# Implementing the Hypothetical Monopolist SSNIP Test With Multi-Product Firms

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In this note, we describe the implementation of the "hypothetical monopolist SSNIP test" for market definition in the context of merger cases where firms produce multiple differentiated products. Recent articles by Michael Katz and Carl Shapiro and by Daniel O'Brien and Abraham Wickelgren have set out a framework for carrying out the hypothetical monopolist SSNIP test for market definition under the assumption that the firms maximize profits before the merger.[1] They use the information derived from profit-maximization as an input into the analysis of the incentives of the hypothetical monopolist. We take their framework as a given in our analysis.[2]

Those articles assume that each firm sells only a single product. In this note, we extend their analysis for the case of multi-product firms. We discuss the appropriate test and illustrate it with an example. As we demonstrate below, when multi-product firms sell substitute products, the relevant market under the hypothetical monopolist SSNIP test tends to be broader. In the Appendix, we provide a more general and technical description of the test.

## Basic Framework

It is common for firms to produce multiple differentiated products that are substitutes for at least some consumers. For example, GM produces Cadillacs and Buicks; Phillip Morris produces Marlboro and Parliament cigarette brands; Verizon offers a variety of cell phone models and calling plans. Industrial products, such as microprocessors and certain chemicals, also fit this paradigm. Where firms produce multiple differentiated products, the proper analysis of market definition should take this structure into account.

Consider the following simple market structure, for example. Three firms each sell brands of three products that are substitutable to some degree for some consumers. Suppose that the three sellers are The Coca-Cola Company, PepsiCo, and Cadbury Schweppes, and the three products are Cola (e.g., Coke, Pepsi, and RC Cola), Lemon-Lime (e.g., Sprite, Sierra Mist, and 7Up), and

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[1] Michael Katz & Carl Shapiro, *Critical Loss: Let's Tell the Whole Story*, Antitrust, Spring 2003, at 49; Daniel O'Brien & Abraham Wickelgren, *A Critical Analysis of Critical Loss Analysis*, 71 Antitrust L.J. 161 (2003). *See also* Øystein Daljord, Lars Sørgard & Øyvind Thomassen, *The SSNIP Test and Market Definition with the Aggregate Diversion Ratio: A Reply to Katz and Shapiro*, J. Competition L. & Econ. (forthcoming 2008); Joseph Farrell & Carl Shapiro, *Improving Critical Loss Analysis*, Antitrust Source, Feb. 2008, http://www.abanet.org/antitrust/at-source/08/02/Feb08-Farrell-Shapiro.pdf.

[2] In particular, we are assuming that it is useful to define a relevant market. As many economics commentators have pointed out, it is not clear that relevant market definition is even necessary for unilateral effects analysis, other than perhaps for evaluating safe harbors. *See, e.g.*, Jonathan B. Baker & Steven C. Salop, *Should Concentration Be Dropped from the Merger Guidelines, in* Perspectives on Fundamental Antitrust Theory (ABA Section of Antitrust Law 2001) (*reprinted in* 33 U. West Los Angeles L. Rev. 3 (2001)). More generally, for a discussion of simulation techniques for evaluating competitive effects, see Gregory Werden, *Simulating the Effects of Differentiated Products Mergers: A Practical Alternative to Structural Merger Policy*, 5 Geo. Mason L. Rev. 363 (1997).

Orange Flavor (e.g., Fanta, Tropicana Twister, and Sunkist). Assume that all the brands are substitutes to some degree. For example, the three brands sold by Coca-Cola (e.g., Coke, Sprite, and Fanta) are imperfect substitutes for one another, as are the three brands of the other two firms.

Consider next a proposed merger between two of these firms. The antitrust authorities might want to determine whether the three flavors (cola, lemon-lime, and orange) constitute separate, distinct markets or whether they jointly constitute a single, broader carbonated soft drink market.[3]

## The Hypothetical Monopolist SSNIP Test

To answer this market definition question, the Merger Guidelines dictate the use of the "hypothetical monopolist SSNIP test."[4] According to that test, product X is a relevant market if a profit-maximizing hypothetical monopolist of product X could impose a small but significant, nontransitory increase in price (SSNIP) above the current prices of the brands of product X. In the example, Cola would be a relevant market if a hypothetical monopolist of Cola would raise the prices of Coke, Pepsi, and RC Cola by at least a SSNIP.

Following the Guidelines, the hypothetical monopolist is assumed to be the only present and future producer of the relevant product (e.g., cola). Most importantly, the hypothetical monopolist is also assumed to produce and sell only the relevant product and not any other products (e.g., lemon-lime or orange).[5]

However, unlike the hypothetical monopolist, the three firms in the real world do produce and sell brands of the other products. This fact has important implications for evaluating market definition under the Merger Guidelines' test. As explained in the Katz-Shapiro and O'Brien-Wickelgren articles, the pricing incentives of the hypothetical monopolist are related to the factors that have determined the current price level in the premerger world. In the context of multi-product firms, the current equilibrium price level of product X depends in part on the fact that at least some of the firms selling their brand of product X also sell brands of products Y and Z, which are somewhat substitutable for product X. Therefore, to determine the profitability of a price increase by the hypothetical monopolist, we begin by analyzing how premerger prices are affected by the presence of multi-product firms.

## Premerger Competition and Prices

We assume that the firms engage in Bertrand (unilateral) price competition. In setting its prices, each firm rationally takes into account that the brands that it sells are substitutes to some degree. In the premerger world, when Firm A unilaterally sets the price of its brand of product X, it takes into account the fact that raising price would reduce the sales of its brand of product X by a certain amount, but that a fraction of the lost sales would be diverted to its own brands of products Y and Z, as well as to the brands of all three products sold by its competitors. (For example, when Coca-Cola evaluates the profitability of raising the price of Coke, it rationally would take into

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[3] As discussed above, this may not be necessary for a unilateral effects analysis.

[4] U.S. Dep't of Justice & Fed. Trade Comm'n, Horizontal Merger Guidelines § 1.11 (1992, revised 1997), http://www.ftc.gov/bc/docs/horizmer.htm [hereinafter Merger Guidelines]. *See also* Fed. Trade Comm'n & U.S. Dep't of Justice, Commentary on the Horizontal Merger Guidelines 5 (2006), http://www.ftc.gov/opa/2006/03/mergercom.shtm [hereinafter Merger Commentary].

[5] As stated in the Merger Guidelines, "Specifically, the Agency will begin with each product (narrowly defined) produced or sold by each merging firm and ask what would happen if a hypothetical monopolist of that product imposed at least a 'small but significant and nontransitory' increase in price, but the terms of sale of all other products remained constant." Merger Guidelines, *supra* note 4, § 1.11. Thus, the hypothetical monopolist would own the three brands of (say) cola and no brands of either (say) lime or orange.

account the fact that some fraction of the lost sales of Coke would be recaptured by higher sales of Sprite and Fanta.) These recaptured sales would contribute to the profitability of the increase in the price of Firm A's brand of product X.

The profit-maximizing price of Firm A's brand of product X balances the gains and losses in profits, taking into account both the volume reduction of its brand of product X and the diversion to its own brands of products Y and Z.[6] In the premerger world, Firm A engages in a similar profitability calculus in setting the prices of its brands of products Y and Z, as do Firm B and Firm C in setting their prices, all of which leads to the premerger equilibrium. The hypothetical monopolist's own pricing incentives would be compared to this initial price equilibrium.

## Implications for the Hypothetical Monopolist SSNIP Test

*[W]hen there are multi-product firms selling imperfect substitutes, premerger prices are higher than if firms sell only a single product. Intuitively, a multi-product firm has a unilateral incentive to set higher prices than would a single-product firm.*

Our key analytic point is that when there are multi-product firms selling imperfect substitutes, premerger prices are higher than if firms sell only a single product. Intuitively, a multi-product firm has a unilateral incentive to set higher prices than would a single-product firm.[7] This is because the customers who switch to the other products sold by that same firm are not lost customers. In contrast, a hypothetical monopolist solely of product X would not have this "multi-product incentive" to keep prices at this relatively high current level; this hypothetical single-product monopolist does not sell any brands of the other two products. This factor alone would tend to lead the hypothetical monopolist of product X to reduce its profit-maximizing price below the initial, premerger level. Of course, a hypothetical monopolist of product X would also own and sell all the brands of product X. This fact alone would tend to lead the hypothetical monopolist of product X to increase its profit-maximizing price above the initial, premerger level.

Therefore, in evaluating the overall profitability of raising the price of one brand of product X, both factors must be taken into account. The hypothetical monopolist would account for any sales recapture via the diversion from the one brand to all the other brands of product X. But the hypothetical single-product monopolist would not account for any recapture of the one brand's sales via the diversion to the other brands of products Y and Z. This latter recapture is ignored even though that recapture previously was taken into account by the standalone firms when setting prices in the premerger world.

For example, a hypothetical monopolist of cola would recognize that raising the price of Coke would divert some sales to Pepsi Cola and RC Cola—diversion that would be recaptured. This is the basis of the hypothetical monopolist's standard incentive to raise price. (In contrast, in the premerger world, Coca-Cola clearly does not have this incentive to raise price because the diversion to Pepsi and RC Cola is a complete loss to Coca-Cola.) However, the hypothetical cola monopolist would not take into account any recapture of diversion to Sprite and Fanta. (In contrast, in the premerger world, Coca-Cola clearly does have this incentive to raise the price of its cola brand because of the recapture of this diversion.)

Thus, multi-product firms have an incentive to set higher prices than would single-product firms that only sold one of the products. They take into account a price-raising factor that a hypo-

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[6] The profit-maximizing price also depends on the brand's own-price elasticity and the margins of the other brands sold by Firm A. See the Appendix for further discussion.

[7] In this unilateral effects framework, we assume that these higher prices are the result of multi-product profit-maximization, not coordinated interaction among the firms. Therefore, there is no *Cellophane* Fallacy issue raised. *See* United States v. E.I. du Pont de Nemours & Co., 351 U.S. 377 (1956). This is consistent with the approach of the Merger Guidelines when there is no coordinated interaction. Merger Guidelines, *supra* note 4, § 1.11 (use prevailing prices unless premerger circumstances are strongly suggestive of coordinated interaction).

thetical monopolist of a single product would not take into account. Therefore, when there are multi-product firms involved in market definition analysis, the hypothetical monopolist SSNIP test tends to define broader markets, relative to the situation with only single-product firms, all things being equal.

### Accounting for Multi-Product Firms in the SSNIP Test

This analysis affects the basic methodology for implementing the hypothetical monopolist SSNIP test. To illustrate, suppose that an analyst is evaluating whether product X is a relevant market and first evaluates whether it would be profit-maximizing for the hypothetical monopolist of product X to impose a single-brand SSNIP; that is, a price increase for only one of the brands of product X.[8]

In evaluating the profitability of this single-brand SSNIP, the hypothetical monopolist would take into account the profit recapture from the customer diversion to the brands of product X previously produced by Firm B and Firm C. As discussed above, however, the hypothetical monopolist would not take into account any recapture of diversion to Firm A's brands of products Y and Z because of the assumption that the hypothetical monopolist controls only a single product.

*[A] single product is more likely to constitute a separate relevant market if consumers' loyalty to the product exceeds their loyalty to the seller of the brand of the product.*

The profitability of the single-brand SSNIP by the hypothetical monopolist thus depends crucially on the relative magnitudes of (a) the diversion from the brand of product X produced by Firm A to the brands of product X produced by Firm B and Firm C versus (b) the diversion from the brand of product X produced by Firm A to the brands of products Y and Z sold by Firm A. If the latter diversion is larger, then the hypothetical monopolist would have an incentive to reduce the price of the brand of product X produced by Firm A. This factor would make it less likely that product X would comprise a separate market. (If the above relationship between diversion within the candidate market and diversion within the firm applies to each of the brands of product X, then clearly product X is not a relevant market.)

This same single-brand SSNIP test also can be carried out for the prices of each of the brands of product X sold by Firm B and Firm C. The test also can be implemented for other candidate product markets (e.g., product Y, products X and Y jointly, etc.). Thus, this test compares the diversion inside the firm across its various products versus the diversion outside the firm across brands of the same product. This comparison can be expressed intuitively as follows: a single product is more likely to constitute a separate relevant market if consumers' loyalty to the product exceeds their loyalty to the seller of the brand of the product.

This test can be implemented for a SSNIP of a single brand of a product and, under some assumptions, for a uniform SSNIP for all the brands in the candidate market.[9] Like the Katz-Shapiro and O'Brien-Wickelgren tests, our generalized test also requires data on diversion ratios and margins, and can be implemented for a SSNIP of any size.

The formal analysis in the Appendix leads to a very simple test when the firms are symmetric: assume initially that all three firms earn fully identical margins (that is, identical on each brand and identical to each other). For simplicity, assume also that there are no economies of scope in pro-

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[8] The Merger Guidelines contemplate this type of single brand SSNIP. Merger Guidelines, *supra* note 4, § 1.11. *See also* Katz & Shapiro, *supra* note 1. However, it may have limited practical importance if the agency is concerned about a broader competitive effect, that is, a price increase for all the brands. In short, market definition should not be divorced from the ultimate competitive effects analysis. *See, e.g.*, Steven C. Salop, *The First Principles Approach to Antitrust,* Kodak*, and Antitrust at the Millennium*, 68 ANTITRUST L.J. 187 (2000).

[9] Generally, the hypothetical monopolist would maximize profits by imposing different SSNIPs on different brands. The analysis of such non-uniform SSNIPs would require use of a simulation model.

ducing the three products. Under these symmetry conditions, a candidate market comprises a relevant market with respect to a uniform SSNIP of X percent, if and only if

$$\delta_M > \frac{\delta_F m + 2X}{m + 2X}$$

where $m$ is the percentage profit-margin, $\delta_M$ is the aggregate diversion ratio from each brand in the candidate market whose price has been increased to all the other brands in the candidate market, and $\delta_F$ is the diversion ratio from each brand whose price has been increased to all the other brands sold by the same firm (inside as well as outside the candidate market).

For example, suppose that the percentage profit-margin is 30 percent and the uniform SSNIP is 5 percent. Assume further that there were only single-brand firms, in which case $\delta_F = 0$. In that situation, the product would comprise a relevant market if the diversion ratio from the brand whose price has been increased to other brands of the same product exceeds 25% (i.e., 10% ÷ {30% + 10%}).[10] However, if instead there were multi-brand firms and the diversion inside the firm from its brand of the product to its other brands was significant, say, $\delta_F$ = 25%, then the critical aggregate diversion ratio would rise substantially. The product would comprise a relevant market only if the diversion ratio $\delta_M$ from one brand of the product to other brands of the same product exceeds 44% (i.e., {25% x 30% + 10%} ÷ {30% + 10%}). This is a large difference that would be determinative in many cases.

Thus, taking into account the fact that firms sell multiple products is essential to implementing the hypothetical monopolist SSNIP test for multi-product firms based on an assumption that firms are maximizing premerger profits. ●

[10] If $\delta_F = 0$, the test is identical to that described in Katz & Shapiro, *supra* note 1, at 53 & 56 n.25.

## APPENDIX

In this Appendix, we carry out a formal analysis of the hypothetical monopolist test when there are multi-product firms.

### *A. Profit Maximization by Multi-Product Firms*

Consider a candidate market with $M$ products.[11] For each product $j \in M$, let $N_j$ be the set of all the products sold by the owner of product $j$, including all the products (if any) that the owner of product $j$ sells *outside* the candidate market. The first-order condition can be written as:

$$m_j - \sum_{k \neq j}^{N_j} \delta_{jk} m_k \frac{p_k}{p_j} = \frac{1}{\eta_{jj}} \qquad (\forall j \in M) \qquad (1)$$

where $m_j = (p_j - c_j)/p_j$ is the percentage profit margin of product $j$, $\delta_{jk} = -(\partial D_k/\partial p_j)/(\partial D_j/\partial p_j)$ is the diversion ratio from product $j$ to product $k$, and $\eta_{jj} = -(\partial D_j/\partial p_j)(p_j/D_j)$ is the own-price elasticity of demand for product $j$.

### *B. Profit Maximization by the Hypothetical Monopolist*

Consider a hypothetical monopolist who owns all the $M$ products in the candidate market and no other product outside the candidate market. Let $X$ be the uniform percentage price increase imposed on each of the $M$ products (and assume that the prices of all the other products outside the candidate market remain constant). Assuming that demand and cost functions are linear, one can evaluate the effect on the hypothetical monopolist's profit of each individual price increase, and then add up those effects.

For each product $j \in M$, the increase in price (from $p_j$ to $(1 + X)p_j$) has three effects on the hypothetical monopolist's profit:

- The price increase reduces the sales volume of product $j$ (from $q_j$ to $(1 - X\eta_{jj})q_j$) and thus tends to reduce the profits of product $j$ by the amount:

$$X\eta_{jj} q_j m_j p_j \qquad (2)$$

- The price increase increases the margin (from $m_j p_j$ to $(m_j + X)p_j$) earned on the remaining volume of product $j$ that is still being sold, and thus tends to increase the profits of product $j$ by the following magnitude:

$$(1 - X\eta_{jj})q_j X p_j \qquad (3)$$

[11] We use $M$ to denote both the number and the set of products included in the candidate market.

- The price increase raises the sales volume of the other products in the candidate market, and thus tends to increase the profits of those other products by the following magnitude:

$$X\eta_{jj}q_j\sum_{k\neq j}^{M}\delta_{jk}(m_k+X)p_k \tag{4}$$

Based on Equations (2), (3) and (4), the net effect of increasing the price of product $j$ on the total profits of the $M$ products included in the candidate market can be written as follows:

$$X\eta_{jj}p_jq_j\left[\sum_{k\neq j}^{M}\delta_{jk}(m_k+X)\frac{p_k}{p_j}-m_j-X+\frac{1}{\eta_{jj}}\right] \tag{5}$$

Using the convention $\delta_{jj}=-1$, the total effect on the hypothetical monopolist's profit of increasing the prices of all the products in the candidate market is equal to:

$$X\sum_{j=1}^{M}p_jq_j\left[\eta_{jj}\sum_{k=1}^{M}\delta_{jk}(m_k+X)\frac{p_k}{p_j}+1\right] \tag{6}$$

Under the assumption that demands and costs are linear, if a price increase of $Z$ is neither profitable nor unprofitable, then the profit-maximizing price increase is $Z/2$. It thus follows from Equation (6) that the profit-maximizing price increase is expressed as follows:

$$X^*=-\frac{\sum_{j=1}^{M}q_j\eta_{jj}\sum_{k=1}^{M}\delta_{jk}m_kp_k+\sum_{j=1}^{M}p_jq_j}{2\sum_{j=1}^{M}q_j\eta_{jj}\sum_{k=1}^{M}\delta_{jk}p_k} \tag{7}$$

In this regard, note that if one assumes that the hypothetical monopolist would raise the price of product $j$ only (and would leave the other prices unchanged), then the profit-maximizing price increase would be equal to (using Equation (5) and then Equation (1)):

$$\begin{aligned}X_j^*&=\frac{1}{2}\left[\sum_{k\neq j}^{M}\delta_{jk}m_k\frac{p_k}{p_j}-m_j+\frac{1}{\eta_{jj}}\right]\\&=\frac{1}{2}\left[\sum_{k\neq j}^{M}\delta_{jk}m_k\frac{p_k}{p_j}-\sum_{k\neq j}^{N_j}\delta_{jk}m_k\frac{p_k}{p_j}\right]\end{aligned} \tag{8}$$

This expression shows that the profitability of a SSNIP (from the perspective of the hypothetical monopolist) tends to be lower when firms sell multiple products either inside or outside the candidate market.

### *C. Analysis of the Symmetric Case*

In the situation where there is symmetry across brands and across firms, Equations (1) and (6) reduce to:

$$(1-\delta_F)m=1/\eta \tag{9}$$

$$X\,M\,pq(\eta(\delta_M-1)(m+X)+1) \tag{10}$$

Here, $\delta_F$ is the total diversion ratio from any product in the candidate market to all the products sold by the same firm (inside as well as outside the candidate market), while $\delta_M$ is the aggregate diversion ratio from any product in the candidate market to all the other products in the candidate market.

Using Equation (9) to substitute for $\eta$, Equation (10) is positive (and thus the price increase is profitable for the hypothetical monopolist) if and only if:

$$(\delta_M - 1)(m + X) + (1 - \delta_F)m > 0 \qquad (11)$$

In the special case with single-product firms (i.e., $\delta_F = 0$), Equation (11) can be written as $\delta_M > X/(m + X)$. This is the standard formula of Katz-Shapiro (2003) and O'Brien-Wickelgren (2003). The more general formula proposed in this note is:

$$\delta_M > \frac{\delta_F m + X}{m + X} \qquad (12)$$

Thus, when there are multi-product firms (i.e., $\delta_F > 0$), a SSNIP of X percent is less likely to be profitable (*ceteris paribus*) and thus the candidate market is less likely to be a relevant market.

This also can be seen from the hypothetical monopolist's profit-maximizing price increase, i.e., Equation (7), which in the symmetric case reduces to:

$$X^* = \frac{(\delta_M - \delta_F)m}{2(1 - \delta_M)} \qquad (13)$$

It follows from Equation (13) that, in industries with multi-product firms (i.e., $\delta_F > 0$), the profit-maximizing price increase $X^*$ is lower and thus the candidate market is less likely to be a relevant market.

Two other properties of this analysis are worth noting. First, this analysis may not lead to very narrow markets even for very small SSNIP*s*. For example, for an infinitesimal price increase (i.e., $X = 0$), Equation (12) reduces to $\delta_M > \delta_F$. In other words, the hypothetical monopolist has an incentive to raise price at all only if the diversion within the candidate market is higher than the diversion within the firm. As previously expressed in the text, a single product is more likely to constitute a separate relevant market if consumers' loyalty to the product exceeds their loyalty to the seller of the brand of the product.

Second, if one were to assume that the hypothetical monopolist raises the price only of single product *j* (and leaves the other prices unchanged), then it follows from Equation (8) that the profit-maximizing price increase would be equal to the following:

$$X_j^* = (\delta_M - \delta_F)m/2 \qquad (14)$$

This result can be useful in analyzing the profitability of price discrimination and the existence of relevant markets based on that price discrimination.