Gauging Parallel Accommodating Conduct Concerns with the CPPI [+]

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Abstract

The 2010 Merger Guidelines give greater prominence to the concept of parallel accommodating conduct. Parallel accommodating conduct (PAC) has a long history in oligopoly theory, dating back more than seventy years. It is a type of coordinated conduct that does not require an agreement. Instead, it involves a firm engaging in a certain conduct, with the expectation that one or more other firms will follow that same conduct. For example, PAC could involve two leading firms raising their prices in parallel – but without an agreement – over and above the prices determined by their unilateral pricing incentives. One firm would raise price above the Bertrand equilibrium level and the other firm would simply follow. We have formulated an index for gauging PAC concerns, which we call the "Coordinated Price Pressure Index," or CPPI. The CPPI corresponds to the largest price increase that the two coordinating firms would be willing both to initiate and follow. A larger CPPI implies an incentive of the two firms for larger PAC price increases, which suggests more serious PAC concerns, *ceteris paribus*. We also have applied the CPPI to merger analysis. The impact of a merger on the magnitude of the potential parallel price increases is measured by the increase in the CPPI (or "Delta CPPI").

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I. Introduction

Analysis of competitive effects from a merger typically distinguishes between unilateral effects and coordinated effects. One recent development in unilateral effects analysis has been the adoption of measures of upward pricing pressure by the 2010 Horizontal Merger Guidelines. These measures (indices) provide a quantitative indication of the potential unilateral effects of a merger without the attempt to correctly implement a full-blown merger simulation model. In this note we propose a related measure of upward pricing pressure for one form of coordinated effects concern, parallel accommodating conduct.

We refer to our proposed index as the "Coordinated Price Pressure Index" or CPPI. We first show that the CPPI is an intuitive measure for gauging the concerns for parallel accommodating conduct between two firms in a market. We then discuss how the merger-induced increase in the CPPI can be used to score the incremental impact of a merger on these concerns. We also illustrate these ideas in the context of the proposed AT&T/T-Mobile merger, by deriving the CPPI (and the merger-induced increase in the CPPI) for the potential of parallel accommodating conduct between AT&T and Verizon.

Parallel accommodating conduct (PAC) has a long history in oligopoly theory, dating back more than seventy years.[1] PAC is a type of coordinated conduct that does not require an agreement. Instead, it involves a firm engaging in a certain conduct, with the expectation that one or more other firms will follow that same conduct. For example, PAC could involve two leading firms

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[1] *See, e.g.*, Robert L. Hall and Charles J. Hitch, *Price Theory and Business Behavior*, 2 Oxford

raising their prices in parallel over and above the prices determined by their unilateral pricing incentives. One firm would raise price above this level and the other firm would simply follow.

The language of the 2010 Merger Guidelines gives greater prominence to the concept of parallel accommodating conduct, explicitly identifying these PAC effects as a form of coordinated interaction. As stated in the 2010 Merger Guidelines:

> Coordinated interaction alternatively can involve parallel accommodating conduct not pursuant to a prior understanding. Parallel accommodating conduct includes situations in which each rival's response to competitive moves made by others is individually rational, and not motivated by retaliation or deterrence nor intended to sustain an agreed-upon market outcome, but nevertheless emboldens price increases and weakens competitive incentives to reduce prices or offer customers better terms.[2]

While the 2010 Merger Guidelines suggest an index for gauging upward pricing pressure for unilateral effects, they do not suggest an index for gauging coordinated effects, whether through PAC or any other flavor of coordinated interaction. To help evaluate this type of conduct in merger analysis, we have formulated such an index.

Our proposed coordination index for PAC is formulated as follows. Suppose that two firms engage in PAC.[3] We define the CPPI as the largest price increase that the two firms could sustain with PAC. A larger CPPI implies an incentive of the two firms for larger PAC price increases, which suggests more serious PAC concerns, *ceteris paribus*.

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[2] U.S. Department of Justice and the Federal Trade Commission, *Horizontal Merger Guidelines* (August 19, 2010) at 24-25.

[3] In this note, we focus on potential parallel accommodating conduct by two leading firms.

Suppose that there is a merger in the market in which Firm A acquires a third firm (say, Firm C). In this situation, the CPPI for Firms A (now merged with C) and B may rise.[4] If so, the merger would increase the magnitude of the potential parallel price increases. Thus, the increase in the CPPI ("Delta CPPI") can be used as a measure of the parallel accommodating conduct concerns raised by the merger.

We want to clarify that we have formulated this CPPI to gauge upward pricing pressure for only one form of coordinated effects (PAC), within the confines of a particular oligopoly model (price competition with differentiated products) and under particular assumptions (e.g., PAC by only two firms). However, PAC obviously is neither the only type of oligopoly conduct that might occur in a market nor the only oligopoly model used by economists. For example, there are numerous oligopoly models that economists use in antitrust, including the Bertrand model with differentiated products; the Cournot model with homogeneous products; the Stigler defection/punishment model; the Stackelberg leader/follower model; and the dominant firm/perfectly competitive fringe model; among others.[5]

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[4] However, the CPPI need not increase. As discussed below, this differs from the gross upward pricing pressure index (GUPPI), which always implies an increase in prices.

[5] For the Bertrand model, *see, e.g.*, Jerry Hausman, Gregory Leonard, and J. Douglas Zona, *Competitive Analysis with Differenciated Products*, 34 Annales D'Economie et de Statistique 159 (1994). For the Cournot model, *see*, *e.g.*, Joseph Farrell and Carl Shapiro, *Horizontal Mergers: An Equilibrium Analysis*, 80 American Economic Review 107 (1990). For the Stigler defection/punishment model, *see*, *e.g.*, James W. Friedman, *A Noncooperative Equilibrium for Supergames*, 38 Review of Economic Studies 1 (1971). For the Stackelberg model, *see*, *e.g.*, Marcel Boyer and Michel Moreaux, *On Stackelberg Equilibria with Differentiated Products: The Critical Role of the Strategy Space*, 36 Journal of Industrial Economics 217 (1987). For the dominant firm model, *see*, *e.g.*, Gautam Gowrisankaran and Thomas J. Holmes, *Mergers and the Evolution of Industry Concentration: Results from the Dominant-Firm Model*, 35 RAND Journal of Economics 561 (2004). The Merger Guidelines refer to some of these oligopoly models. In particular, the analysis of unilateral conduct in differentiated-product markets in the 1992 and

Therefore, we are not claiming that the CPPI is the only possible index that could be formulated to score coordinated effects concerns. Instead, the CPPI described here is one useful index for gauging the effects of a merger on concerns about coordinated interaction through PAC in industries with differentiated products.[6] Nor are we saying that this is the only relevant oligopoly model for which a CPPI could be formulated. We expect that other indices can and will be formulated for other coordinated effects models.[7]

The CPPI for parallel accommodating conduct is related to the GUPPI score for unilateral effects. In fact, under certain assumptions, the formula for the CPPI has a close similarity to the simultaneous GUPPI that scores unilateral effects when the merging firm raises the prices of the products of both merging firms simultaneously.[8]

2010 Merger Guidelines is consistent with the Bertrand model with differentiated products. The concepts of the "value of diverted sales" in the 2010 Merger Guidelines and the associated "gross upward pricing pressure index" (GUPPI) can be derived from that Bertrand model. Similarly, the treatment of coordinated effects in the 1982 and 1992 Merger Guidelines is consistent with the Stigler defection/punishment model.

[6] There can be multiple possible indices for a given oligopoly model. For example, gross upward pricing pressure for mergers involving unilateral effects in Bertrand markets with differentiated products have been scored with the single-product GUPPI, the simultaneous GUPPI, and the CMCR. *See* Jerry Hausman, Serge Moresi, and Mark Rainey, *Unilateral Effects of Mergers with General Linear Demand*, 111 Economics Letters 119 (2011) for the simultaneous GUPPI, and Gregory J. Werden, *A Robust Test for Consumer Welfare Enhancing Mergers among Sellers of Differentiated Products*, 44 J. Indus. Econ. 409 (1996) for the CMCR. *See* also the Indicative Price Rise used in Office of Fair Trading, *Anticipated Acquisition of the Online DVD Rental Subscription Business of Amazon Inc. by LOVEFiLM International Limited* (2008) at 13-14; and testimony by Alison Oldale in Federal Trade Commission, *Horizontal Merger Guidelines Review Project*, Hearing, Washington D.C. (December 3, 2009).

[7] *See, e.g.*, Douglas Bernheim and Michael D. Whinston, *Multimarket Contact and Collusive Behavior*, 21 RAND Journal of Economics 1 (1990).

[8] Hausman, Moresi, and Rainey, *Unilateral Effects of Mergers with General Linear Demand*, *supra* note 6.

Furthermore, like the GUPPI, the CPPI is an index; it is neither a prediction of the post-merger price increase, nor is it intended to capture every detail of the equilibrium outcome of a dynamic oligopoly model.[9] Thus, the CPPI should be used in conjunction with other evidence.

For example, the existence of a positive CPPI alone does not imply that PAC necessarily will occur in a market. In fact, beginning at the pre-merger Bertrand equilibrium point, the CPPI *always* suggests that the two firms have an incentive to engage in PAC. However, it often is the case that PAC does not occur. There may be various impediments to successful PAC, such as lack of information; fear of entry or repositioning; or incentives to secretly or openly cut prices after engaging in PAC.

The level of the CPPI does not indicate the impact of a merger. There is a pre-merger CPPI as well as a post-merger CPPI. To gauge the effect of a merger on the PAC concern, we calculate the Delta CPPI, which is the increase in the CPPI as a result of a merger.[10] A notable feature of the Delta CPPI from a merger is that it can be negative. This means that the incentives of Firms A and B to engage in PAC might decrease following a merger of Firms A and C. This differs from the GUPPI for unilateral effects, which cannot be negative. Note that, like the GUPPI, the Delta CPPI does not take into account the potential for downward pricing pressure from merger-specific cost efficiencies.

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[9] This is in contrast to methods of coordinated effects analysis that require a full merger simulation model that attempts to describe the full equilibrium in a market. See, for example, William E. Kovacic, Robert C. Marshall, Leslie M. Marx, and Steven P. Schulenberg, *Quantitative Analysis of Coordinated Effects*, 76 Antitrust Law Journal 397 (2010).

[10] Thus, the CPPI is like the HHI, in that both the level and change in the level are relevant for merger evaluation. .

Finally, it is important to raise a caution flag related to a variant of the Cellophane Fallacy: the CPPI and the Delta CPPI also may be lower when coordination already is occurring. When there already is coordination, there is less incentive for further PAC. In this case, while a merger may not lead to more PAC, it could help to entrench the coordination already existing. Thus, a low CPPI and Delta CPPI do not eliminate coordinated effects concerns from a merger, when there is evidence of pre-merger coordination.

The remainder of this paper is organized as follow. Section II describes the assumptions and the formula of the CPPI. Section III considers several extensions of the CPPI. Section IV analyzes the impact of a merger on the CPPI and presents some illustrative examples. Section V applies the CPPI to the proposed merger of AT&T and T-Mobile. Section VI discusses the use of the CPPI as a tool for merger analysis. The Appendix shows the technical derivation of the CPPI formula in more detail.

II. Largest Sustainable Increase in Price and the CPPI

The formula for the CPPI is derived from a simple model of oligopoly interaction between two firms. Formally, we define the "largest sustainable increase in price" (LSIP) as the maximum price increase that one firm is willing to initiate *and* the other firm is willing to match (holding the prices of all the other firms constant). In general, this maximum price increase depends on which firm is initiating the price increase. Thus, there is a $LSIP_A$ for a price increase initiated by Firm A and a $LSIP_B$ for a price increase initiated by Firm B. We then define the CPPI as the smaller of these two LSIPs. This ensures that the CPPI corresponds to the largest price increase that both firms would be willing to initiate, which in turn implies that both firms would benefit from PAC. The CPPI thus gauges the potential price increase that the two firms could achieve through parallel accommodating conduct. This section explains this methodology in more detail.

A. Assumptions

Consider a differentiated-products market in which firms compete in price. We assume that the two leading firms – say, Firm A and Firm B – each contemplate raising prices through PAC.[11] We analyze the incentives of Firms A and B to raise prices in parallel, beginning either at the (static) Bertrand equilibrium or some higher price level.

We specifically model this PAC process as follows. In period 1, one of the two firms (say, Firm A) decides whether or not to initiate a price increase by raising the price of its product by some percentage amount (say, 10%). If Firm A raises its price, then Firm B decides in period 2 *either* to match the price increase (and also raise its price by 10%) *or* keep its price unchanged at the initial level.[12] If Firm B does not match the price increase, then Firm A rescinds its price increase in period 3 and reduces its price back to its initial level. If instead Firm B matches the price increase initiated by Firm A, then these parallel price increases by Firms A and B become permanent.

To simplify the analysis for the purposes of deriving a CPPI, we make several assumptions. First, we assume that Firm A takes the current price of Firm B as given and does not expect

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[11] For simplicity, we assume that each of these two firms sells a single product and faces a constant marginal cost of production.

[12] The PAC model is different from the Stackelberg leader-follower model. First, the timing assumption of the PAC model involves the leader increasing its price a period before the follower matches or not. During that period, the leader loses a relatively high volume of sales because the follower will not match the price increase until the next period. In contrast, the Stackelberg model assumes that the follower responds to the leader's price increase very quickly within the same period. Second, the percentage price increases in the PAC model are assumed to be identical. In contrast, the follower in the Stackelberg model does not typically raise price by the same amount as the leader.

Firm B to initiate a price increase. We thus gauge Firm A's incentive to initiate a price increase relative to the status quo.

Second, we restrict the choice set of Firm B either to match exactly the price increase of Firm A (through an identical *percentage* price increase) or keep its price unchanged. We do not permit Firm B to raise its price by less than the increase in Firm A's price. We thus gauge Firm B's incentive to accommodate through a strictly parallel (*i.e.*, identical) percentage price increase.

Third, we initially consider only one round of price increases through PAC. That is, we do not allow Firms A and B to increase price gradually through several rounds of PAC. Instead, we restrict the analysis to the maximum parallel price increase that Firms A and B could achieve through a single round of PAC.[13] This implies that, in situations in which the two firms engaging in PAC are substantially different from one another, the largest sustainable increase in price may be substantially different depending on which of the two firms initiates the price increase. However, we take into account differences in the incentives to raise prices by measuring the CPPI as the smaller of the maximum price increases initiated by either firm.

Fourth, we consider only PAC by two firms, and we do not account for potential responses by other firms in the market. We gauge the incentives of Firms A and B to engage in PAC, holding the prices of other firms constant. For this reason, our analysis may understate the potential for parallel price increases, though it is less clear whether the Delta CPPI from a merger would understate the incremental impact of the merger.

[13] In Section III, we derive the *stable* PAC price level, which is the price level achievable by PAC at which firms would have no incentive to engage in any further PAC price increases.

Thus, for these and the other reasons discussed earlier, the CPPI is not intended to be a precise *prediction* of the likely accommodating price increases. The CPPI is not a full-blown market equilibrium model. Instead it is an *index* for gauging potential competitive concerns about parallel accommodating conduct. The CPPI and the Delta CPPI are useful to gauge how a merger affects the firms' incentives to engage in PAC and how significant these coordinated effects can be. Thus, the Delta CPPI is analogous to the GUPPI defined in the Merger Guidelines to score unilateral effects concerns.

B. The Basic CPPI Formula for PAC

We use m_A, e_A, and q_A to denote respectively the initial percentage margin, own price elasticity, and sales volume of the product sold by Firm A. Similarly, m_B, e_B, and q_B denote the initial percentage margin, own price elasticity, and sales volume of the product of Firm B. The diversion ratio from product A to product B is denoted by DR_{AB} and the diversion ratio from product B to product A is denoted by DR_{BA}.

As shown in the Appendix, assuming linear demand, the maximum price increase that Firm A is willing to initiate (supposing for the moment that Firm B will match) is given by:

$$S_A^I = \frac{\delta F_{BA} - \theta_A}{1 - \delta F_{BA}} m_A \tag{1}$$

$$\text{where} \quad F_{BA} = \frac{DR_{BA} q_B e_B}{q_A e_A} \quad \text{and} \quad \theta_A = 1 - \frac{1}{m_A e_A}$$

In Equation (1), the parameter δ denotes the discount factor used by Firm A to calculate the net present value of its profits.[14] The parameter F_{BA} is the "gain/loss ratio" for Firm A, i.e., Firm A's output increase (per period) when firm B will match firm A's price increase ($se_B q_B DR_{BA}$) divided by Firm A's output reduction when it initiates the price increase ($se_A q_A$). The parameter θ_A reflects the extent to which Firm A might already be engaging in some form of coordination with Firm B or other firms.[15]

The maximum price increase that Firm B is willing to match (when Firm A initiates the price increase) is given by:[16]

$$S_B^M = \frac{\delta F_{AB} - \theta_B}{1 - F_{AB}} m_B \tag{2}$$

$$\text{where} \quad F_{AB} = \frac{DR_{AB} q_A e_A}{q_B e_B} \quad \text{and} \quad \theta_B = 1 - \frac{1}{m_B e_B}.$$

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[14] If the (instantaneous) rate of return that the firm could earn by investing profits is equal to r, and the risk premium for engaging in PAC is equal to ρ, then the discount factor is equal to $\delta = \exp\{-(r+\rho)\Delta\}$ where Δ denotes the length of a period. We define the "length of a period" as the amount of time that would need to elapse before a firm would be able to observe a price change by its rival and be confident that the rival is trying to initiate a price increase through PAC (as opposed to having changed its price for some other reason). Thus, the value of δ can be close to 1 when such response period is very short or when the firm's risk-adjusted required rate of return is very low.

[15] If the market is initially in Bertrand equilibrium, then the Lerner condition implies $m_A e_A = 1$ and $\theta_A = 0$. If instead Firms A and B are already engaging in any type of pricing coordination, then $m_A e_A > 1$ and $\theta_A > 0$ (similarly, $m_B e_B > 1$ and $\theta_B > 0$). The CPPI will tend to be lower if the firms are already engaged in some type of coordinated conduct. This raises interpretation issues, as discussed in more detail below.

[16] For simplicity, we assume that Firm B uses the same discount factor as Firm A.

In general, when Firm A initiates the price increase, the maximum price increase that Firm A is willing to initiate (*i.e.*, S_A^I) can be higher or lower than the maximum price increase that Firm B is willing to match (*i.e.*, S_B^M). We define the largest sustainable increase in price when Firm A ($LSIP_A$) initiates the price increase as the smaller of the two maximum price increases:

$$LSIP_A = \min \{S_A^I, S_B^M\} \tag{3}$$

Similarly, we define the largest sustainable increase in price when Firm B initiates the price increase ($LSIP_B$) as:

$$LSIP_B = \min \{S_B^I, S_A^M\} \tag{4}$$

where S_B^I denotes the maximum price increase that Firm B is willing to initiate and S_A^M denotes the maximum price increase that Firm A is willing to match. These maximum price increases are given respectively by rewriting Equations (1) and (2) with the roles of Firms A and B reversed.

<u>Example 1</u>

Suppose that the market initially is at the Bertrand equilibrium.[17] Firms A and B have equal volume shares and each earn a margin of $m_A = m_B = m = 40\%$.[18] Suppose that the diversion

[17] In Bertrand markets, the Lerner conditions imply $m_A e_A = m_B e_B = 1$. We use these conditions in the footnotes explaining Examples 1 to 3.

ratio between them is $DR_{AB} = DR_{BA} = DR = 25\%$ and assume a discount factor equal to $\delta = 80\%$. The maximum price increase that either firm is willing to initiate is equal to $S_A^I = S_B^I = 10\%$,[19] while the maximum price increase that either firm is willing to match is $S_A^M = S_B^M = 10.67\%$.[20] Therefore, the largest sustainable increase in price is $LSIP_A = LSIP_B = 10\%$, which does not depend on which firm initiates the price increase.

<u>Example 2</u>

Assume instead that the diversion ratio from Firm B to Firm A is higher, $DR_{BA} = 50\%$, but the other assumptions remain the same as in Example 1. The maximum price increase that Firm A is willing to initiate now is $S_A^I = 26.67\%$, while the maximum price increase that Firm A is willing to match is S_A^M =32%.[21] The corresponding maximum price increases for Firm B are the same as in Example 1.[22] As a result, for a price increase initiated by Firm A, the binding constraint for $LSIP_A$ is given by the incentive of Firm B to match. That is, $LSIP_A = \min\{26.67\%, 10.67\%\} = 10.67\%$. For a price increase initiated by Firm B, $LSIP_B$ is constrained by the incentive of Firm B to initiate. That is, $LSIP_B = \min\{10\%, 32\%\} = 10\%$.

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[18] Equal volume shares and equal margins imply $F_{AB} = DR_{AB}$ and $F_{BA} = DR_{BA}$. We use these conditions in the footnotes explaining Examples 1 to 3.

[19] Equation (1) yields: $S_A^I = S_B^I = \frac{\delta DR}{1-\delta DR} m = \frac{(80\%)(25\%)}{1-(80\%)(25\%)}(40\%) = 10\%$.

[20] Equation (2) yields: $S_A^M = S_B^M = \frac{\delta DR}{1-DR} m = \frac{(80\%)(25\%)}{1-(25\%)}(40\%) = 10.67\%$.

[21] Equation (1) yields: $S_A^I = \frac{\delta DR_{BA}}{1-\delta DR_{BA}} m = \frac{(80\%)(50\%)}{1-(80\%)(50\%)}(40\%) = 26.67\%$. Similarly, Equation (2) yields: $S_A^M = \frac{\delta DR_{BA}}{1-DR_{BA}} m = \frac{(80\%)(50\%)}{1-(50\%)}(40\%) = 32\%$.

[22] This follows because Equations (1) and (2) from Firm B's perspective depend on the diversion ratio from Firm A to Firm B, DR_{AB}, which has not changed relative to Example 1.

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We define the CPPI as the smaller of the two LSIPs, that is:

$$CPPI = \min\ \{LSIP_A,\ LSIP_B\} \tag{5}$$

As shown in the Appendix, it follows from the previous equations that $S_i^I < S_i^M$ for each Firm $i \epsilon \{A, B\}$. That is, the maximum price increase that a firm is willing to initiate is always smaller than the maximum price increase that the *same* firm is willing to match. As a result, in situations where a firm initiating a price increase (say Firm A) is constrained by the incentive of its rival (Firm B) to match (*i.e.*, $LSIP_A = S_B^M$), it is necessarily the case that when Firm B initiates the price increase the largest sustainable increase in price will be constrained by the incentive of Firm B to initiate, not the incentive of Firm A to match (*i.e.*, $LSIP_B = S_B^I$).

This property is exhibited in Example 2. When Firm A initiates the price increase, the largest sustainable increase in price is determined by the incentive of Firm B to match (10.67%). When instead Firm B initiates the price increase, the largest sustainable increase in price is determined by the incentive of Firm B to initiate (10%).

Given this relationship, the CPPI is the smaller of the two maximum parallel price increases that the two firms would be willing to initiate. As a result, Equation (5) reduces to the following basic CPPI formula:

$$CPPI = \min\{S_A^I, S_B^I\} \tag{6}$$

C. Further Properties of the CPPI

We next discuss several significant properties of the CPPI.

1. Symmetric Firms

If Firms A and B are symmetric, so that $F_{AB} = F_{BA} = DR$, $\theta_A = \theta_B = \theta$, and $m_A = m_B = m$, then $S_A^I = S_B^I$ and the CPPI can be written as:

$$CPPI \ = \ \frac{\delta DR - \theta}{1 - \delta DR} \, m$$

If the initial prices of these symmetric firms are Bertrand equilibrium prices (*i.e.*, $em = 1$, which implies $\theta = 0$), then this formula becomes

$$CPPI \ = \ \frac{\delta DR}{1 - \delta DR} \, m \tag{7}$$

In the next section, we discuss the relationship between this formula and the hypothetical monopolist test in the limiting case when there is no discounting (*i.e.*, $\delta = 1$).

2. Initial Coordination

If there is initial coordination instead of Bertrand competition (*i.e.*, $\theta_A > 0$,and $\theta_B > 0$) then the CPPI is reduced. This property also is not surprising. If there is already coordination occurring, there is less incentive for further PAC.[23] This property raises a caution. If this property were overlooked, the CPPI (and the Delta CPPI) could fall victim to a variant of the Cellophane

[23] For example, if the two firms were symmetric and perfectly coordinating, and if there were no discounting, then the resulting CPPI would equal zero.

Fallacy. It would overlook the potential for a merger helping to maintain or entrench pre-merger coordination. Thus, where there is evidence of pre-merger coordination, the CPPI may have less usefulness in ruling out coordination effects concerns.[24] However, a high Delta CPPI would still raise concerns.

3. Just-Profitable versus Profit-Maximizing Price Increases

The CPPI is defined as the maximum price increase that is profitable. This definition raises the standard issue in antitrust analysis regarding the distinction between the "just-profitable" and the "profit-maximizing" price increase. Under our assumption of linear demand, the profit-maximizing price increase is one-half of the just-profitable (*i.e.*, break-even) price increase. We show in the Appendix that the profit-maximizing PAC price increase is equal to one-half of the expression in equation (6). This is similar to the well-recognized property of the GUPPI.[25]

III. Alternative CPPI Formulations for PAC

In this section, we first consider an alternative CPPI definition, which revolves around identifying the stable price level at which firms would have no incentive to initiate any PAC-type price changes. This leads to a CPPI formula somewhat different from Equation (6). We then look at the limiting case with no discounting (for example when matching occurs very quickly) and show that the two CPPI definitions, and hence the two CPPI formulae, coincide. In addition,

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[24] This is analogous to the point made in the Merger Guidelines that the hypothetical monopolist test for market definition should not use the current price if there is evidence of pre-merger coordination. *See* the 1992 Merger Guidelines at Section 1.11 and the 2010 Merger Guidelines at Section 4.1.2.

[25] *See*, *e.g.*, Carl Shapiro, *The 2010 Horizontal Merger Guidelines: From Hedgehog to Fox in Forty Years*, 77 Antitrust Law Journal 701 (2010) at 729, 750.

this limiting case yields a CPPI formula that is very closely related to the hypothetical monopolist test for market definition.

A. Stable PAC Price Increase

In the previous sections we defined the CPPI by considering only a single round of price increases and matching. However, it may be that at the profit-maximizing prices corresponding to the CPPI, firms would have an incentive to engage in further rounds of price increases and matching. This possibility suggests the following alternative CPPI definition related to a stable PAC price increase.

Consider the prices that are achievable via PAC. Given initial prices p_A, p_B, these are the price pairs obtained when both firms raise their prices by the same percentage level, S. We can look for the critical percentage level S^* which would make the price pair $p_A(1+S^*), p_B(1+S^*)$ stable, in the sense that neither firm has an incentive to initiate a further price change (neither increase nor decrease) under the expectation that its price change will be matched. This critical percentage level can thus serve as an alternative CPPI definition.

We show in the Appendix that neither firm will initiate a further price increase or decrease when both firms have raised their prices by S^*, where

$$S^* = \min \left(\frac{\delta F_{BA} - \theta_A}{2-(1+\delta)F_{BA}} m_A, \ \frac{\delta F_{AB} - \theta_B}{2-(1+\delta)F_{AB}} m_B \right). \qquad (8)$$

Comparing (8) and (6), and recalling that the profit-maximizing price increase corresponding to the CPPI is equal to one half of the CPPI, the stable price increase is larger than the price that would be achieved by a single round of PAC price increases. This suggests that, if Firms A and B are able to coordinate, they would have an incentive to engage in further price increases after an initial round of PAC. This also suggests that one could use $2S^*$ as an alternative CPPI formula.[26] In the particular case with $\theta_i = 0$, the alternative CPPI formula is

$$\text{Stable CPPI} = \min \left(\frac{2\delta F_{BA}}{2-(1+\delta)F_{BA}} m_A, \ \frac{2\delta F_{AB}}{2-(1+\delta)F_{AB}} m_B \right). \qquad (9)$$

This formula is used in Section V when we calculate CPPIs for the wireless market.

B. CPPI when there is No Discounting

The discount factor would be equal to $\delta = 1$, under two alternative scenarios. First, if the two firms are infinitely patient (say, because the interest rate were zero), then they would not discount. Second, and perhaps more relevant, the discount factor would be equal to unity if the response time between the initial price increase by one firm and the revelation of the decision by the other firm to match or not is extremely short.

The no discounting assumption leads to an important equivalence result. Comparing (6) and (8), one can see that $\delta = 1$ implies that the basic CPPI definition assuming a single-round PAC (*i.e.*, equation (6)) is equivalent to (twice) the alternative CPPI definition based on stable PAC (*i.e.*, equation (8)). In other words, the basic CPPI for PAC is stable when there is no discounting.

The no discounting assumption also has another interesting implication when the two firms considering PAC are symmetric and are initially in Bertrand equilibrium. In the limiting case with no discounting, the CPPI for two coordinating firms is equal to the simultaneous GUPPI that would be used to score unilateral effects if the two coordinating firms were to merge:

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[26] The use of $2S^*$ (instead of S^*) makes this alternative formula comparable to the benchmark CPPI of equation (6) and to the GUPPI.

$$CPPI = \frac{DR}{1 - DR} m \tag{10}$$

For example, under the assumptions of Example 1 ($DR = 25\%$ and $m = 40\%$) the CPPI and the simultaneous GUPPI both would be equal to 13.33%.[27]

This equivalence indicates that the potential price increase from parallel accommodating conduct by the two firms is the same as the potential price increase from unilateral effects that would arise if the two firms were to merge. This relationship between the CPPI and the simultaneous GUPPI is not surprising. The simultaneous GUPPI assumes that a merged firm raises the prices of both products simultaneously, while taking into account the feedbacks between the products.[28] If there is no discounting and the two firms are symmetric, then the maximum PAC price increase by two independent firms is equal to the maximum price increase that would remain profitable following a merger of those two firms.

As we have shown, in the case with no discounting and symmetric firms, the CPPI for coordination between Firms A and B is equal to the simultaneous GUPPI for a hypothetical merger of Firms A and B. Following the acquisition of Firm C by Firm A, the Delta CPPI is equal to the increase in the simultaneous GUPPI for a hypothetical merger of Firm A (which now owns Firm C) and Firm B.

The same expression as in Equation (10) also is sometimes used for the hypothetical monopolist test for market definition, under the assumption that the hypothetical monopolist raises all prices

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[27] That is, *CPPI* = 25% * 40% / (1-25%) = 13.33%.

[28] *See* Hausman, Moresi, and Rainey, *Unilateral Effects of Mergers with General Linear Demand*, *supra* note 6.

uniformly.[29] In other words, if there is no discounting and Firms A and B are symmetric, then the CPPI is equal to (twice) the profit-maximizing uniform SSNIP that a hypothetical monopolist of products A and B would impose. If instead the two firms are asymmetric, the CPPI is lower than (twice) the profit-maximizing uniform SSNIP, because the CPPI assumes no side-payments between the two firms. Thus, the relationship between the CPPI and the hypothetical monopolist test also is not surprising.

IV. Incremental Effect of a Merger: The Delta CPPI

Suppose that Firm A acquires a third firm, Firm C. Thus, after the acquisition ("post-merger") Firm A will supply products A and C. We analyze how the merger of Firms A and C changes the incentives of the leading firms (Firms A and B) to engage in PAC. Specifically, we measure the incremental effect of the acquisition on the CPPI, which we refer to as the "Delta CPPI." The Delta CPPI is the increase in the maximum parallel price increase (that Firms A and B can achieve by engaging in PAC) after the merger of Firms A and C versus before the merger.

In order to simplify the calculation of the post-merger CPPI and make it more comparable with the pre-merger CPPI, we make several assumptions.

Assumption 1: We measure the Delta CPPI as the increase in the CPPI for the coordinating firms relative to the pre-merger price level.

This assumption raises two noteworthy issues. First, as noted earlier, this assumption can lead to a very low CPPI (both pre-merger and post-merger) if the firms are already engaging in coordinated conduct (whether PAC, express collusion or some of other type of coordination).

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[29] The expression in Equation (9) might be called the "Uniform Price GUPPI."

Where there is evidence of significant pre-merger coordination, the CPPI may be less useful in ruling out coordination concerns. The CPPI and the Delta CPPI would not capture or measure the impact of a merger on preventing the breakdown of coordination. That is, the CPPI and the Delta CPPI for the coordinating firms could fall victim to a variant of the Cellophane Fallacy. However, a high Delta CPPI still would raise concerns.

Second, in calculating the post-merger CPPI for the coordinating firms, we abstract from any unilateral effects caused by the merger. Mergers may and often do raise both unilateral and coordinated concerns simultaneously. However, our CPPI measures the "gross" impact of the merger (of Firms A and C) on PAC pricing incentives (by Firms A and B), not the "net" impact over and above the unilateral price effects of the merger. We make this assumption because the purpose of the CPPI is to gauge the effect of the merger on the incentives for PAC, not the overall upward pricing pressure from all causes. This approach is consistent with the Merger Guidelines.[30] This methodology also is somewhat simpler to calculate. Finally, this approach also is necessary because there could be other coordinated effects concerns besides PAC.

Assumption 2: Firm A would raise the prices of products A and C by the same percentage amount if it attempted to engage in PAC with Firm B after the merger.[31]

[30] In addition, if there are adverse unilateral effects of the merger, then the merger could be considered anticompetitive whether or not there are coordinated effects.

[31] This assumption is crucial for the result that the Delta CPPI can be negative. If one assumes that the merged firm would raise the price of product A only, then the Delta CPPI is either zero or positive.

Assumption 3: The post-merger total sales volume of Firm A will increase from q_A to $q_A + q_C$ as a result of the acquisition, where q_C is the current sales volume of Firm C. The diversion ratio from Firm B to the merged firm is equal to the sum of the diversion ratio from B to A and that from B to C. The diversion ratio from the merged firm to Firm B is the share of lost sales from the merged firm following a uniform price increase (of products A and C) that goes to Firm B.

Assumption 4: Product C has the same price and margin as product A. After the acquisition of Firm C, Firm A will face the same elasticity, charge the same price and earn the same margin (for both products A and C) as prior to the acquisition.[32]

Given these assumptions, the post-merger CPPI for the coordinating firms can be calculated using the same formulae given above, but with a larger sales volume for Firm A, a higher diversion ratio from Firm B to Firm A, and a revised diversion ratio from Firm A to Firm B.[33]

The post-merger CPPI will always be higher than the pre-merger CPPI if the binding constraint facing PAC post-merger will be the pricing incentive of the non-merging firm (*i.e.*, Firm B). This is because the merger of Firms A and C creates a positive "diversion effect" on Firm B's incentive to initiate a price increase. Intuitively, Firm B will have a stronger incentive to initiate a price increase post-merger because the short-term cost to Firm B of initiating the price increase is the same as pre-merger but the long-term benefit from Firm A matching the price increase will be higher. The long-term benefit to Firm B will be higher because more output will be diverted

[32] These assumptions could be relaxed, although it would make the formulae more complicated.

[33] Under our assumptions, the diversion ratio from Firm B to the merged firm is always higher than the diversion ratio from Firm B to Firm A pre-merger. The diversion ratio from the merged firm to Firm B, however, is not necessarily higher than the diversion ratio from Firm A to Firm B pre-merger.

to Firm B when the merged firm will match the price increase and raise the prices of products A *and* C.

However, the post-merger CPPI for the coordinating firms can be lower than the pre-merger CPPI if, instead, the binding constraint facing PAC post-merger is the pricing incentive of the merging firm (*i.e.*, Firm A). In this case, there are both a positive "diversion effect" and a negative "volume effect" on Firm A's incentive to initiate a price increase. Intuitively, Firm A can have a weaker incentive to initiate a price increase post-merger because raising the prices of products A and C will lead to a larger short-term loss of sales volume (in units) than raising the price of only product A pre-merger. When Firm B will match the price increase, if the volume diverted from Firm B to Firm A will increase less than proportionally (relative to the short-term loss of output incurred by Firm A), the long-term benefit to Firm A from Firm B matching the price increase will not be sufficiently higher and, as a result, Firm A will have a weaker incentive to initiate a price increase than it had pre-merger. In other words, if the negative volume effect dominates the positive diversion effect, the merged firm will have a reduced incentive to initiate a price increase. Under these circumstances, it is possible that the post-merger CPPI for the coordinating firms could be lower than the pre-merger CPPI and, therefore, that the Delta CPPI could be negative.[34]

The above results are illustrated through the following example.

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[34] This is similar to Comte et al. (2002) who also find a negative "output effect." *See* Olivier Compte, Frederic Jenny, and Patrick Rey, *Capacity Constraints, Mergers and Collusion*, 46 European Economic Review 1 (2002). In their model, the merged firm might find it too costly to punish cheaters (because it has more output) while in our model the merged firm might find it too costly to initiate a price increase.

Example 3

In this example, the binding constraint on PAC is the pricing incentive of the merging firm (*i.e.*, Firm A) both pre-merger and post-merger. The CPPI for the coordinating firms does not change post-merger because the "volume effect" and the "diversion effect" exactly counterbalance one another. Consider again the assumptions of Example 1 (*i.e.*, Bertrand competition, $DR = 25\%$ and $m = 40\%$). Assume further that Firms A and B each have a market share of 20%. Assume next that Firm A merges with Firm C, which has a market share of 10% and also earns a margin of 40%. Assume that the diversion ratios between the merged firm and Firm B are $DR_{A'B} = 2/7$ and $DR_{BA'} = 3/8$ (where the subscript A' now denotes the merged firm).[35] Post-merger, the maximum price increase that Firm B is willing to initiate is equal to 20.87%,[36] while the maximum price increase Firm B is willing to match is 24%.[37] For the merged firm, the maximum price increases that it is willing to initiate and match remain the same as for Firm A in Example 1 and are equal to 10% and 10.67%, respectively.[38]

These assumptions lead to the result that the CPPI for the coordinating firms does not change after the merger. When the merged firm initiates the price increase, the largest sustainable increase in price is equal to 10%, coinciding with the pre-merger $LSIP_A$. When Firm B initiates

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35 These assumptions are consistent with proportional diversion and a retention ratio of 100%.

36 Using $F_{A'B} = \frac{q_{A'}DR_{A'B}}{q_B} = \frac{(30)(\frac{2}{7})}{(20)} = 3/7$, Equation (1) yields $S_B^I = \frac{\delta F_{A'B}}{1-\delta F_{A'B}}m = \frac{(80\%)(\frac{3}{7})}{1-(80\%)(\frac{3}{7})}(40\%) = 20.87\%.$

37 Equation (2) yields: $S_B^M = \frac{\delta F_{A'B}}{1-F_{A'B}}m = \frac{(80\%)(\frac{3}{7})}{1-(\frac{3}{7})}(40\%) = 24\%.$

38 Notice that $F_{BA'} = \frac{q_B DR_{BA'}}{q_{A'}} = \frac{(20)(\frac{3}{8})}{(30)} = 25\%$, which coincides with F_{BA} in Example 1.

the price increase, the largest sustainable increase in price is equal to 10.67%, which is constrained by the merged firm's incentive to match. Thus, the value of the CPPI remains at 10% both pre-merger and post-merger, and hence the Delta CPPI is zero. (It is straightforward to modify Example 3 and show that the Delta CPPI can be either positive or negative. For example, if one assumes a higher diversion ratio from Firm B to the merged firm, say, $DR_{BA'} = 50\%$, then the CPPI increases from 10% pre-merger to 14.55% post-merger. If instead one assumes $DR_{BA'} = 27.5\%$, the CPPI decreases from 10% pre-merger to 7.0% post-merger.)

V. Application: The Proposed AT&T/T-Mobile Merger

In this section we apply the CPPI to the proposed merger between AT&T and T-Mobile.[39] This analysis suggests that the merger could raise substantial coordinated effects concerns.

The American wireless market has four national competitors—Verizon, AT&T, Sprint, and T-Mobile, along with several other suppliers providing regional service or that specialize in prepaid service. On March 20, 2011 AT&T entered into an agreement to buy T-Mobile from Deutsche Telekom AG.[40] As of this writing, that proposed merger is still under review by the DOJ and the FCC. We focus here on the potential for the merger to facilitate coordination between the two largest wireless carriers, AT&T and Verizon, in the postpaid market.

The 2010 postpaid wireless market shares for AT&T and Verizon are about 32% and 39% respectively. Assuming proportional diversion and a retention ratio of 100%, these would lead to a diversion ratio from AT&T to Verizon of 57.4%, and a diversion ratio from Verizon to AT&T

[39] As noted earlier, the authors are consulting for Sprint on the proposed AT&T/T-Mobile merger.

[40] See http://www.att.com/gen/press-room?pid=19358&cdvn=news&newsarticleid=31703&mapcode=corporate|financial.

of 52.5%. Given the T-Mobile market share of about 11%, after the proposed merger of AT&T and T-Mobile the market share of the merged firm would be 43%. Hence, under proportional diversion the diversion ratio from the merged firm to Verizon would be 68.4%, and the diversion ratio from Verizon to the merged firm would be 70.5%.

Tables 1 and 2 present the stable PAC CPPIs and Delta CPPI for coordination by AT&T and Verizon for illustrative retention ratios of 60%, 80%, and 100%, and illustrative margins of 70% and 40%. Table 1 assumes a discount factor δ equal to 100%. For comparison purposes, Table 2 assumes a discount factor δ equal to 90%.

Given these proportional diversion ratios, the merger would raise concerns about parallel accommodating conduct. The pre-merger CPPIs are high. Moreover, the Delta CPPIs also are high. For example, for a 70% margin, an 80% retention ratio, and a 100% discount factor, the CPPI rises from 42.3% to 73.3%, giving a Delta CPPI of 31.0%. For a 40% margin and an 80% retention ratio, the CPPI rises from 24.2% to 41.9%, giving a Delta CPPI of 17.7%. The concerns would be even greater if materials reviewed in the course of a merger investigation showed evidence of pre-merger parallel accommodating conduct or other forms of coordination.

Comparing Tables 1 and 2, the lower discount factor reduces the CPPIs and Delta CPPIs. With a 70% margin and an 80% retention ratio but a 90% discount factor, the CPPI rises from 36.9% to 62.7%, giving a Delta CPPI of 25.8%. For a 40% margin and an 80% retention ratio, the CPPI rises from 21.1% to 35.8%, giving a Delta CPPI of 14.7%.

Table 1: AT&T-Verizon CPPI Results for Stable PAC in the Postpaid Market (Discount Factor = 100%)

	Margin = 70%			Margin = 40%		
Retention Ratio	60%	80%	100%	60%	80%	100%
Stable Price Increases Pre-Merger						
For AT&T Incentives	43.6%	73.3%	124.1%	24.9%	41.9%	70.9%
For Verizon Incentives	27.5%	42.3%	62.2%	15.7%	24.2%	35.6%
Stable Price Increases Post-Merger						
For AT&T Incentives	43.6%	73.3%	124.1%	24.9%	41.9%	70.9%
For Verizon Incentives	57.9%	106.5%	215.0%	33.1%	60.9%	122.9%
Pre-Merger CPPI	27.5%	42.3%	62.2%	15.7%	24.2%	35.6%
Post-Merger CPPI	43.6%	73.3%	124.1%	24.9%	41.9%	70.9%
Delta CPPI	16.0%	31.0%	61.9%	9.2%	17.7%	35.4%

Notes:
CPPI = Coordination Pricing Pressure Index.

Table 2: AT&T-Verizon CPPI Results for Stable PAC in the Postpaid Market (Discount Factor = 90%)

	Margin = 70%			Margin = 40%		
Retention Ratio	60%	80%	100%	60%	80%	100%
Stable Price Increases Pre-Merger						
For AT&T Incentives	38.0%	62.7%	102.6%	21.7%	35.8%	58.6%
For Verizon Incentives	24.3%	36.9%	53.6%	13.9%	21.1%	30.6%
Stable Price Increases Post-Merger						
For AT&T Incentives	38.0%	62.7%	102.6%	21.7%	35.8%	58.6%
For Verizon Incentives	50.0%	89.1%	167.7%	28.6%	50.9%	95.9%
Pre-Merger CPPI	24.3%	36.9%	53.6%	13.9%	21.1%	30.6%
Post-Merger CPPI	38.0%	62.7%	102.6%	21.7%	35.8%	58.6%
Delta CPPI	13.7%	25.8%	49.0%	7.8%	14.7%	28.0%

Notes:
CPPI = Coordination Pricing Pressure Index.

VI. Conclusions

The CPPI is an intuitive measure for gauging the parallel accommodating conduct concerns involving two firms in a market. The CPPI also can be used to score the incremental impact of a merger on these concerns. The main advantage of using indices like the CPPI is that they can be implemented with limited information, that is, prices, volumes, margins and diversion ratios.

The formula for the CPPI also uses the discount factor used by each coordinating firm to compare the long-term gain from coordination and the short-term cost of initiating (or matching) a price increase. A discount factor of 1 is most clearly applicable when there is little impediment to firms quickly matching price increases. But, given the potential difficulties in determining the actual cost of capital and response times, one alternative is to apply the CPPI formula using the discount factor equal to 1, at least as a starting point for coordinated effects analysis. Moreover, there is no need to distinguish between a single round of PAC and the stable PAC price level, because the two concepts coincide if $\delta = 1$.

Another important parameter in the formula for the CPPI measures the extent to which the two firms are already engaging in some form of coordination. Absent evidence to the contrary, the natural assumption might be that the two firms are not engaging in any coordination initially, and the market is in Bertrand equilibrium.

The results reported here are for a specific type of post-merger coordination—parallel accommodating conduct. In other mergers, the hypothesized coordinated conduct may be different. Nevertheless, we expect that the general approach of formulating pricing pressure indices to gauge the propensity for post-merger coordinated conduct can fruitfully be adapted to other situations. It will be interesting to compare the results of the various indices.

Technical Appendix

This appendix describes the formal analysis of the model of parallel accommodating conduct (PAC) used in this Note and derives the formula for the coordination pricing pressure index (CPPI).

We consider a differentiated-product market with linear demand and assume that two firms, say, Firm A and Firm B, might engage in PAC. For each Firm $i \epsilon \{A, B\}$, define the initial price (p_i), margin (m_i), sales volume (q_i), and own-price elasticity of demand (e_i). Let DR_{AB} be the diversion ratio from Firm A to Firm B, and DR_{BA} be the diversion ratio from Firm B to Firm A. Note that, if the market initially is at the Bertrand equilibrium, then the Lerner condition implies $e_i m_i = 1$. However, we allow the market to begin at initial prices other than the Bertrand equilibrium prices.

Suppose that, in period 1, Firm A were to initiate a price increase by raising the price of its product from p_A to $(1 + s)p_A$ and this price increase were not followed immediately by any other firms. With linear demand, this price increase would lead to a reduction in Firm A's sales volume by $se_A q_A$ units in period 1.[41]

The price increase likely would have an adverse impact on Firm A's profits in period 1. It would lead to incremental profits on the volume that Firm A would continue to sell, but lost profits on

[41] We are assuming that, in period 1, Firm A would lose only a fraction of its volume of sales following the price increase, i.e., $se_A < 1$.

the customers who would leave Firm A following the price increase. The net lost profits for Firm A in period 1 would amount to:

$$
\begin{aligned}
L &= se_A q_A p_A m_A - (q_A - se_A q_A) s p_A \\
&= s q_A p_A (e_A m_A - 1 + se_A)
\end{aligned}
\tag{A.1}
$$

Now consider the effect of Firm A's price increase on Firm B's profits in period 1. The price increase would lead to an increase in Firm B's sales volume of $se_A q_A DR_{AB}$ units. Thus, in period 1, Firm B would gain additional profit equal to:

$$
G = se_A q_A DR_{AB} m_B p_B \tag{A.2}
$$

In period 2, suppose that Firm B could decide either to match Firm A's price increase or keep its price unchanged. For simplicity, assume that in either case all other firms would maintain their prices at their initial levels. If Firm B decided not to match the price increase, then Firm B would obtain the gain G in period 2. However, Firm B would understands that if it did not match the price increase, then Firm A would reduce its price in period 3 from $(1 + s)p_A$ back down to p_A. Thus, evaluated at period 2, the strategy of not matching the price increase would give Firm B a gain of G in period 2 (the same gain as it obtained in period 1) and no gain thereafter.

If instead Firm B decided to match the price increase, then in period 2 the sales volume of Firm B would fall by $se_B q_B - se_A q_A DR_{AB}$ units relative to its initial output.[42] Firm B would lose profits from these customers, but would gain additional profit on the remaining customers following the price increase. The net change in Firm B's profit in period 2 would amount to:

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[42] We are assuming that Firm B would lose at most a fraction of its volume of sales, i.e., $se_B q_B - se_A q_A DR_{AB} < q_B$.

$$\Delta_B = (q_B - se_B q_B + se_A q_A DR_{AB}) sp_B - (se_B q_B - se_A q_A DR_{AB}) m_B p_B \qquad \text{(A.3)}$$
$$= sq_B p_B (1 - se_B - e_B m_B) + se_A q_A DR_{AB} p_B (s + m_B).$$

We assume that if Firm B decided to match Firm A's price increase, then the two price increases would become permanent. Assuming no response by other firms, it follows that Firm B would have an incentive to match the price increase if and only if:

$$\frac{\Delta_B}{1-\delta} > G \qquad \text{(A.4)}$$

where δ denotes the discount factor used by Firm B to calculate the net present value of a permanent change in profit of Δ_B per period (starting in period 2 and lasting forever).

Using equations (A.2) and (A.3) , one can rewrite equation (A.4) as follows:

$$\frac{DR_{AB} q_A e_A}{q_B} > \frac{se_B + m_B e_B - 1}{s + \delta m_B} \qquad \text{(A.5)}$$

Suppose now that equation (A.4) is satisfied so that, in period 1, Firm A expects that Firm B would match the price increase in period 2. Then in period 1, Firm A would have an incentive to initiate the price increase if and only if the loss incurred by Firm A in period 1 is less than the net present value of the gains realized in all subsequent periods:

$$L < \delta \frac{\Delta_A}{1-\delta} \qquad \text{(A.6)}$$

where Δ_A is the net change in profits that Firm A would receive in each period (starting in period 2) if both Firm A and Firm B raised prices; the derivation is analogous to (A.3):

$$\Delta_A = sq_Ap_A(1 - se_A - e_Am_A) + se_Bq_BDR_{BA}p_A(s + m_A) \tag{A.7}$$

Note that Firm A is assumed to have the same discount factor as Firm B. Using equations (A.1) and (A.7), condition (A.6) can be rewritten as follows:

$$\frac{DR_{BA}q_Be_B}{q_A} > \frac{se_A + m_Ae_A - 1}{\delta(s + m_A)}. \tag{A.8}$$

Equations (A.5) and (A.8) both must be satisfied in order for the price increase to be sustainable through PAC.

We can use these conditions to calculate the largest sustainable increase in price (LSIP); $LSIP_i$ is the maximum price increase that Firm *i* is willing to initiate and that Firm *j* is willing to match through a single round of PAC.

To simplify the resulting equations, we define:

$$F_{ij} = \frac{DR_{ij}q_ie_i}{q_je_j} \quad \text{and} \quad \theta_i = 1 - \frac{1}{m_ie_i} \quad \text{for } i, j \in \{A, B\} \tag{A.9}$$

Firm A would find it profitable to initiate PAC as long as conditions (A.8) and (A.5) are satisfied, and these conditions are satisfied as long as s is sufficiently small. We define S_A^I to be the maximum percentage price increase that firm A would find it profitable to initiate given the expectation that Firm B would match that percentage price increase in the next period; S_A^I is the value of s that satisfies (A.8) with equality. Rearranging (A.8) gives:[43]

[43] This step assumes $1 - \delta F_{BA} > 0$. When this assumption is violated, Firm A would have an incentive to initiate an arbitrarily large percentage price increase. Notice, however, that when

$$S_A^I = \frac{\delta F_{BA} - \theta_A}{1 - \delta F_{BA}} m_A \quad \text{(A.10)}$$

With linear demand, Firm A's profit from a price increase s that is matched in subsequent periods is quadratic in s, and therefore Firm A would initiate a profit-maximizing PAC price increase of $S_A^I/2$.

The maximum price increase that firm B would be willing to match, S_B^M, is given by rearranging (A.5):[44]

$$S_B^M = \frac{\delta F_{AB} - \theta_B}{1 - F_{AB}} m_B \quad \text{(A.11)}$$

We define $LSIP_A$ as the smaller of these two maximum price increases:

$$LSIP_A = \min \{S_A^I, S_B^M\} \quad \text{(A.12)}$$

$LSIP_A$ is the maximum price increase that Firm A would find profitable to initiate and that Firm B would be willing to match.[45]

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this assumption is violated, we must have $\frac{q_A e_A}{q_B e_B} > 1$. But, then we must have $\frac{q_B e_B}{q_A e_A} < 1$, which would in turn imply $1 - \delta F_{AB} > 0$. Hence, it cannot be the case that both firms have an incentive to initiate an arbitrarily large percentage price increase. Since the CPPI is defined as the minimum of S_A^I and S_B^I, it will always be bounded.

[44] This step assumes $1 - F_{AB} > 0$. When this assumption is violated, Firm B would have an incentive to match an arbitrarily large percentage price increase. Notice, however, that when this assumption is violated, we must have $\frac{q_B e_B}{q_A e_A} > 1$. But, then we must have $\frac{q_A e_A}{q_B e_B} < 1$, which would in turn imply $1 - F_{BA} > 0$. Hence, it cannot be the case that both firms have an incentive to match an arbitrarily large percentage price increase.

Turning next to the possibility that Firm B will initiate PAC, we define $LSIP_B$, which is the maximum price increase that Firm B would be willing to initiate and that Firm A would be willing to match. Using an analogous derivation,

$$S_B^I = \frac{\delta F_{AB} - \theta_B}{1 - \delta F_{AB}} m_B \tag{A.13}$$

$$S_A^M = \frac{\delta F_{BA} - \theta_A}{1 - F_{BA}} m_A \tag{A.14}$$

$$LSIP_B = \min \{S_B^I, S_A^M\} \tag{A.15}$$

where S_B^I denotes the maximum price increase that Firm B would be willing to initiate (assuming that Firm A would match) and S_A^M denotes the maximum price increase that Firm A would be willing to match.[46]

Note that, because (A.10) and (A.14) differ only by an additional δ in the denominator, $S_A^I < S_A^M$ for all $\delta < 1$. Similarly, comparing (A.11) and (A.13), $S_B^I < S_B^M$.

We define the CPPI as the smaller of the two LSIPs:[47]

[45] The profit-maximizing price increase that Firm A would initiate is given by min $\{S_A^I/2, S_B^M\}$.

[46] The profit-maximizing price increase that Firm B would initiate is given by min $\{S_B^I/2, S_A^M\}$.

[47] The profit-maximizing price increase corresponding to the CPPI is equal min{min $\{S_A^I/2, S_B^M\}$, min $\{S_B^I/2, S_A^M\}$}, which reduces to one half of the CPPI. This assumes that $CPPI/2 < \min\{1/e_A, 1/e_B\}$. This assumption is satisfied in the application to the proposed AT&T and T-Mobile merger in Section V.

$$CPPI = \min\{LSIP_A,\ LSIP_B\} \tag{A.16}$$

Since the maximum price increase that a firm would be willing to match is always at least as large as the largest increase in price that the same firm would be willing to initiate, the CPPI will always be equal to either S_A^I or S_B^I. The CPPI thus is the smaller of these two price increases:[48] [49]

$$CPPI = \min\{S_A^I, S_B^I\} \tag{A.17}$$

If Firms A and B are symmetric, so that $F_{AB} = F_{BA} = DR$, $\theta_A = \theta_B = \theta$, and $m_A = m_B = m$, then equations (A.10) and (A.13) are identical, and the CPPI can be written as:

$$CPPI = \frac{\delta DR - \theta}{1 - \delta DR}\, m \tag{A.18}$$

If in addition Firms A and B begin in a Bertrand equilibrium, so that the Lerner condition $em = 1$ implies $\theta = 0$, then the CPPI simplifies to:

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[48] The CPPI is an equilibrium in the sense that each firm would prefer to maintain this price increase rather than switching back to its initial price. That is, suppose that after the PAC succeeds, Firm A were to contemplate reducing its price back down to the initial level, knowing that it would be followed a period later by Firm B. Starting from a PAC with a price increase s, the condition for when reverting to the initial price would not be profitable is identical to the condition for matching a PAC price increase s starting from the initial price. Thus, as long as PAC established a price increase $s < S_A^M$, Firm A would have no incentive to deviate from PAC and switch back to its initial price. This implies that Firm A would not deviate from a PAC at the level of the CPPI, since $CPPI \le S_A^I < S_A^M$. Analogously, Firm B would not deviate from the CPPI price increase since $CPPI \le S_B^I < S_B^M$.

[49] More formally, one can define a game in which each firm has two choices, to price at the current price or at the LSIP price increase level. The following strategy for each firm constitutes a subgame perfect equilibrium of the game: price high at the LSIP price initially, price at the LSIP level if the other firm prices at the LSIP level, and price at the current market price otherwise.

$$CPPI \; = \; \frac{\delta DR}{1 - \delta DR} \; m \tag{A.19}$$

Derivation of the stable PAC price level

For each firm $i \epsilon \;\; A, B$, define price (p_i), margin (m_i), sales volume (q_i), and own-price elasticity of demand (e_i). We want to determine what conditions are required for p_A, p_B to constitute a stable PAC. Let DR_{AB} be the diversion ratio from Firm A to Firm B, and DR_{BA} be the diversion ratio from Firm B to Firm A. Firm A would inititate PAC at a ε% higher price level, assuming that price increase would be matched, if and only if

$$\begin{aligned} V(\varepsilon) = \quad & \Pi((1+\varepsilon)p_A, p_B) + \frac{\delta}{1-\delta}\Pi((1+\varepsilon)p_A, (1+\varepsilon)p_B) \\ & -\frac{1}{1-\delta}\Pi(p_A, p_B) > 0 \end{aligned} \tag{A.20}$$

Assuming linear demand,

$$V(\varepsilon) = \frac{\varepsilon q_A p_A}{1-\delta}(1 - e_A m_A - \varepsilon e_A) + \frac{\delta \varepsilon q_B p_A e_B D_{BA}}{1-\delta}(\varepsilon + m_A) \tag{A.21}$$

Taking derivatives with respect to ε,

$$\begin{aligned} \frac{dV(\varepsilon)}{d\varepsilon} &= \frac{q_A p_A}{1-\delta}(1 - e_A m_A - 2\varepsilon e_A) + \frac{\delta q_B p_A e_B D_{BA}}{1-\delta}(2\varepsilon + m_A) \\ \frac{d^2V(\varepsilon)}{d\varepsilon^2} &= \frac{-2p_A}{1-\delta}(q_A e_A - \delta q_B e_B D_{BA}) \\ &= \frac{-2p_A q_A e_A}{1-\delta}(1 - \delta F_{BA}), \end{aligned} \tag{A.22}$$

where

$$F_{ij} = \frac{DR_{ij} q_i e_i}{q_j e_j} \quad for\, i,j \in \{A,B\}. \tag{A.23}$$

Firm A will not initiate a small change in PAC as long as

$$\left.\frac{dV(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} = \frac{q_A p_A}{1-\delta}(1 - e_A m_A) + \frac{\delta q_B p_A e_B m_A D_{BA}}{1-\delta} = 0 \tag{A.24}$$

This will be satisfied whenever

$$\frac{1}{e_A} - (1 - \delta F_{BA}) m_A = 0. \tag{A.25}$$

Note that a necessary condition for (A.25) is that $\delta F_{BA} < 1$. When that is true, it follows from (A.22) that $V(\varepsilon)$ is concave. Consequently, if (A.25) is satisfied, there is no alternative price level that Firm A would be willing to initiate for PAC. Thus prices that satisfy (A.25) are a candidate for PAC, although one would also need to verify that Firm B would not initiate a price change starting at that price level.

Now consider a market that starts at arbitrary values of price (p_i), margin (m_i), sales volume (q_i), and own-price elasticity of demand (e_i), for $i \epsilon \{A, B\}$. As before, define DR_{AB} as the diversion ratio from Firm A to Firm B, DR_{BA} as the diversion ratio from Firm B to Firm A, and let[50]

[50] Note that with linear demand, the diversion ratio does not change as prices change.

$$F_{ij} = \frac{DR_{ij} q_i e_i}{q_j e_j} \quad for\, i, j \in \{A, B\}. \tag{A.26}$$

Define Φ to be the locus of price pairs that are feasible when each firm raises its price by s% starting from (p_A, p_B):

$$\Phi = \{(x_A, x_B) \mid x_A = (1+s)p_A \text{ and } x_B = (1+s)p_B \text{ for some } s\} \tag{A.27}$$

.

For a particular $(x_A, x_B) \in \Phi$, with s given by $x_A = (1+s)p_A$, the margin, output, and elasticity for Firm A are

$$m_A = \frac{m_A + s}{1+s} \tag{A.28}$$

$$q_A = q_A(1 - s e_A(1 - F_{BA})) \tag{A.29}$$

$$e_A = \frac{e_A(1+s)}{1 - s e_A(1 - F_{BA})}, \tag{A.30}$$

with similar expressions for Firm B. The condition for x_A, x_B to be a stable PAC from the perspective of Firm A is that (A.25) be satisfied at x_A, x_B. Substituting (A.26), (A.28), (A.29), and (A.30) into (A.25), the condition characterizing a candidate stable PAC is

$$(\frac{1}{e_A} - 2s - m_A + sF_{BA}) + \delta F_{BA}(s + m_A) = 0, \tag{A.31}$$

which is satisfied when

$$S_A = \frac{\delta F_{BA} - \theta_A}{2 - (1 + \delta)F_{BA}} m_A, \tag{A.32}$$

$$\text{where} \quad \theta_i = 1 - \frac{1}{m_i e_i} \quad \text{for } i, j \in \{A, B\} \tag{A.33}$$

A similar analysis for Firm B shows that the price level at which Firm B would not initiate any further changes in price levels (assuming that Firm A would match) is

$$S_B = \frac{\delta F_{AB} - \theta_B}{2 - (1 + \delta)F_{AB}} m_B. \tag{A.34}$$

If the market is initially in a Bertrand equilibrium, then $m_i e_i = 1$, $\theta_i = 0$, and the candidate stable price increase levels become

$$S_A = \frac{\delta F_{BA}}{2 - (1 + \delta)F_{BA}} m_A \quad \text{and} \quad S_B = \frac{\delta F_{AB}}{2 - (1 + \delta)F_{AB}} m_B. \tag{A.35}$$

The stable PAC equilibrium corresponds to a price increase $S^* = \min\{S_A, S_B\}$. At this price level, one firm (without loss of generality, Firm A) will not initiate any further price increases. To show that this price level is stable, we also need to show that Firm B will also not raise prices. There are two possibilities. One is that Firm B would raise prices unilaterally even if that price change is not matched by Firm A. The second is that Firm A, even though it would not initiate further PAC with a price increase above S_A, might be willing to match a price increase initiated by Firm B. In the first case, as long as the current PAC equilibrium is at a price above Firm B's Bertrand equilibrium price, Firm B would prefer to match Firm A rather than setting a permanently higher price. For the second case, we need to modify the derivation of the condition for initiating a price increase to instead consider the decision to match a price increase. Suppose

Firm B has initiated at ε% price increase above the current PAC price level. If Firm A matches, prices remain ε% higher, whereas if Firm A does not match, it gets increased profits for one period because of diversion from Firm B, and thereafter prices return to the current level. The net gain from matching for Firm A is

$$
\begin{aligned}
M(\varepsilon) &= \Pi(p_A, (1+\varepsilon)p_B) + \frac{\delta}{1-\delta}\Pi(p_A, p_B) - \frac{1}{1-\delta}\Pi((1+\varepsilon)p_A, (1+\varepsilon)p_B) \\
&= \frac{\varepsilon p_A}{1-\delta}[q_A - e_A m_A q_A + \delta e_B q_B D_{BA} m_A - \varepsilon(e_A q_A + \delta e_B q_B D_{BA})] \qquad \text{(A.36)} \\
&= \frac{\varepsilon p_A e_A q_A}{1-\delta}[\frac{1}{e_A} - \ \ 1 - \delta F_{BA} \ \ m_A - \varepsilon(1 + \delta F_{BA})] \\
&= -\frac{\varepsilon^2 p_A e_A q_A}{1-\delta}(1 + \delta F_{BA}) < 0,
\end{aligned}
$$

after substituting (A.25). Thus the gain for Firm A from matching a further price increase is negative. Meanwhile, the derivation of S_A implies that Firm A would initiate a price decrease at a higher price, which Firm B would have an incentive to match rather than maintaining a higher price. Thus no price level above S^* can be a stable PAC price level.