs from the upstream monopolist. Clearly, if the downstream firms were producing unrelated products, so were not competing against one another, the vertical merger would eliminate double marginalization (between the two merging firms) and would lead to a lower price by the integrated downstream firm. In addition, the merger would have no effect on the input price that the upstream monopolist charges to the unintegrated downstream firm, and thus no effect on the output price of the unintegrated downstream firm.

If, instead, the downstream firms were producing perfect substitutes, and the downstream sector is perfectly competitive, then this vertical merger would have no price effects. Indeed, pre-merger there would be no double marginalization since the downstream firms would be selling at marginal cost. As a result, pre-merger the upstream monopolist would be able to maximize and extract the vertically integrated profits of the industry. The merger, therefore, would affect neither the input price paid by the downstream rival nor the output prices charged by the downstream firms. This is the “single monopoly profit” result.

This note deals with the in-between case where the downstream firms are producing imperfect substitutes, although as we emphasize below, the analysis is general enough to encompass both extremes: perfect competition, or no competition, between the two downstream firms. The vertical merger has the following three effects.

First, there is the “elimination of double marginalization" effect described above. This effect tends to lower the output price of the integrated downstream firm. In addition, it tends to lower the demand for the product of the downstream rival, and thus tends to lower the output price of the downstream rival.

Second, the reduction in the demand for the rival product (that results from the elimination of double marginalization) implies a reduction in the rival firm’s demand for the input, and thus tends to induce the upstream monopolist to lower the input price to the downstream rival.[6] This “reduced demand” effect tends to lower the output price of the downstream rival (which in turn tends to lower the demand faced by the integrated downstream firm and thus the output price of the integrated downstream firm). Clearly, the “reduced demand” effect and the “elimination of double marginalization” effect are pro-competitive.

Third, there is the standard “raising rivals’ costs" effect. All else equal, post-merger the upstream monopolist has an incentive to raise the input price to the unintegrated downstream firm as a means to raise the output price of that firm and thus increase the profits of the

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6 In this model, the “reduced demand effect” arises from the elimination of double marginalization between U and D1. More generally, it arises from the realization of any merger-induced efficiency that leads to a reduction in the output price of D1 (and hence a reduction in the input demand of D2).

integrated downstream firm. The “raising rivals’ costs" effect tends to raise all the prices and is anti-competitive.

These two pro-competitive and one anti-competitive effects raise the issue of whether the net effect in this upstream monopoly model is pro-competitive or anti-competitive on balance.

Section 3 describes the basic economic model and its two main assumptions: 1) the downstream firms face symmetric linear demand functions, and 2) the integrated firm becomes a price leader post-merger. Under these conditions, the model predicts that the vertically integrated company has no incentive to raise the input price to the unintegrated downstream competitor. That is, the input price is the same as pre-merger. In addition, the model shows that both firms' output prices fall post-merger. Thus, consumer welfare increases from the merger. In other words, the two pro-competitive effects dominate the anti-competitive effect on balance, when there is an upstream monopolist.

Sections 4 and 5 discuss two variants of the basic model. In Section 4, it is assumed that the integrated downstream firm does not become a price leader post-merger. This strengthens the pro-competitive result. That is, post-merger the upstream monopolist *lowers* the input price charged to the unintegrated downstream rival. In this case, the vertical merger also leads to a reduction of both output prices, and hence to an increase in consumer welfare.

Section 5 relaxes the assumption of symmetric demand. Even if demand is asymmetric, the pro-competitive results of the basic model stated above continue to hold, as long as the "cross-slopes" are equal.[7] However, for more general demand systems, it is possible to construct examples where the vertically integrated firm raises the upstream (input) price to the unintegrated downstream rival. Under certain conditions, the unintegrated downstream firm may increase its output price post-merger, even if the integrated downstream firm reduces its own price. In extreme cases of asymmetric demand, the vertically integrated company may find it

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7 In the case with two downstream firms (D1 and D2), the cross slopes are equal if a $1 price increase in D1’s product increases the demand for D2’s product by the same amount as a $1 price increase in D2’s product increases the demand for D1’s product.

profitable to totally foreclose the downstream rival (by refusing to supply it with the input or, equivalently, by setting the input price at a prohibitive level). Thus, consumer welfare may fall when demand is asymmetric in a particular way. This occurs when the "cross-slope" of the integrated downstream firm sufficiently exceeds the "cross-slope" of the unintegrated downstream firm.

3. The Basic Model

3.1. Pre-Merger Equilibrium

Consider a market with one upstream supplier (U) and two downstream firms (D1 and D2). U sells an input to D1 at a price denoted by W_1, and D1 then sells a product to consumers at a price denoted by P_1. Similarly, D2 purchases an input from U at price W_2, and then sells a product at price P_2. The model allows for differentiation between the two downstream products, although the degree of differentiation can vary from perfect homogeneity to complete differentiation. The downstream firms' demand functions are denoted $D_1(P_1, P_2)$ and $D_2(P_2, P_1)$. For simplicity, assume that U can supply the input at zero marginal cost, and the marginal costs of D1 and D2 are equal to W_1 and W_2, respectively.

It is assumed that each downstream firm takes the input prices charged by U as given, and sets its output price according to the Bertrand price competition model.[8] In other words, the output prices P_1 and P_2 are functions of the input prices W_1 and W_2:[9]

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8 Alternatively, firms might engage in Cournot competition in which firms choose quantities rather than prices and prices are subsequently set by a "Walrasian auctioneer." As with the Bertrand model we examine in this note, a vertical merger with Cournot competition and homogeneous products results in lower prices for consumers (assuming symmetric constant marginal costs and strictly concave profit functions). Although the vertical merger results in the upstream monopolist no longer supplying D2 in this Cournot formulation, consumers are better off after the merger: they enjoy lower prices, and with homogeneous downstream products, there is no reduction in variety.

9 Formally, D1 takes W_1 and P_2 as given and chooses P_1 to maximize its profit $(P_1-W_1)D_1(P_1,P_2)$. This leads to D1's best response function $P_1=B_1(P_2,W_1)$. Similarly, D2 has a best response function $P_2=B_2(P_1,W_2)$. Solving these two equations for P_1 and P_2 gives Eq. (1).

$$P_1 = R_1(W_1, W_2) \quad \text{and} \quad P_2 = R_2(W_2, W_1). \tag{1}$$

When U sets the input prices W_1 and W_2, U takes into account that higher input prices lead to higher output prices, which implies a reduction in the demand for the output, and hence a reduction in the volume of inputs demanded by D1 and D2. Formally, U chooses W_1 and W_2 to maximize its total profits, which are given by:

$$\Pi_U = W_1 D_1(P_1, P_2) + W_2 D_2(P_2, P_1), \tag{2}$$

where P_1 and P_2 are given by Eq. (1).

3.2. Symmetric Linear Demand

It is useful to illustrate the above model by considering the case of a linear demand system. Under linear demand, it is possible to obtain explicit solutions for the input and output prices. In addition, one can compare the pre-acquisition prices with the post-acquisition prices, and determine sufficient conditions for the acquisition to increase consumer welfare.

First, consider the case in which D1 and D2 face symmetric demands:

$$D_1 = a - bP_1 + d(P_2 - P_1) \quad \text{and} \quad D_2 = a - bP_2 + d(P_1 - P_2), \tag{3}$$

where a, b and d are positive parameters. Note that d measures the degree of product substitutability, with the products becoming perfect substitutes when d tends to infinity, and unrelated products when d tends to zero. In this case, the pre-merger equilibrium prices are:

$$W_1 = W_2 = \frac{a}{2b} \quad \text{and} \quad P_1 = P_2 = \frac{a(3b+d)}{2b(2b+d)}. \tag{4}$$

For later use, the pre-merger profits of U and D1 are equal to:

$$\Pi_U = \frac{a^2(b+d)}{2b(2b+d)} \quad \text{and} \quad \Pi_{D1} = \frac{a^2(b+d)}{4(2b+d)^2}. \tag{5}$$

(Appendix 1 provides a detailed derivation of Eq. (4) and (5).)

3.3. Post-Merger Equilibrium

3.3.1. Total foreclosure is not profitable

An important issue is whether the vertically integrated company will have an incentive to totally foreclose D2 (by either refusing to supply the input or raising the price of the input to a prohibitive level). As shown below, in the case of symmetric linear demand, the vertically integrated company will have no incentive to totally foreclose D2.

If the vertically integrated company totally forecloses D2, then the demand for D1 increases to:[10]

$$D_1 = (a - bP_1)(1 + \frac{d}{b+d}), \tag{6}$$

and the vertically integrated company can charge the "monopoly price", i.e.,

$$P_1 = \frac{a}{2b}. \tag{7}$$

Note that this price is lower than the pre-acquisition price charged by D1 due to the elimination of double marginalization between U and D1. More importantly, if the vertically integrated company totally forecloses D2, the total profits of U and D1 are lower than pre-acquisition. Indeed, at the above monopoly price, the demand for D1 is equal to $a(b+2d)/[2(b+d)]$, and hence the total profits of the vertically integrated company are equal to:

$$\Pi_V = \frac{a^2(b+2d)}{4b(b+d)}. \tag{8}$$

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10 Solving D_2=0 for P_2 and then substituting into D1's demand leads to Eq. (6).

As shown in Appendix 2, Π_V is smaller than the sum of the pre-merger profits in Eq. (5).

As shown next, if the vertically integrated company does not totally foreclose D2, then the total profits of U and D1 are higher than pre-acquisition. Therefore, the vertically integrated company will have no incentive to totally foreclose D2.

3.3.2. Raising rival's cost is not profitable

If the vertically integrated company does not totally foreclose D2, it may in principle still have an incentive to raise the input price to D2. However, the following argument shows that the vertically integrated company will actually have no incentive to raise the input price to D2. Let

$$P_2 = B_2(P_1, W_2) \tag{9}$$

be the best response of D2 to the prices set by the vertically integrated company. Assume that the vertically integrated company becomes a "price leader" post-acquisition, that is, it sets its output price P_1 and its input price W_2 before D2 sets its output price P_2. Then, the vertically integrated company chooses P_1 and W_2 to maximize its total profits:

$$\Pi_V = P_1 D_1(P_1, P_2) + W_2 D_2(P_2, P_1), \tag{10}$$

where P_2 is given by Eq. (9).

For the above symmetric linear demand system, the above profit maximization problem leads to the following post-acquisition prices (see Appendix 3):

$$W_2 = P_1 = \frac{a}{2b} \quad \text{and} \quad P_2 = \frac{a(3b+2d)}{4b(b+d)}. \tag{11}$$

This shows that the vertically integrated company will neither raise nor reduce the input price to D2 (i.e., the value of W_2 in Eq. (11) is equal to that in Eq. (4)). In addition, the vertically integrated company will lower its output price by eliminating the downstream markup (i.e., the value of P_1 in Eq. (11) is equal to the value of W_1 in Eq. (4)). Finally, D2 will lower its output price as well.[11]

It follows that the total profits of the vertically integrated company are equal to:

$$\Pi_V = \frac{a^2(3b+4d)}{8b(b+d)}. \tag{12}$$

As shown in Appendix 3, this amount is greater than the sum of the pre-acquisition profits given in Eq. (5).

To summarize, under symmetric linear demand, the model produces the following results:

1. *The vertical merger will not affect the input price charged to D2.*

2. *The vertical merger will lower the output price of D1.*

3. *Since the output price of D1 will fall, and D2 will face the same input price as pre-merger, D2 also will lower its output price.*

4. *D1's output will increase post-merger while D2's output will decrease.*

(Result 4 is proved in Appendix 3.)

3.4. Intuitive Explanation of the No-Foreclosure Result

The analysis in the previous section showed that the vertically integrated company will have no incentive to foreclose D2. That is, the vertically integrated company will optimally decide to sell the input to D2 for the same price as pre-acquisition. To gain some intuition about this result, it is useful to first compare the following two market structures for the post-acquisition world:

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11 For all b>0 and d>0, the value of P_2 in Eq. (11) is smaller that of P_2 in Eq. (4).

Market structure A. First, the vertically integrated company chooses P_1 and W_2 to maximize the total profits of D1 and U. Then, given P_1 and W_2, D2 chooses P_2 to maximize its own profit.

This is the market structure used in the previous section. The implicit assumption is that D1 becomes a price leader post-acquisition, i.e., it can precommit itself to charge a given output price P_1 before D2 sets its output price P_2. As shown in Appendix 4, this market structure is equivalent to the following:

Market structure B. First, U chooses W_1 and W_2 to maximize the total profits of U and D1. Then, given W_1 and W_2, D1 and D2 simultaneously choose P_1 and P_2 to maximize their own downstream profits.

Under this alternative and equivalent market structure, the vertically integrated company runs its downstream division independently. That is, it continues to charge an input price W_1 to D1, and then D1 sets its output price P_1 to maximize its downstream profits only. In other words, the output prices P_1 and P_2 are given by the same functions of the input prices W_1 and W_2 as pre-acquisition, i.e.:

$$P_1 = R_1(W_1, W_2) \quad \text{and} \quad P_2 = R_2(W_2, W_1). \tag{13}$$

U chooses W_1 and W_2 to maximize the total profits of U and D1, i.e.:

$$\Pi_V = P_1 D_1(P_1, P_2) + W_2 D_2(P_2, P_1), \tag{14}$$

where P_1 and P_2 are given by Eq. (13). It follows that the only difference with respect to the pre-acquisition market structure is that post-acquisition U chooses the input prices to maximize the total profits of U and D1. (The output prices change only to the extent that the input prices change.)

Intuitively, there are two opposing effects on the input price W_2 that U charges to D2. On the one hand, by eliminating double marginalization between U and D1, the vertical merger tends to reduce the output price P_1 of D1. This in turn tends to reduce the demand faced by D2, and thus D2's demand for the input, which puts pressure on U to lower the input price W_2 that it charges to D2. Note that a lower W_2 and a lower P_1 tend to induce D2 to lower P_2. Therefore, this "reduced demand effect" tends to increase consumer welfare. On the other hand, U has an incentive to raise the input price W_2 to D2 because an increase in W_2 increases D1's downstream profits. This "raising rival's cost effect" tends to increase output prices and reduce consumer welfare. In general, the net effect on W_2 is likely to be ambiguous.

However, for the case of a symmetric linear demand system, the above analysis showed that the net effect on the input price W_2 charged to D2 is zero. This can be best understood by looking at the first-order condition with respect to W_2. Pre-acquisition, it reads:

$$D_2 + W_1\left(\frac{\partial D_1}{\partial P_1}\frac{\partial R_1}{\partial W_2} + \frac{\partial D_1}{\partial P_2}\frac{\partial R_2}{\partial W_2}\right) + W_2\left(\frac{\partial D_2}{\partial P_1}\frac{\partial R_1}{\partial W_2} + \frac{\partial D_2}{\partial P_2}\frac{\partial R_2}{\partial W_2}\right) = 0, \tag{15}$$

while post-acquisition it becomes:

$$\frac{\partial R_1}{\partial W_2}D_1 + D_2 + P_1\left(\frac{\partial D_1}{\partial P_1}\frac{\partial R_1}{\partial W_2} + \frac{\partial D_1}{\partial P_2}\frac{\partial R_2}{\partial W_2}\right) + W_2\left(\frac{\partial D_2}{\partial P_1}\frac{\partial R_1}{\partial W_2} + \frac{\partial D_2}{\partial P_2}\frac{\partial R_2}{\partial W_2}\right) = 0. \tag{16}$$

In comparing Eq. (15) and (16), note that the latter has an additional term, i.e., $(\partial R_1 / \partial W_2)D_1$, because post-acquisition U takes into account that raising W_2 tends to increase output prices and hence D1's downstream profit. This is the "raising rival's cost effect" and calls for an increase in W_2 post-acquisition.[12] Note also that the second term in Eq. (16), i.e., D_2, is smaller than the first term in Eq. (15) since D2's output falls post-acquisition (see result 4 at the end of the previous section). This is the "reduced demand effect" and calls for a decrease in W_2. It turns out that the "raising rival's cost effect" and the "reduced demand effect" cancel each other. Finally, note that the last two terms in Eq. (16), i.e.,

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12 Intuitively, since the additional term is strictly positive, it tends to increase the left-hand side of Eq. (16), ceteris paribus. From the second-order conditions, the left-hand side of Eq. (16) is strictly decreasing in W_2, and thus the additional term calls for an increase in W_2.

$$P_1\left(\frac{\partial D_1}{\partial P_1}\frac{\partial R_1}{\partial W_2}+\frac{\partial D_1}{\partial P_2}\frac{\partial R_2}{\partial W_2}\right)+W_2\left(\frac{\partial D_2}{\partial P_1}\frac{\partial R_1}{\partial W_2}+\frac{\partial D_2}{\partial P_2}\frac{\partial R_2}{\partial W_2}\right),$$

are equal to the last two terms in Eq. (15), i.e.,

$$W_1\left(\frac{\partial D_1}{\partial P_1}\frac{\partial R_1}{\partial W_2}+\frac{\partial D_1}{\partial P_2}\frac{\partial R_2}{\partial W_2}\right)+W_2\left(\frac{\partial D_2}{\partial P_1}\frac{\partial R_1}{\partial W_2}+\frac{\partial D_2}{\partial P_2}\frac{\partial R_2}{\partial W_2}\right),$$

and hence call for neither an increase nor a decrease in W_2. Indeed, the post-acquisition output price P_1 is equal to the pre-acquisition input price W_1 (see the discussion following Eq. (11)) and the linearity of the demand system implies that all the partial derivatives in Eq. (16) are constant and equal to those in Eq. (15). It follows that the net effect of the vertical merger on W_2 is zero.

4. Variant I: D1 has no ability to precommit its output price post-acquisition

Consider the above basic model, but suppose that the market structure of the post-merger world is the following:

Market structure C. First, U chooses W_2 to maximize the total profits of U and D1. Then, given W_2, D1 and D2 simultaneously choose P_1 and P_2. In doing so, D1 maximizes the total profits of D1 and U, while D2 maximizes its own profit.

This market structure is different from that in the basic model. Here, the vertically integrated company does not become a price leader in the downstream market post-acquisition. One can show that the vertically integrated company still has no incentive to totally foreclose D2. In fact, the vertically integrated company finds it profitable to *lower* the input price W_2 to D2. (See Appendix 5.) As a result, the output prices P_1 and P_2 fall after the acquisition, although not as much as in the basic model.

5. Variant II: Asymmetric demand

The results of the basic model still hold if one relaxes the symmetric demand assumption in the following way:

$$D_1 = a_1 - b_1 P_1 + d(P_2 - P_1) \quad \text{and} \quad D_2 = a_2 - b_2 P_2 + d(P_1 - P_2)\,. \tag{17}$$

That is, the two demand functions can have different intercepts and different own slopes. As long as the cross slopes are equal (i.e., the effect on D_1 of a \$1 increase in P_2 is the same as the effect on D_2 of a \$1 increase in P_1), the results of the basic model continue to hold.[13]

However, for the general linear demand system:

$$D_1 = a_1 - b_1 P_1 + d_1(P_2 - P_1) \quad \text{and} \quad D_2 = a_2 - b_2 P_2 + d_2(P_1 - P_2)\,, \tag{18}$$

the merger effects become ambiguous. For example, suppose that the demand functions are given by:

$$D_1 = 1 - P_1 + 0.5(P_2 - P_1) \quad \text{and} \quad D_2 = 1 - P_2 + 0.1(P_1 - P_2)\,. \tag{19}$$

Notice that the cross slopes are asymmetric (i.e., d_1=0.5 and d_2=0.1). In particular, the demand for D1's output is relatively sensitive to P_2, while the demand for D2's output is not very sensitive to P_1. In this example, the vertically integrated company still has no incentive to totally foreclose D2, but it now has an incentive to raise the price W_2 that it charges to D2 (from 0.54 to 0.58). The output price P_1 of the integrated downstream firm still falls (from 0.69 to 0.47), but now the output price P_2 of the unintegrated downstream firm increases slightly (from 0.76 to 0.77).

If one keeps increasing the asymmetry between the two cross slopes (i.e., increasing d_1 and lowering d_2), one can generate extreme cases where the demand for D1's output is very sensitive to P_2, while the demand for D2's output is almost independent of P_1. Then, one can show that the vertically integrated company may find it profitable to totally foreclose D2, and the output price of D1 may increase. Intuitively, the more sensitive D1's demand is to the price of its competitor the bigger is the "raising rival's cost effect." The less sensitive the demand of D2

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13 The results of this section are explained in Appendix 6 and Appendix 7.

is to P_1 the smaller is the "reduced demand effect." In the extreme case where D2's demand does not depend on P_1, there is no "reduced demand effect" and thus the vertically integrated firm will raise the input price W_2 to D2. In addition, if the "raising rival's cost effect" is sufficiently large, then total foreclosure can arise.

Appendix 1: Pre-Merger Equilibrium for Symmetric Linear Demand

This appendix derives the equilibrium prices and profits given in Eq. (4) and (5).

The symmetric demand functions faced by D1 and D2 are given by:

$$D_1(P_1, P_2) = a - bP_1 + d(P_2 - P_1) \quad \text{and} \quad D_2(P_2, P_1) = a - bP_2 + d(P_1 - P_2), \tag{A.1}$$

and the profit functions of D1 and D2 are given by:

$$\Pi_{D1} = (P_1 - W_1)D_1(P_1, P_2) \text{ and } \Pi_{D2} = (P_2 - W_2)D_2(P_2, P_1). \tag{A.2}$$

Given the input prices W_1 and W_2, D1 and D2 engage in Bertrand price competition. The equilibrium output prices, P_1 and P_2, are determined by the following two first-order conditions:

$$\frac{\partial \Pi_{D1}}{\partial P_1} = D_1(P_1, P_2) - (b+d)(P_1 - W_1) = 0 \quad \text{and}$$

$$\frac{\partial \Pi_{D2}}{\partial P_2} = D_2(P_2, P_1) - (b+d)(P_2 - W_2) = 0. \tag{A.3}$$

Solving (A.3) gives the equilibrium output prices P_1 and P_2 as functions of the input prices W_1 and W_2:

$$P_1^* = R_1(W_1, W_2) = \frac{a}{2b+d} + \frac{2(b+d)^2}{(2b+d)(2b+3d)}W_1 + \frac{d(b+d)}{(2b+d)(2b+3d)}W_2 \quad \text{and}$$

$$P_2^* = R_2(W_2, W_1) = \frac{a}{2b+d} + \frac{2(b+d)^2}{(2b+d)(2b+3d)}W_2 + \frac{d(b+d)}{(2b+d)(2b+3d)}W_1. \tag{A.4}$$

U chooses the input prices W_1 and W_2 to maximize its total profits while taking into account that the output prices depend on the input prices according to Eq. (A.4). U's problem can thus be written as:

$$\underset{W_1,W_2}{Max}\,\Pi_U(W_1,W_2) = W_1 D_1(R_1(W_1,W_2),R_2(W_2,W_1)) + W_2 D_2(R_2(W_2,W_1),R_1(W_1,W_2)). \quad \text{(A.5)}$$

The first-order condition with respect to W_1 reduces to:

$$a(2b+3d) - 2(2b^2+4bd+d^2)W_1 + 2d(b+d)W_2 = 0. \quad \text{(A.6)}$$

Using symmetry, in equilibrium U charges the same input price to D1 and D2, i.e., W_1=W_2. Substituting this into Eq. (A.6) leads to the pre-merger equilibrium input prices:

$$W_1^0 = W_2^0 = \frac{a}{2b}. \quad \text{(A.7)}$$

Then, substituting Eq. (A.7) into Eq. (A.4) gives the pre-merger equilibrium output prices:

$$P_1^0 = P_2^0 = \frac{a(3b+d)}{2b(2b+d)}. \quad \text{(A.8)}$$

The pre-merger equilibrium profits of U are obtained by substituting Eq. (A.7) into Eq. (A.5):

$$\Pi_U^0 = \frac{a^2(b+d)}{2b(2b+d)}. \quad \text{(A.9)}$$

The pre-merger equilibrium profits of D1 and D2 are obtained by substituting Eq. (A.7) and (A.8) into Eq. (A.2):

$$\Pi_{D1}^0 = \Pi_{D2}^0 = \frac{a^2(b+d)}{4(2b+d)^2}. \quad \text{(A.10)}$$

Appendix 2: Post-Merger Equilibrium with Total Foreclosure of D2

This appendix derives the post-merger equilibrium prices and profits in the case of total foreclosure of D2, as given in Eq. (7) and (8). It also proves that the total profits of U and D1 are smaller in the case of total foreclosure of D2 than the sum of their pre-merger profits.

If post-merger U totally forecloses D2, consumers can no longer buy D2's product. The demand for D1's product can be derived from a constrained utility maximization problem such that the consumption of D2's product is restricted to be zero. The resulting new demand for D1's product is given by:

$$\tilde{D}_1(P_1) = D_1(P_1, P_2^c) \tag{A.11}$$

where $D_1(\cdot)$ is the same demand function as in Eq. (A.1) and P_2^c is the "choke price" of D2's product (i.e., the smallest price that makes the demand for D2's product equal to zero). For any given P_1, P_2^c is the solution of the equation $D_2(P_2^c, P_1) = 0$, i.e.:

$$P_2^c = \frac{a + dP_1}{b + d}\,. \tag{A.12}$$

Substituting Eq. (A.12) into Eq. (A.11) gives the demand for D1's product under total foreclosure of D2:

$$\tilde{D}_1(P_1) = \frac{(b + 2d)(a - bP_1)}{b + d}\,, \tag{A.13}$$

which is the same as in Eq. (6).

The total profits of U and D1 are given by:

$$\Pi_V = P_1 \tilde{D}_1(P_1)\,. \tag{A.14}$$

The vertically integrated company chooses P_1 to maximize Eq. (A.14). The optimal price is given by the first-order condition:

$$\frac{\partial \Pi_V}{\partial P_1} = \tilde{D}_1(P_1) - \frac{b(b + 2d)}{b + d} P_1 = 0\,. \tag{A.15}$$

Solving Eq. (A.15) gives the optimal output price of the vertically integrated company (assuming total foreclosure of D2):

$$P_1^f = \frac{a}{2b}, \tag{A.16}$$

which is the same as in Eq. (7).

Finally, the total profits of the vertically integrated company are obtained by substituting Eq. (A.16) into Eq. (A.14):

$$\Pi_V^f = \frac{a^2(b+2d)}{4b(b+d)}, \tag{A.17}$$

which is the same as Eq. (8). The difference between the post-merger and pre-merger total profits of U and D1 is thus given by:

$$\Pi_V^f - \Pi_U^0 - \Pi_{D1}^0 = -\frac{a^2b^2}{4(b+d)(2b+d)^2} < 0. \tag{A.18}$$

Notice that when the products of D1 and D2 are perfect substitutes, i.e., $d \to \infty$, the difference between the post-merger and pre-merger total profits of U and D1 becomes zero. This is because pre-merger U is already obtaining the integrated monopoly profit and thus will gain nothing from merging with D1. Clearly, the merger will have no effects on prices. This is also true in all the variants of the basic model. Hereafter, we only consider the case of imperfect substitutes, i.e., $0 \le d < \infty$.

Appendix 3: Post-Merger Equilibrium without Total Foreclosure for Market Structure A

This appendix derives the post-merger equilibrium prices and profits without total foreclosure of D2 and under market structure A. It also proves that the total profits of U and D1 are greater than the sum of their pre-merger profits.

Under market structure A, the vertically integrated company sets its output price P_1 and the input price W_2 to D2 before D2 sets its output price P_2. Given P_1 and W_2, D2 sets P_2 to maximize its profit as given in Eq. (A.2). The best response of D2 is given by:

$$P_2^{**} = B_2(P_1, W_2) = \frac{a + dP_1 + (b+d)W_2}{2(b+d)}. \quad \text{(A.19)}$$

The problem of the vertically integrated company can thus be written as:

$$\underset{P_1, W_2}{Max}\, \Pi_V = P_1 D_1(P_1, B_2(P_1, W_2)) + W_2 D_2(B_2(P_1, W_2), P_1). \quad \text{(A.20)}$$

The first-order conditions are:

$$a(2b+3d) - 2(2b^2 + 4bd + d^2)P_1 + 2d(b+d)W_2 = 0 \quad \text{and}$$

$$\frac{a}{2} + dP_1 - (b+d)W_2 = 0. \quad \text{(A.21)}$$

Solving Eq. (A.21) gives the equilibrium prices of the vertically integrated company:

$$P_1^a = W_2^a = \frac{a}{2b}. \quad \text{(A.22)}$$

Then, substituting Eq. (A.22) into Eq. (A.19) gives the output price of D2:

$$P_2^a = \frac{a(3b+2d)}{4b(b+d)}. \quad \text{(A.23)}$$

The total profits of the vertically integrated company are obtained by substituting Eq. (A.22) into Eq. (A.20):

$$\Pi_V^a = \frac{a^2(3b+4d)}{8b(b+d)}. \quad \text{(A.24)}$$

The difference between the post-merger and pre-merger total profits of U and D1 is thus given by:

$$\Pi_V^a - \Pi_U^0 - \Pi_{D1}^0 = \frac{a^2(2b^2 + 4bd + d^2)}{8(b+d)(2b+d)^2} > 0. \tag{A.25}$$

Finally, the output prices can be substituted into the demand equations to obtain the equilibrium quantities of the downstream firms. Post-merger, the outputs of the downstream firms are

$$Q_1^a = D_1(P_1^a, P_2^a) = \frac{a(2b+3d)}{4(b+d)} \text{ and } Q_2^a = D_2(P_2^a, P_1^a) = \frac{a}{4}. \tag{A.26}$$

Similarly, the pre-merger output levels can be obtained as

$$Q_1^0 = D_1(P_1^0, P_2^0) = \frac{a(b+d)}{2(2b+d)} \text{ and } Q_2^0 = D_2(P_2^0, P_1^0) = \frac{a(b+d)}{2(2b+d)}. \tag{A.27}$$

It is then straightforward to show that D1's output increases post-merger and D2's output decreases post-merger.

Appendix 4: Equivalence of Market Structure A and Market Structure B

This appendix proves that the equilibrium prices (W_2, P_1, and P_2) under market structure A are identical to the equilibrium prices under market structure B. It follows that the equilibrium outputs of D1 and D2 are the same under either market structure and so are the equilibrium profits of the vertically integrated company and D2.

Before proving the equivalence of the two market structures, it is useful to briefly describe each market structure separately.

Description of market structure A

Under market structure A, D2 sets its price P_2 to maximizes its profits while taking P_1 and W_2 as given. D2's best-response function can be written as:

$$P_2^{**} = B_2(P_1, W_2). \tag{A.28}$$

The vertically integrated company chooses its prices P_1 and W_2 to maximize the integrated profits while taking into account D2's response as given in Eq. (A.28). The equilibrium profits of the vertically integrated company under market structure A can thus be written as:

$$\Pi_V^a(P_1^a, W_2^a) = \underset{P_1, W_2}{Max} \; P_1 D_1(P_1, P_2^{**}) + W_2 D_2(P_2^{**}, P_1), \tag{A.29}$$

where P_1^a and W_2^a are the equilibrium prices chosen by the vertically integrated company. The equilibrium price of D2 is then given by:

$$P_2^a = B_2(P_1^a, W_2^a). \tag{A.30}$$

Description of market structure B

Under market structure B, given W_1 and W_2, D1 and D2 simultaneously choose P_1 and P_2 to maximize their own downstream profits while taking each other's price as given. The best-response functions of D1 and D2 can be written as:

$$P_1^{**} = B_1(P_2, W_1) \text{ and } P_2^{**} = B_2(P_1, W_2). \tag{A.31}$$

Notice that D2's best-response function under market structure B is the same as that under market structure A. The downstream equilibrium prices can be obtained by solving (A.31), and can be written as:

$$P_1^{*} = R_1(W_1, W_2) \text{ and } P_2^{*} = R_2(W_1, W_2). \tag{A.32}$$

Given the downstream equilibrium as given in Eq. (A.32), U chooses W_1 and W_2 to maximize the integrated profits of U and D1. The equilibrium profits of the vertically integrated company are thus given by:

$$\Pi_V^b(W_1^b, W_2^b) = \underset{W_1, W_2}{Max} \;\; P_1^* D_1(P_1^*, P_2^*) + W_2 D_2(P_2^*, P_1^*)\,, \tag{A.33}$$

where W_1^b and W_2^b are the equilibrium prices chosen by U. The downstream equilibrium prices are then given by:

$$P_1^b = R_1(W_1^b, W_2^b) \text{ and } P_2^b = R_2(W_1^b, W_2^b)\,. \tag{A.34}$$

Alternatively, the equilibrium price of D2 can be written as

$$P_2^b = B_2(P_1^b, W_2^b)\,. \tag{A.35}$$

Proof of the equivalence of the two market structures

The first part of the proof consists in showing that $\Pi_V^a(P_1^a, W_2^a) = \Pi_V^b(W_1^b, W_2^b)$. This is done in two steps by first showing that $\Pi_V^a(P_1^a, W_2^a) \geq \Pi_V^b(W_1^b, W_2^b)$ and then showing that $\Pi_V^a(P_1^a, W_2^a) \leq \Pi_V^b(W_1^b, W_2^b)$.

Under market structure A, if the vertically integrated company chooses its prices P_1 and W_2 as $P_1 = P_1^b$ and $W_2 = W_2^b$, then D2's price is equal to $B_2(P_1^b, W_2^b) = P_2^b$ and the profit of the vertically integrated company is equal to $P_1^b D_1(P_1^b, P_2^b) + W_2^b D_2(P_2^b, P_1^b) = \Pi_V^b(W_1^b, W_2^b)$. In other words, under market structure A, the vertically integrated company can achieve the same level of profits as under market structure B (by setting the prices P_1 and W_2 at the same level as under market structure B, which ensures that P_2 will also be at the same level). Therefore, $\Pi_V^a(P_1^a, W_2^a) \geq \Pi_V^b(W_1^b, W_2^b)$.

Under market structure B, if the integrated company chooses its prices W_1 and W_2 such that $W_2 = W_2^a$ and $R_1(W_1, W_2^a) = P_1^a$, then D2's price is equal to $B_2(P_1^a, W_2^a) = P_2^a$ and the profit of the vertically integrated company is equal to $P_1^a D_1(P_1^a, P_2^a) + W_2^a D_2(P_2^a, P_1^a) = \Pi_V^a(W_1^a, W_2^a)$. In other words, under market structure B, the vertically integrated company can achieve the same level of profits as under market structure A (by setting W_2 at the same level as under market structure A and by setting W_1 at the level that induces the same P_1 as under market structure A, which in turn ensures that P_2 will also be at the same level). Thus, $\Pi_V^b(W_1^b, W_2^b) \geq \Pi_V^a(P_1^a, W_2^a)$.

Finally, under linear demand, the equilibrium is unique under both market structure A and market structure B. Therefore, $\Pi_V^a(P_1^a, W_2^a) = \Pi_V^b(W_1^b, W_2^b)$ implies $P_1^a = P_1^b$, $W_2^a = W_2^b$ and $P_2^a = P_2^b$. (W_1 is irrelevant under market structure A.)

Appendix 5: Post-Merger Equilibrium under Market Structure C

First, let us consider the case where the vertically integrated company does not totally foreclose D2. Under market structure C, given W_2, D1 chooses P_1 to maximize the total profits of U and D1, that is:

$$\Pi_V = P_1 D_1(P_1, P_2) + W_2 D_2(P_2, P_1) \tag{A.36}$$

while D2 behaves as pre-merger. (Note that W_1 is irrelevant since it no longer affects D1's pricing behavior.) From the first-order conditions, one obtains the equilibrium output prices as functions of W_2:

$$P_1^{***} = \frac{a}{2b+d} + \frac{3d(b+d)}{(2b+d)(2b+3d)} W_2 \quad \text{and}$$

$$P_2^{***} = \frac{a}{2b+d} + \frac{2b^2 + 4bd + 3d^2}{(2b+d)(2b+3d)} W_2 . \tag{A.37}$$

U chooses W_2 to maximize the total profits of U and D1, while taking into account that the output prices depend on W_2 according to Eq. (A.37). From the first-order condition, one obtains the equilibrium input price W_2:

$$W_2^c = \frac{a(2b+3d)(4b^2 + 6bd + 3d^2)}{2b(b+d)(8b^2 + 16bd + 9d^2)} . \tag{A.38}$$

Recall that the pre-merger input price to D2 is $W_2^0 = \frac{a}{2b}$. The difference between W_2^c and W_2^0 is:

$$W_2^c - W_2^0 = \frac{-ad^2}{2(b+d)(8b^2+16bd+9d^2)} < 0. \qquad \text{(A.39)}$$

This shows that, under market structure C, the vertically integrated company lowers the input price to D2 post-merger. (We will show below that it has no incentive to totally foreclose D2.)

Substituting Eq. (A.38) into Eq. (A.37) gives the equilibrium output prices:

$$P_1^c = \frac{a(2b+3d)(4b+3d)}{2b(8b^2+16bd+9d^2)} \quad \text{and} \quad P_2^c = \frac{a(12b^3+32b^2d+30bd^2+9d^3)}{2b(b+d)(8b^2+16bd+9d^2)}. \qquad \text{(A.40)}$$

The difference between the post-merger and pre-merger output prices is:

$$P_1^c - P_1^0 = -\frac{a(8b^2+12bd+7d^2)}{2(2b+d)(8b^2+16bd+9d^2)} < 0 \quad \text{and}$$

$$P_2^c - P_2^0 = -\frac{ad(4b^2+7bd+4d^2)}{2(2b+d)(8b^3+24b^2d+25bd^2+9d^3)} < 0. \qquad \text{(A.41)}$$

Substituting Eq. (A.38) and (A.40) into Eq. (A.36) gives the total profits of U and D1:

$$\Pi_V^c = \frac{a^2(12b^3+40b^2d+45bd^2+18d^3)}{4b(b+d)(8b^2+16bd+9d^2)}. \qquad \text{(A.42)}$$

The difference between the post-merger and pre-merger total profits of U and D1 is:

$$\Pi_V^c - \Pi_U^0 - \Pi_{D1}^0 = \frac{a^2(8b^4+32b^3d+43b^2d^2+24bd^3+4d^4)}{4(b+d)(2b+d)^2(8b^2+16bd+9d^2)} > 0. \qquad \text{(A.43)}$$

Finally, if the vertically integrated company decides to totally foreclose D2, then it will obtain the same equilibrium profit as in Appendix 2, which we know is less than the pre-merger total profits of U and D1. Therefore, in equilibrium, the vertically integrated company will not choose to totally foreclose D2.

Appendix 6: Merger Effects for Asymmetric Linear Demand with Symmetric Cross-Slopes

We solved the model using *Mathematica*. The *Mathematica* notebook is available upon request.

Appendix 7: A Numerical Example for a Demand System with Asymmetric Cross-Slopes

We solved the model using *Mathematica*. The *Mathematica* notebook is available upon request.

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