Market Definition in Merger Analysis ♣

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Abstract

In this short and mainly expository article, we explain the "hypothetical monopolist test" that has become the standard methodology for identifying relevant antitrust markets in merger cases, and discuss two approaches to implementing the test. We then focus on the implementation of the test when firms offer multiple products or services, either inside or outside the candidate market, and discuss the "hypothetical cartel test" introduced in the 2010 U.S. Merger Guidelines.

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1. Introduction

Merger analysis includes defining the "relevant market" and then calculating the market shares of the various firms in the market.[1] Market definition is not an end in itself. The purpose of defining a market is to help frame the analysis of competitive interaction, gauge a firm's power over price and output, as well as measure market concentration. In some merger cases, it may be possible to prove market power directly, in which case defining the relevant market may be unnecessary. For example, when the U.S. Federal Trade Commission ("FTC") blocked the first proposed Staples-Office Depot merger in 1997, there was evidence that Staples charged higher prices in areas where there was no Office Depot.[2] That evidence suggested that the merger of Staples and Office Depot would lead to higher prices, irrespective to the presence of other providers of office supplies.

In cases that go to trial, the definition of the relevant market is often litigated intensely. Jonathan Baker notes that "Throughout the history of U.S. antitrust litigation, the outcome of more cases has surely turned on market definition than on any other substantive issue."[3] For example, when the FTC sought a preliminary injunction to halt

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[1] *See, e.g.,* Brown Shoe Co. v. United States, 370 U.S. 294, 324 (1962). In this Chapter, we focus on mergers. Market definition in monopolization cases raises additional issues, including the Cellophane Fallacy and its variants. For example, *see* Steven C. Salop, *The First Principles Approach to Antitrust, Kodak, and Antitrust at the Millenium*, 68 ANTITRUST LAW JOURNAL 187 (2000).

[2] *See, e.g*., Serdar Dalkir and Frederick R. Warren-Boulton, *Prices, Market Definition, and the Effects of Merger: Staples-Office Depot (1997)*, in John E. Kwoka and Lawrence J. White (Eds.), THE ANTITRUST REVOLUTION: ECONOMICS, COMPETITION, AND POLICY (1999).

[3] Jonathan B. Baker, *Market Definition: An Analytical Overview*, 74 ANTITRUST LAW JOURNAL 129 (2007) at 129.

the second proposed Staples-Office Depot merger in 2016, the FTC and the merging

parties disagreed over the breadth of that market.[4] The FTC argued that the relevant

market was certain "consumable office supplies" purchased by large business entities, a

product market that included (among other products) pens, binder clips, note pads, and

copy paper. The merging parties attempted to include ink and toner as well as other

goods (such as janitorial products) into a broader market definition, which would have

significantly diminished the merging parties' market shares.[5] Similarly, defining product

and geographic markets is frequently a source of contention in hospital merger cases.[6]

In merger practice, market definition analysis can be assisted by the allegations of

potential anticompetitive effects just as market definition can shed light on the

competitive effects. For example, the 2010 Horizontal Merger Guidelines of the U.S.

Department of Justice and the Federal Trade Commission note:

> Evidence of competitive effects can inform market definition, just as market
> definition can be informative regarding competitive effects. For example,
> evidence that a reduction in the number of significant rivals offering a group

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[4] *See* FTC et al. v. Staples, Inc., and Office Depot, Inc., Civil Action No. 15-2115 (EGS) (May 10, 2016),
§IV (B), https://www.ftc.gov/system/files/documents/cases/051016staplesopinion.pdf.

[5] The Court rejected the defendants' argument because competitive conditions for the FTC's scope of
"consumable office supplies" in its proposed market were substantially different from those for goods like
ink and toner. The merging parties also argued that Amazon was on the verge of entering the sale of
"consumable office supplies" and should be included as a participant in the relevant antitrust market. That
also would have significantly diminished their market shares. The FTC argued that there was no evidence
that Amazon was a participant in the relevant market or would become one in the near term. The Court
agreed. *Id*. at §§IV.C.1 and IV.I.1.

[6] *See*, *e.g.*, Greg Vistnes, *Hospitals, Mergers, and Two-Stage Competition*, 67 ANTITRUST LAW
JOURNAL 671 (2000); Sean May and Monica Noether, *Unresolved Questions Relating to Market
Definition in Hospital Mergers*, 59 ANTITRUST BULLETIN 479 (2014); David Dranove and Christopher
Ody, *Evolving Measures of Provider Market Power*, 2 AMERICAN JOURNAL OF HEALTH
ECONOMICS 145 (2016).

of products causes prices for those products to rise significantly can itself establish that those products form a relevant market.[7]

The remainder of this Chapter is organized as follows. Section 2 explains the "hypothetical monopolist test" that has become the standard methodology for identifying relevant antitrust markets in merger cases. Section 3 then discusses two approaches to implementing the test. Section 4 addresses the implementation of the test when firms offer multiple products or services, either inside or outside the candidate market. It also discusses the "hypothetical cartel test" introduced in the 2010 Merger Guidelines.[8] Section 5 concludes. The formal and technical details are contained in the Appendix.

2. The Hypothetical Monopolist Test

In U.S. merger analysis, market definition focuses solely on potential demand side substitution.[9] For making this evaluation, the Merger Guidelines introduced the concept of the "hypothetical monopolist test" ("HMT") to define relevant antitrust markets. That test was developed to make more concrete the concept of "reasonable interchangeability"

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[7] U.S. Department of Justice and Federal Trade Commission, HORIZONTAL MERGER GUIDELINES (Issued August 19, 2010) (hereinafter "Guidelines" or "2010 Merger Guidelines") at §4, *available at* http://www.justice.gov/atr/public/guidelines/hmg-2010.pdf. *See* also the discussion in Salop, *supra* note 1. In colloquial terms, the plaintiff's allegations can be used to identify a candidate market in which those allegations would make economic sense. That market then can be tested via the market definition exercise.

[8] Guidelines, *supra* note 7 at n. 4.

[9] This is clear in the current version of the Guidelines, *supra* note 7 at §4:

Market definition focuses solely on demand substitution factors, i.e., on customers' ability and willingness to substitute away from one product to another in response to a price increase or a corresponding non-price change such as a reduction in product quality or service. The responsive actions of suppliers are also important in competitive analysis. They are considered in these Guidelines in the sections addressing the identification of market participants, the measurement of market shares, the analysis of competitive effects, and entry.

of products by customers and the role of the "cross-price elasticity of demand" as a metric for product substitutability.[10]

The HMT made its first appearance in the 1982 Horizontal Merger Guidelines of the U.S. Justice Department.[11] In the current 2010 Merger Guidelines, the HMT still has a central role:

> The hypothetical monopolist test is applied to a group of products together with a geographic region to determine a relevant market … Specifically, the test requires that a hypothetical profit-maximizing firm, not subject to price regulation, that was the only present and future seller of those products ("hypothetical monopolist") likely would impose at least a small but significant and non-transitory increase in price ("SSNIP") on at least one product in the market, including at least one product sold by one of the merging firms.[12]

The wording of the Guidelines ("on at least one product in the market") creates flexibility with respect to whether the HMT (alternatively, the "SSNIP test") might involve raising the price of one product or raising some or all the prices in the candidate

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[10] For example, in *Brown Shoe,* the Court stated: "The outer boundaries of a product market are determined by reasonable interchangeability of use or the cross-elasticity of demand between the product itself and substitutes for it." Brown Shoe Co. v. United States, 370 U.S. 294, 325 (1962).

[11] *See* U.S. Department of Justice, HORIZONTAL MERGER GUIDELINES (1982) at n. 6. https://www.justice.gov/archives/atr/1982-merger-guidelines. An interesting discussion of the genesis of the HMT can be found in Gregory J. Werden, *The 1982 Merger Guidelines and the Ascent of the Hypothetical Monopolist Paradigm*, 71 ANTITRUST LAW JOURNAL 253 (2003).

[12] Guidelines, *supra* note 7 at §§4 and 4.1.1. Although this discussion is framed in terms of product market definition, the same HMT applies to geographic market definition. For geographic markets based on customer locations, the Guidelines at §4.2.2 state:

> The hypothetical monopolist test requires that a hypothetical profit-maximizing firm that was the only present or future seller of the relevant product(s) to customers in the region would impose at least a SSNIP on some customers in that region. A region forms a relevant geographic market if this price increase would not be defeated by substitution away from the relevant product or by arbitrage, e.g., customers in the region travelling outside the region to purchase the relevant product. In this exercise, the terms of sale for products sold to all customers outside the region are held constant.

market by a SSNIP. It also creates flexibility with respect to whether the HMT might

involve raising several prices uniformly (i.e., raise each price by the same percentage)

versus non-uniformly.[13] This has led to a number of variants of the HMT being used in

practice that we will discuss here.

The antitrust agencies often use a SSNIP of 5%.[14] They also generally use a

uniform SSNIP on all the products in the candidate market.[15] If the hypothetical

monopolist in some candidate market were to find it profit-maximizing to raise price by

at least 5%, then that candidate market would comprise a relevant antitrust market. If

instead the hypothetical monopolist were to find it profit-maximizing to raise price by

less than 5%, then the candidate market would fail the test.

It is important to stress that the HMT does *not* lead to a unique market

definition.[16] For example, consider a hypothetical merger between a wine producer and a

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[13] This flexibility reflects the fact that the Guidelines, while intuitive, are not based directly on any explicit model of competition and welfare effects. *See*, e.g., Michael D. Whinston, *Antitrust Policy toward Horizontal Mergers*, in Mark Armstrong and Robert Porter (eds), HANDBOOK OF INDUSTRIAL ORGANIZATION, 2007, North-Holland, Amsterdam.

[14] The U.S. agencies nearly always use a SSNIP of 5% in merger enforcement actions. However, what constitutes a "small but significant" price increase depends on the industry, and the agencies may use a SSNIP that is larger or smaller than 5%. *See* Guidelines, *supra* note 7 at §4.1.2. For example, in *Whole Foods*, the FTC argued that a 1% SSNIP was appropriate to define "markets characterized by high volume sales but low profit margins," and Judge Friedman noted that the economic experts from both parties (Dr. Murphy and Dr. Scheffman) "agreed that in some cases a hypothetical price increase as low as 1% may be appropriate." *See* FTC v. Whole Foods Market, Inc., No. 07-1021 (D.D.C. Aug. 16, 2007) at 28 and n. 9. https://www.ftc.gov/sites/default/files/documents/cases/2007/08/0710114dcopinpub.pdf.

[15] Agencies and expert witnesses sometimes use a non-uniform SSNIP or a single-product SSNIP on only one product in the candidate market. *See infra* section 3.c.

[16] Guidelines, *supra* note 7 at §4.1.1. ("The hypothetical monopolist test ensures that markets are not defined too narrowly, but it does not lead to a single relevant market. The Agencies may evaluate a merger in any relevant market satisfying the test, guided by the overarching principle that the purpose of defining the market and measuring market shares is to illuminate the evaluation of competitive effects.") The Guidelines also allow for a different relevant antitrust market for each product sold by the merging firms. Guidelines, *supra* note 7 at §§4 and 4.1. ("When a product sold by one merging firm (Product A) competes

vodka producer. Suppose that wine and beer satisfy the HMT and thus constitute a relevant market. Suppose further that wine, beer and vodka also satisfy the HMT and thus they also constitute a relevant market. In this example, the market definition that includes wine, beer and vodka is more appropriate for gauging the potential competitive effects of this merger than would the market definition that includes only wine and beer. This is because the latter market definition would include only one of the merging firms and thus would miss the possibility that the merger might have anticompetitive effects.[17]

The Guidelines state that the relevant market cannot exclude a product that is a closer substitute for the merging products than is another product included in the relevant market.[18] In the previous example of a merger between a wine producer and a vodka producer, suppose that wine, beer and vodka constitute a relevant market and also that whiskey is a closer substitute for wine and vodka than is beer.[19] Then, whiskey also must be included in the relevant market.

3.   **Critical Loss Analysis**

The HMT is often implemented with a methodology called Critical Loss analysis.

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against one or more products sold by the other merging firm, the Agencies define a relevant product market around Product A to evaluate the importance of that competition. Such a relevant product market consists of a group of substitute products including Product A. Multiple relevant product markets may thus be identified.")

[17] Once the relevant market is defined, one then can identify the market participants and calculate their market shares. In a merger case, one also would evaluate efficiencies, entry conditions and other factors to assess the competitive effect of the merger in that market.

[18] Guidelines, *supra* note 7 at §4.1.1 and Example 6.

[19] Whiskey is a closer substitute if, in response to a SSNIP on wine and vodka, greater revenues are diverted to whiskey than to beer. *See* Guidelines, *supra* note 7 at §4.1.1.

The original critical loss methodology involves estimating the reduction in the sales volume of the hypothetical monopolist following a SSNIP, without taking into account the information about the price sensitivity of demand that may be inferred from the pre-merger margins of the firms. Over the years, a new, modern version of critical loss analysis that takes this information into account has emerged and has been recognized in the 2010 Merger Guidelines.[20]

a. Original Critical Loss Analysis

An initial effort to render the HMT operational was developed by Barry C. Harris and Joseph J. Simons.[21] Their "critical loss" test and variations on it have been widely used in "testing" market definitions. The intuition behind the test is straightforward.

Consider a candidate market where several firms are selling products.[22] Consider a hypothetical monopolist that would be the sole seller of these products and would face the same demand and costs as the original firms. The profits of the hypothetical monopolist also would be the same as the profits of the original firms, if the monopolist were to charge the same prices as the original firms. If instead the monopolist were to raise the prices of all the products in the candidate market by some uniform SSNIP of *X*

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[20] Guidelines, *supra* note 7 at §4.1.3. ("Unless the firms are engaging in coordinated interaction … high pre-merger margins normally indicate that each firm's product individually faces demand that is not highly sensitive to price.")

[21] Barry C. Harris and Joseph J. Simons, *Focusing Market Definition: How Much Substitution is Necessary?*, 12 RESEARCH IN LAW AND ECONOMICS 207 (1989).

[22] Customers can be either "final consumers" that purchase the products for their own consumption or "intermediate producers" that purchase the products as inputs for production of other products.

percent,[23] suppose the monopolist would experience a percentage loss in sales of L, as some customers would substitute to others products outside the candidate market. If the loss in sales due to the price increase is sufficiently small, then the price increase would be profitable, and the candidate market would be considered a relevant antitrust market. If instead the loss in sales is sufficiently large, then the price increase would not be profitable and so the candidate market would fail the test.[24]

Specifically, a price increase of X percent on all the products in the candidate market is profitable to the hypothetical monopolist if the loss in total sales, L, satisfies:

$$L \quad < \quad X/(m + X) \tag{1}$$

where m denotes the variable percentage margin of the original firms.[25] The term $X/(m + X)$ is the so-called *critical loss* and L is the *actual loss*, where these factors are expressed as percentages of the original number of units sold. If, as in the expression above, the actual loss is less than the critical loss, then the price increase X would be profitable and the candidate market would be considered a relevant market.[26] If instead the actual loss is greater than the critical loss, then the price increase would be

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[23] The critical loss test is often implemented assuming a uniform SSNIP on all the products in the candidate market. For the alternative test where the price of only one product is raised, *see* James Langenfeld and Wenqing Li, *Critical Loss Analysis in Evaluating Mergers*, 46 ANTITRUST BULLETIN 299 (2001).

[24] At the time their test was developed, the convention was that the candidate market would be expanded to include other products until the critical loss test was satisfied. However, this procedure was based on the assumption that only the "smallest market" satisfying the test was relevant to merger analysis. The test also was premised on the assumption that the prices of all products would be raised uniformly. These assumptions are no longer required, though they are often followed. *See supra* note 16 and the earlier discussion on the Merger Guidelines' flexibility with respect to whether the HMT involves a uniform or non-uniform SSNIP.

[25] *See* Appendix A. When firms have different margins, m denotes the revenue-weighted average margin.

[26] Of course, it is possible that it is not the only relevant antitrust market satisfying the HMT.

unprofitable and the candidate market would fail the test.

Finding that a particular price increase is *profitable* does not mean that it is *profit-maximizing*. Katz and Shapiro note that the Merger Guidelines require that the SSNIP be profit-maximizing and that as a rough approximation, "the profit-maximizing price increase is half as large as the maximal price increase that yields profits above their pre-merger level. Therefore, if a 10 percent price increase would cause the hypothetical monopolist's profits to be higher than their pre-merger level, then the profit-maximizing price increase is at least 5 percent."[27]

Thus, the uniform SSNIP test in condition (1) can be restated as follows: if condition (1) is satisfied, then a uniform price increase of X percent is *profitable* to the hypothetical monopolist, so that the *profit-maximizing* uniform price increase would be at least $X/2$ percent, and thus the candidate market satisfies the HMT.

To illustrate, for $X=10\%$ and $m=40\%$, the critical loss would be 20% (i.e., 10/(40+10)). If a 10% price increase resulted in a demand reduction of less than 20%, the price increase would be profitable, the profit-maximizing price increase would be at least 5%, and the candidate market would satisfy the HMT.[28]

Implementation of the HMT using critical loss analysis requires an estimate of the actual loss. One possible approach might be to rely on testimony and documentary

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[27] Michael Katz and Carl Shapiro, *Critical Loss: Let's Tell the Whole Story*, 17 ANTITRUST 49 (2003) at 52. As a result, if the SSNIP threshold used by the antitrust agencies is, say, 5% then the "profitable" critical loss test analyzed here must be implemented using a price increase X equal to 10%. *See* also the "profit-maximizing" critical loss test in Gregory Werden, *Beyond Critical Loss: Tailoring Applications of the Hypothetical Monopolist Paradigm*, mimeo, U.S. Department of Justice (2002).

[28] If the margin is higher, say 70% instead of 40%, then the critical loss is lower, i.e., 12.5% instead of 20%. As we will explain shortly, that lower critical loss does *not* imply that a 10% price increase would be less profitable (or more unprofitable) when the current margin is 70% than when it is 40%.

evidence that the actual loss would be smaller (or larger) than the critical loss. Another

approach might be to estimate econometrically the elasticity of demand for the products

in the candidate market (i.e., the percentage reduction in the total quantity demanded in

the candidate market following a 1% price increase of all the products in the candidate

market). In that case, the actual percentage loss could be approximated as $L = EX$, i.e.,

the estimated elasticity E (in absolute value) *times* the percentage price increase X.[29]

Note that reliably estimating the demand elasticity (E) econometrically may raise a

variety of contentious issues.[30]

b. *Modern Critical Loss Analysis*

The potential difficulty in estimating the actual loss can result in analysts using

simplified and often misplaced substitutions to products outside the candidate market that

end up overstating the actual loss and suggesting an overly broad market definition. This

is because the original critical loss methodology does not take into account the

information about the price sensitivity of demand that one may be able to infer from the

profit-maximizing margins set by the firms pre-merger. If the pre-merger competitive

interaction among the firms can be reliability assumed (e.g., unilateral price competition

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[29] $L = EX$ is a linear approximation. If demand is convex in prices, then a linear approximation would overstate the actual loss. For example, for constant elasticity demand, the correct formula for the actual loss is $L = 1 - (1 + X)^{-E}$. That formula however would understate the actual loss, if demand becomes more elastic as price increases, as in the case of linear demand.

[30] As Katz and Shapiro observe, even if (say) merger partners with high margins were to estimate a high demand elasticity, and so the actual loss would seemingly be greater than the critical loss, they would have to explain how a high margin can be consistent with a high demand elasticity. *See* Katz and Shapiro, *supra* note 27 at 52.

a la Bertrand), the analyst can use the observed pre-merger margins to obtain information about the price sensitivity of demand, and thus to estimate the actual loss more precisely.

To illustrate these points, suppose that the merging parties have high margins. Katz and Shapiro note that the merging parties frequently try to define an overly broad market by arguing that high margins imply that any lost sales would reduce profits substantially, and so even a hypothetical monopolist controlling a group of products could not profitably raise price.[31] Katz and Shapiro then identify the flaw in this seemingly reasonable argument: "This story is incomplete because high margins also tend to imply that the actual loss is small, and thus a price increase might be profitable even when the critical loss is small."[32]

Katz and Shapiro then suggest an alternative approach for estimating whether the actual loss is smaller or larger than the critical loss, that takes into account the fact that a high margin typically implies that the firm faces a low demand elasticity.[33] Their approach involves estimating what they originally called the "aggregate diversion ratio," which is referred to as the "recapture percentage" in the 2010 Merger Guidelines.[34]

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[31] *Id.* at 50. *See* also Daniel O'Brien and Abraham Wickelgren, *A Critical Analysis of Critical Loss Analysis*, 71 ANTITRUST LAW JOURNAL 161 (2003).

[32] *Id.* Profit-maximization generally implies that the margin of a firm is inversely related to the price elasticity of demand faced by the firm. Thus, a larger margin is strong evidence that the firm is facing a lower demand elasticity and therefore a lower customer responsiveness to any given price change than is the case with a smaller margin.

[33] Katz and Shapiro, *supra* note 27 at 53-54. *See* also O'Brien and Wickelgren, *supra* note 31. The 2010 Merger Guidelines point to the same inverse relationship between margin and demand elasticity (*see supra* note 20).

[34] The recapture percentage is "the percentage of sales lost by one product in the candidate market, when its price alone rises, that is recaptured by other products in the candidate market, with a higher recapture percentage making a price increase more profitable for the hypothetical monopolist." Guidelines, *supra* note 7 at §4.1.3.

The estimation of the recapture percentage begins by considering a price increase for only one of the products in the candidate market by the hypothetical monopolist. For simplicity, suppose that each firm in the candidate market sells only one product and the hypothetical monopolist considers raising only the price of product A sold by firm A.

When the hypothetical monopolist raises the price of product A by X percent, some customers will reduce their purchases of A and switch to the other products controlled by the hypothetical monopolist in the candidate market, while other customers will substitute to products outside the candidate market. The fraction of the sales lost by A that are diverted to other products within the candidate market controlled by the hypothetical monopolist is the "recapture percentage." For example, denoting this recapture percentage by δ_M, suppose that an increase in the price of A causes A to lose 100 units of output. If in response some customers shift 60 units to other products in the candidate market, then δ_M is equal to 60%. To put it differently, 40% of the sales lost by A are lost to "outside goods" (i.e., products outside of the candidate market).

The greater is the recapture percentage when the price of A increases, the smaller is the diversion from A to outside goods and the more likely will a price increase of A be profitable. As we discuss below, this analysis suggests the relevance of a *single-product* SSNIP test, that is, where the hypothetical monopolist would increase the price of one product only. This test can be particularly useful for candidate markets with strong asymmetries among products. The single-product SSNIP test must be carried out for each product in the candidate market. According to the Guidelines, the candidate market

is a relevant market if the test is satisfied for "at least one product in the market, including at least one product sold by one of the merging firms."[35]

To illustrate, consider a merger of firms A and B, and a candidate market comprising solely of the two merging products, A and B. Suppose that the hypothetical monopolist would find it profit-maximizing to raise the price of A by more than a SSNIP, while holding the price of B constant. According to the Guidelines, this profitable SSNIP for A would be sufficient to define the candidate market to be a relevant market.

This single-product SSNIP test is most applicable if product B is very different from product A and faces more competition from outside goods than does A. In this situation, it is possible that the hypothetical monopolist would find it profit-maximizing to raise the price of A by more than a SSNIP, but would *not* find it profit-maximizing to raise the prices of *both* A and B simultaneously by more than a SSNIP. In other words, in highly asymmetric candidate markets, a uniform SSNIP may be unprofitable while a single-product SSNIP would be profitable for one of the products.[36] Thus, the HMT would be satisfied for the single-product SSNIP test but not for the uniform SSNIP test.

In candidate markets without significant asymmetries, however, the agencies often use the *uniform* SSNIP test where the hypothetical monopolist would raise the prices of all the products in the candidate market together and by the same SSNIP (e.g. 5%). The greater are the recapture percentages (when the price of one product increases

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[35] Guidelines, *supra* note 7 at §4.1.1.

[36] *See e.g.* Øystein Daljord and Lars Sørgard, *Single-Product versus Uniform SSNIPs*, 32 INTERNATIONAL REVIEW OF LAW & ECONOMICS 142 (2011).

unilaterally) for each product, the more likely will a uniform SSNIP be profitable.[37] Note

also that, in a perfectly symmetric candidate market, the uniform SSNIP test may be

satisfied while each of the single-product SSNIP tests may fail.

Assuming completely symmetric products for simplicity, formal analysis shows

that a uniform SSNIP of *X* percent on all the products in the candidate market is

profitable (i.e., the actual loss is smaller than the critical loss) if and only if:[38]

$$\delta_M \;\; > \;\; X/(m+X) \qquad\qquad\qquad (2)$$

As stated by Katz and Shapiro (and using their original "aggregate diversion ratio" term),

> *If and only if the aggregate diversion ratio is larger than the critical loss,*
> *then the actual loss is less than the critical loss and thus a hypothetical*
> *monopolist would find a SSNIP profitable.*[39]

To summarize, if condition (2) is satisfied, then the hypothetical monopolist

would find a uniform price increase of *X* percent on all products in the candidate market

profitable (thus, the profit-maximizing price increase would be at least *X*/2 percent) and

the candidate market would satisfy the HMT.[40]

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[37] As shown in Appendix B, the uniform SSNIP test is relatively cumbersome when the recapture percentages differ across products. As a practical matter, if the products are relatively symmetric, the agency might assume that the recapture percentage is the same for all of the products.

[38] For the formal derivation of condition (2), *see* Joseph Farrell and Carl Shapiro, *Improving Critical Loss Analysis*, ANTITRUST SOURCE (Feb. 2008), at Proposition 1. Farrell and Shapiro explain that this test assumes symmetric single-product firms and either linear demand or a small price increase. They also extend the test to account for a number of other factors. *See also* Joseph Farrell and Carl Shapiro, *Recapture, Pass-Through, and Market Definition*, 76 ANTITRUST LAW JOURNAL 585 (2010).

[39] Katz and Shapiro, *supra* note 27 at 53 (emphasis in original).

[40] The appropriate value of *X* to be used for the SSNIP test is discussed earlier. *See supra* notes 14 and 27.

Katz and Shapiro suggest that the data needed for estimating the aggregate diversion ratio often will be readily available from information relied upon by many businesses. Evidence on the aggregate diversion ratio may be gleaned from surveys of customer switching patterns or past customer responses to changes in prices or product availability. Econometric evidence based on demand responses to price changes can also be applied.[41]

c. The Uniform SSNIP Test versus Single-Product SSNIP Test

As discussed by Farrell and Shapiro, condition (2) is the appropriate uniform SSNIP test for a symmetric candidate market.[42] A different condition would apply for the single-product SSNIP test, where the hypothetical monopolist is assumed to impose a SSNIP on just one of the products in the candidate market. As Daljord, Sørgard and Thomassen show, a single-product SSNIP of *X* percent imposed on just one product is *profitable* to the hypothetical monopolist if and only if the following condition holds:[43]

$$\delta_M \;>\; X/m \qquad\qquad\qquad (3)$$

where m denotes the average margin of the other products in the candidate market (i.e., the average margin of all the products in the candidate market excluding the product

[41] Farrell and Shapiro (2008), *supra* note 38 at 6. *See* also Guidelines, *supra* note 7 at §4.1.3.

[42] Farrell and Shapiro (2008), *supra* note 38 at n. 17.

[43] Øystein Daljord, Lars Sørgard and Øyvind Thomassen, *The SSNIP Test and Market Definition with the Aggregate Diversion Ratio: A Reply to Katz and Shapiro*, 4 JOURNAL OF COMPETITION LAW & ECONOMICS 263 (2008). *See* also James Langenfeld and Wenqing Li, *Asymmetric Price Increase in Critical Loss Analysis: Surreply to Daljord, Sørgard and Thomassen*, 10 JOURNAL OF COMPETITION LAW & ECONOMICS 769 (2014).

subject to the SSNIP).[44]

If condition (3) is satisfied, the hypothetical monopolist would find it profitable to raise the price of that product by X percent (so that the profit-maximizing price increase would be at least $X/2$ percent) and the candidate market would satisfy the HMT. As noted earlier, this single-product SSNIP test might be used in asymmetric situations where a single-product SSNIP is apt to be more profitable than is a uniform SSNIP.[45]

The HMT is very often implemented using a uniform SSNIP test, and only seldom using a single-product SSNIP test. For example, in the recent Sysco/U.S. Foods matter, the FTC's expert witness, Dr. Mark Israel, defined a relatively narrow market using the uniform SSNIP test. The defendants' experts, Dr. Timothy Bresnahan and Dr. Jerry Hausman, testified that Dr. Israel's estimate of the aggregate diversion ratio (δ_M) was unreliable and his estimate of the margin (m) was incorrect.[46]

Dr. Hausman also testified that the uniform SSNIP test (condition (2)) used by Dr. Israel was incorrect and led to an overly narrow market definition. He further opined that the single-product SSNIP test (condition (3)) was the more appropriate test in an asymmetric market like food distribution, which involves suppliers with different costs,

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[44] Condition (3) implicitly assumes that the products in the candidate market have equal prices. If prices are asymmetric, then condition (3) becomes $\delta_M > (X/m)(P/p)$, where P denotes the price of the product subject to the SSNIP and p denotes the average price of the other products in the candidate market. This condition can be rewritten as: GUPPI $= \delta_M m\, p/P > X$. That is, the candidate market is a relevant antitrust market if a merger of all the products in the candidate market would lead to a GUPPI of at least X percent. For a discussion of the relationship between unilateral effects (GUPPI) and market definition (SSNIP test), *see* Serge Moresi, *The Use of Upward Price Pressure Indices in Merger Analysis*. ANTITRUST SOURCE (Feb. 2010).

[45] Farrell and Shapiro (2008), *supra* note 38 at n. 17. *See* also Daljord and Sørgard, *supra* note 36.

[46] CRA and Dr. Moresi provided support for Dr. Hausman's testimony.

different types of customers, and a different mix of products. Using the proper margins

and the single-product SSNIP test, Dr. Hausman calculated that the critical value of the

aggregate diversion ratio (i.e., recapture percentage) exceeded 100 percent, which would

make it impossible for the HMT to be satisfied. This is because the aggregate diversion

ratio logically *cannot* exceed 100%. Thus, Dr. Hausman concluded that the relevant

product market was broader than the market defined by Dr. Israel.[47]

4. Market Definition with Multi-Product Firms

The HMT involves relatively simple calculations when firms sell a single product.

The test becomes more complex when firms sell multiple products either inside or

outside the candidate market (or both), as shown in Moresi, Salop and Woodbury

("MSW).[48] The 2010 Merger Guidelines recognize the complexity created by multi-

product firms and state that, under certain conditions, the U.S. Agencies might use a

different SSNIP test, which we refer to as the "hypothetical cartel test:"

> If the pricing incentives of the firms supplying the products in the candidate
> market differ substantially from those of the *hypothetical monopolist*, for
> reasons other than the latter's control over a larger group of substitutes, the

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[47] *See FTC v. Sysco Corp.*, F. Supp. 3d, 2015 WL 3958568 (D.D.C. June 23, 2015) at pp. 33-41
(https://www.ftc.gov/system/files/documents/cases/150623syscomemo.pdf). "Having weighed the
competing expert testimonies and considered them in light of the evidentiary record as a whole, the court
finds Dr. Israel's aggregate diversion analysis and conclusion to be more persuasive than that advanced by
Defendants' expert, Dr. Hausman." *Id*. at p. 39. In a recent case in Greece, the Hellenic Competition
Commission used the single-product SSNIP test to define the relevant market for production and
distribution of white milk in Greece, but it also checked that the uniform SSNIP test was satisfied as well.
See Hellenic Competition Commission, Decision No 515/VI/2011, Appendix 1 at pp. 117-125. *See* also
Panagiotis Fotis, *Economic Tools for Merger Appraisal: A Theoretical and Empirical Standpoint*, 3
JOURNAL OF REVIEWS ON GLOBAL ECONOMICS 24 (2014).

[48] Serge Moresi, Steven C. Salop and John R. Woodbury, *Implementing the Hypothetical Monopolist
SSNIP Test With Multi-Product Firms*, ANTITRUST SOURCE (Feb. 2008).

Agencies may instead employ the concept of a *hypothetical profit-maximizing cartel* comprised of the firms (with all their products) that sell the products in the candidate market. This approach is most likely to be appropriate if the merging firms sell products outside the candidate market that significantly affect their pricing incentives for products in the candidate market.[49]

In the remainder of this chapter, we will describe the implementation of this *hypothetical cartel test* ("HCT"). We show its necessity by comparing it to the outcome of applying the HMT for multi-product firms, as described in MSW. The HMT assumes that the hypothetical monopolist owns *only* the products inside the candidate market. The HMT thus is most likely to be appropriate when firms' pricing incentives in the candidate market are not substantially affected by their sales outside the candidate market.

In this analysis, we apply the modern critical loss approach and take into account the information on the price sensitivity of demand that can be inferred from the firms' profit-maximizing behavior pre-merger. That information is useful for the analysis of the incentives of a hypothetical cartel as well as a hypothetical monopolist. In order to obtain such information, we continue to assume that firms sell differentiated products and engage in Bertrand price competition pre-merger (i.e., each firm takes the prices of other firms as given and unilaterally sets the prices of its products to maximize its total profits.) This analysis shows that when firms sell multiple *substitute* products, the relevant market under the HCT tends to be narrower than under the HMT. When firms sell multiple *complementary* products, the relevant market under the HCT analogously tends to be

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[49] Guidelines, *supra* note 7 at n. 4 (italics added). That footnote goes on to state: "This could occur, for example, if the candidate market is one for durable equipment and the firms selling that equipment derive substantial net revenues from selling spare parts and service for that equipment." This reference focuses on the case where the products that the firms sell outside the candidate market are *complements* for the products inside the candidate market. However, the argument applies to substitute products as well. Our discussion will begin with the case of substitutes.

broader than under the HMT. In Appendix B, we provide a more general, technical description of the two tests.

a. Basic Framework

It is common for firms to produce multiple differentiated products that are substitutes for at least some customers. For example, GM produces Cadillacs and Buicks, and Verizon offers a variety of mobile phone models and calling plans. Industrial products like microprocessors and certain chemicals also fit this paradigm. Where firms produce multiple differentiated products, the proper analysis of market definition should take this structure into account.

For example, consider the following simple market structure. Suppose that three firms each sell brands of three products that are substitutable to some degree for some customers. Suppose that the three sellers are The Coca-Cola Company, PepsiCo and Dr. Pepper Snapple Group, and each seller produces a brand of each of three products, Cola, Lemon-Lime and Orange. Coca-Cola sells Coke, Sprite and Fanta. PepsiCo sells Pepsi, Mist Twst and Tropicana Twister. And Dr Pepper Snapple Group sells RC Cola, 7UP and Sunkist. Assume that all nine of the brands are substitutes to some degree.

Consider next a proposed merger between two of these firms. The agencies might define the market broadly as all carbonated soft drinks and include the sales of all three flavors. However, in some cases, the agency might want to determine whether a narrower set of products constitutes a distinct relevant market. For example, the agency might want to test whether Cola flavor by itself constitutes a narrow relevant product market separate from the other flavors.

b. *SSNIP Tests: Hypothetical Monopolist and Hypothetical Cartel*

In this illustrative example, Cola products constitute a relevant product market if a hypothetical Cola monopolist (or hypothetical Cola cartel) would have the incentive to raise the prices of Coke, Pepsi and RC Cola by at least a SSNIP. The difference between the HMT and HCT involves how the non-Cola products are treated. Under the HMT, the hypothetical monopolist is assumed to produce and sell *only* the Cola products and does not sell any Lemon-Lime or Orange products. In contrast, under the HCT, the hypothetical cartel is assumed to produce and sell both the Cola and non-Cola products of the three companies.

Under both tests, suppose that only the Cola prices are raised, not the prices of the non-Cola products. In this situation, a SSNIP on Cola products likely would be more profitable to a hypothetical cartel than it would be to a hypothetical monopolist, because the non-Cola products would benefit from a SSNIP imposed on Cola products. It follows that when firms sell products outside the candidate market that are substitutes for the products that they sell inside the candidate market, the HCT tends to define narrower markets than does the HMT.

c. *Pre-Merger Competition and Prices*

As discussed in Section 2.b, the pricing incentives of the hypothetical monopolist (or hypothetical cartel) are related to the factors that determine the profit-maximizing prices pre-merger. Therefore, to determine the profitability of a price increase by a hypothetical monopolist or cartel, it is necessary to analyze how pre-merger prices are affected by the presence of multi-product firms that engage in Bertrand price competition.

In setting its pre-merger prices, each firm rationally would take into account that the brands that it sells are substitutes to some degree. For example, in the pre-merger world, when Coca-Cola evaluates the profitability of raising the price of Coke, it takes into account the fact that raising price would reduce the sales of Coke by a certain amount, but it also takes into account the fact that a fraction of the lost sales of Coke would be recaptured by higher sales of Sprite and Fanta. Those recaptured sales are a factor that tends to elevate the profit-maximizing pre-merger price of Coke above the price that would prevail if Coca-Cola were not also selling Sprite and Fanta.[50] In the pre-merger world, Coca-Cola engages in a similar profitability calculus in setting the prices of its Sprite and Fanta brands, as do the other two firms in setting the prices of their brands, all of which leads to the pre-merger equilibrium.

The pricing incentives of the hypothetical monopolist or hypothetical cartel then would be compared to this initial price equilibrium. In making this comparison, the fact that pre-merger prices are higher than if the firms were selling only a single product is key. Intuitively, a multi-product firm has a unilateral incentive to set higher prices than would a single-product firm.[51] This is because the customers who switch to the other products sold by that same firm are *not* lost customers.

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[50] Coca-Cola's profit-maximizing price of Coke balances the gains and losses in profits when raising the price of Coke, taking into account both the reduction of Coke sales volume (which depends on Coke's own-price elasticity) and the diversion of sales to its Sprite and Fanta brands (which depends on the cross-price elasticities of Sprite and Fanta with respect to the price of Coke). The profit-maximizing price of a firm's brand also depends on the margins of the other brands sold by the firm. *See* Appendix B for further discussion.

[51] In this unilateral effects framework, we assume that these higher prices are the result of multi-product profit-maximization, not coordinated interaction among the firms. Therefore, there is no *Cellophane Fallacy* issue raised. This is consistent with the approach of the Guidelines when there is no coordinated interaction. Guidelines, *supra* note 7 at §4.1.2 (use prevailing prices unless pre-merger circumstances are strongly suggestive of coordinated interaction).

d. *Implications for the HMT and HCT*

We analyze the HMT first. The HMT assumes that the hypothetical monopolist only produces and sells the products in the candidate market. To illustrate, consider the incentives of a hypothetical Cola-only monopolist—that is, a hypothetical monopolist that does not sell the other flavors—to raise the price of Coke, *relative to the pre-merger incentives of Coca-Cola*. Suppose that the Cola-only hypothetical monopolist would raise the price of Coke, while holding the prices of Pepsi and RC Cola constant. On the one hand, the Cola monopolist would have an incentive to *raise* the price of Coke because it would recapture sales that Coke lost to Pepsi and RC Cola. This is standard upward pricing pressure flowing from the recapture of diverted sales. On the other hand, the Cola monopolist would have an incentive to *lower* the price of Coke below the price that was charged by Coca-Cola because the hypothetical monopolist would not recapture the sales that Coke lost to Sprite and Fanta. Because of the impact of substitution to the other flavors, the hypothetical Cola monopolist might actually have an incentive to lower the price of Coke, relative to the price set by Coca-Cola. This would occur if the diversion to Sprite and Fanta were to exceed the diversion to Pepsi and RC Cola.

The incentives of the hypothetical Cola-only monopolist to raise the price of Pepsi and RC Cola are analogous. The monopolist would lack incentives to raise the prices of these Cola brands if the diversion from the Cola brand to the Lemon-Lime and Orange brands of the same company exceed the diversion from the Cola brand to the Cola brands produced by the other two companies.

This is the potential flaw in the HMT. Suppose the proposed merger involved Coca-Cola acquiring Dr. Pepper Snapple Group. The HMT might fail to identify a Cola-

only market in a situation where a real-world merger might lead to higher Cola prices. The real-world merged firm, unlike the hypothetical monopolist, would not be a Cola-only producer. It would also own the Lemon-Lime and Orange brands. The real-world merged firm would take into account the recapture of diversion to those other flavors. As a result, the real-world merged firm would have the incentive to raise the price of Cola products, even if the Cola-only hypothetical monopolist would not.

The HCT is designed to prevent this potential problem. The hypothetical cartel would be a firm that produces and sells all three flavors of the companies but contemplates raising the prices only of the Cola brands. This hypothetical cartel would take into account the diversion from Coke to Sprite and Fanta, as Coca-Cola does before the merger, and the diversion from Pepsi and RC Cola to the Lemon-Lime and Orange brands of each of those companies, as those companies do in the pre-merger world. Moreover, the hypothetical cartel also would take into account the diversion from Coke to the other two Cola brands and to the Lemon-Lime and Orange brands that Coca-Cola does not own. As a result, the hypothetical cartel would have greater incentives to raise the price of Cola brands than would the hypothetical Cola-only monopolist. More importantly, the incentives of the hypothetical cartel would more closely reflect the incentives of the real-world merged firm. Thus, the HCT would be the preferred test.

In this example, the HCT is more likely to find that Cola is a relevant market than would the HMT. The example leads to a general statement about relevant markets. If some or all of the firms in the candidate market also sell substitutes outside the candidate market (and none sells complements), then the candidate market is more likely to be a

relevant antitrust market under the HCT than under the HMT. Thus, under these conditions, the HCT tends to permit narrower markets to be defined.

This result is reversed if the products sold outside the candidate market are complements instead of substitutes for the products inside the candidate market. If the outside products are complements, then the hypothetical monopolist would have a greater incentive to raise prices than would a hypothetical cartel. For example, suppose that Coca-Cola and PepsiCo both sell Cola products and pretzels. Suppose that pretzels are complements for Cola drinks. In that case, each individual firm would charge a somewhat lower price pre-merger in order to increase sales of its complementary pretzel brand. A hypothetical Cola-only monopolist would not have that price-lowering incentive, while a hypothetical cartel would. As a result, the HCT would tend to lead to broader markets than the HMT when the other "outside" products sold by the firms are complements.

e. The HCT for Symmetric Firms

Appendix B provides general formulas for conditions (2) and (3) for the *uniform SSNIP test* and the *single-product SSNIP test*, respectively, when firms sell multiple products, some inside and some outside the candidate market. It provides those formulas for both the HMT and the HCT. When there are symmetric firms, each selling the same set of products and price increases are uniform, the formulas are much simpler.[52]

Under the HCT, a candidate market comprises a relevant market with respect to a uniform SSNIP of X%, if and only if:

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[52] Generally, the hypothetical monopolist would maximize profits by imposing different SSNIP*s* on different brands. The analysis of such non-uniform SSNIP*s* would require use of a simulation model.

$$\delta_M > \frac{X + (\delta_F - \delta_N)m}{X + m} \tag{4}$$

where δ_F denotes the total diversion ratio from any product in the candidate market to all the other products sold by the same firm (inside as well as outside the candidate market), and δ_N denotes the aggregate diversion ratio from any product in the candidate market to all the products outside the candidate market sold by the hypothetical cartel. (As before, δ_M denotes the aggregate diversion ratio from any product in the candidate market to all the other products in the candidate market, and m denotes the percentage margin.) For the HMT, one simply sets $\delta_N = 0$ in condition (4).

For example, suppose that the percentage profit-margin is 30% and the uniform SSNIP is 10%. If there were only single-product firms, in which case $\delta_F = \delta_N = 0$, then the HMT and HCT tests would be identical. In that situation, the candidate market would comprise a relevant market if condition (2) is satisfied, that is, if the aggregate diversion ratio δ_M within the candidate market exceeds 25% (i.e., 10% ÷ {30% + 10%}).

However, if instead there were multi-product firms and the diversion inside a firm (from one of its products to its other products) were significant, say, $\delta_F = 20\%$, then the critical value of δ_M under the HMT would rise substantially. In the example, the candidate market would constitute a relevant market under the HMT if δ_M exceeds 40% (i.e., {20% x 30% + 10%} ÷ {30% + 10%}). This increase from 25% to 40% likely would be determinative in many cases. Finally, if the firms inside the candidate market also were selling (imperfect) substitutes outside the candidate market, and the aggregate diversion ratio from an "inside product" to those products was, say, $\delta_N = 10\%$, then the

critical value of δ_M under the HMT would not change but would be 32.5% under the

HCT (i.e., {(20% - 10%) x 30% + 10%} ÷ {30% + 10%}).

Thus, taking into account the fact that firms sell multiple products is essential to implementing both the HMT and HCT for candidate markets with multi-product firms based on an assumption that firms are maximizing pre-merger profits.

5. **Conclusions**

The HMT set out in the 2010 Merger Guidelines is deceptively simple. As we have shown here, the implementation of the test can become more complicated when the details of the industry structure are taken into account in defining the relevant market. However, taking these factors into account can lead to a more refined market definition that better reflects the competitive concerns potentially raised by a proposed merger.

Appendix A: Derivation of the "original" Critical Loss Test

We consider a candidate market where firms are producing a total of Q units at a constant average variable cost of C, and charging a price of P to customers. The firms' total variable profits are equal to $(P - C)Q$. Consider now a hypothetical monopolist ("HM") that would face the same demand and costs as the original firms. The HM's profits also would be $(P - C)Q$ if the HM were to set the same price as the original firms. If instead the HM were to raise the price of all the products in the candidate market by X percent, from the original price P to a price of $(1 + X)P$, the HM would experience a loss in sales units of L percent, from the original amount Q to the amount $(1 - L)Q$. The HM's profits following the price increase thus would be equal to $((1 + X)P - C)(1 - L)Q$.

For the price increase to be profitable to the HM, the profits following the price increase must be larger than the initial profits:

$$\big((1 + X)P - C\big)(1 - L)Q \ > \ (P - C)Q \tag{A1}$$

or equivalently (after rearranging terms):

$$(P - C)LQ \ < \ XP(1 - L)Q \tag{A2}$$

This profitability condition says that, for the candidate market to be a relevant market, the profits lost by the HM from the LQ units of lost sales, i.e., $(P - C)LQ$, must be smaller than the profits gained by the HM from the price increase on the remaining $(1 - L)Q$ units, i.e., $XP(1 - L)Q$. This condition can be simplified and rewritten as equation (1) in Section 2.a of this chapter.

Appendix B: Market Definition Tests with Multi-Product Firms

We consider a candidate market where one or more firms may produce multiple products, including products that are outside the candidate market. The 2010 Merger Guidelines introduced a variant of the Hypothetical Monopolist Test ("HMT") that we have called the Hypothetical Cartel Test ("HCT").[53] This technical appendix analyzes the HCT formally and compares it to the HMT analyzed in Moresi, Salop and Woodbury (MSW).[54]

1. Profit Maximization by Multi-Product Firms

Let $M = \{1, 2, \ldots, M\}$ denote the set of products inside the candidate antitrust market, and let $N = \{M+1, M+2, \ldots, M+N\}$ denote the products outside the candidate market that belong to the firms that own the M products inside the candidate market. For each pair of products (j, k) where $j \in M$ and $k \in M \cup N$, let the indicator ω_{jk} be either 1 or 0 depending on whether the two products belong to the same firm or not, respectively. The first-order conditions for the M products in the candidate market are:

$$m_j \; - \; \sum_{k \neq j}^{M+N} \omega_{jk} \delta_{jk} m_k \frac{p_k}{p_j} \;\; = \;\; \frac{1}{\eta_j} \tag{B1}$$

where $m_j = (p_j - c_j)/p_j$ denotes product j's margin, $\delta_{jk} = -(\partial D_k/\partial p_j)/(\partial D_j/\partial p_j)$ denotes the diversion ratio from product j to product k, and $\eta_j = -(\partial D_j/\partial p_j)(p_j/D_j)$ denotes product j's own-price elasticity of demand (in absolute value).

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[53] *See* Guidelines, *supra* note 7 at n. 4.

[54] *Supra* note 48.

2. The Hypothetical Cartel Test and the Hypothetical Monopolist Test

We consider a hypothetical cartel ("HC") which owns all the M products in the candidate market as well as all the N products that the firms own outside the candidate market. We will also consider a hypothetical monopolist ("HM") which owns only the M products in the candidate market. Let X denote the uniform percentage price increase imposed on each of the M products in the candidate market, and assume that the prices of all the other products outside the candidate market remain constant. Assuming that demand and cost functions are linear, one can evaluate the effect of each individual price increase on the HC's total profits, and then add up those effects.

For each product $j \in M$, the increase in price (from p_j to $(1 + X)p_j$) has three effects on the HC's profit:

- It reduces the sales volume of product j (from q_j to $(1 - X\eta_j)q_j$) and thus tends to reduce the profits of product j by the amount:

$$X\eta_j q_j m_j p_j \tag{B2}$$

- It increases the margin (from $m_j p_j$ to $(m_j + X)p_j$) earned on the remaining volume of product j that is still being sold, and thus tends to increase the profits of product j by the amount:

$$(1 - X\eta_j)q_j X p_j \tag{B3}$$

- It increases the sales volume of each other product k of the HC (from q_k to $q_k + X\eta_j q_j \delta_{jk}$), and thus tends to increase the profits of those other products by the amount:

$$X\eta_j q_j \left(\sum_{\substack{k \neq j}}^{M} \delta_{jk}(m_k + X)p_k + \sum_{k=M+1}^{M+N} \delta_{jk}m_k p_k \right) \qquad \text{(B4)}$$

The only difference between the above HC analysis and the corresponding HM analysis in MSW is the presence of the second summation term in equation (B4).[55] It follows that the HCT tends to define a narrower (broader) relevant market than the HMT if the products outside the candidate market are substitutes (complements) for the products inside the candidate market *ceteris paribus*.[56]

Based on equations (B2), (B3) and (B4), the net effect of increasing the price of product j on the total profits of the $M + N$ products of the HC can be written as follows:

$$X\eta_j q_j p_j \left(-m_j - X + \frac{1}{\eta_j} + \sum_{\substack{k \neq j}}^{M} \delta_{jk}(m_k + X)\frac{p_k}{p_j} + \sum_{k=M+1}^{M+N} \delta_{jk}m_k \frac{p_k}{p_j} \right) \qquad \text{(B5)}$$

Using the convention $\delta_{jj} = -1$, the total effect on the HC's profit of increasing the prices of all the products in the candidate market is equal to:

$$X \sum_{j=1}^{M} q_j p_j \left(1 + \eta_j \sum_{k=1}^{M} \delta_{jk}(m_k + X)\frac{p_k}{p_j} + \eta_j \sum_{k=M+1}^{M+N} \delta_{jk}m_k \frac{p_k}{p_j} \right) \qquad \text{(B6)}$$

Given the assumption that demands and costs are linear, if a price increase of Z is neither profitable nor unprofitable, then the profit-maximizing price increase is $Z/2$.

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[55] Compare equations (B1)-(B4) with equation (1)-(4) in MSW, *supra* note 48.

[56] The diversion ratio from a product inside the candidate market to a product outside the candidate market is positive (negative) if the latter is a substitute (complement) for the former. (The diversion ratios between two products inside the candidate market are positive since the products inside the candidate market are substitutes.)

It thus follows from equation (B6) that the profit-maximizing price increase is:

$$X^* = -\frac{\sum_{j=1}^{M} q_j \eta_j \sum_{k=1}^{M+N} \delta_{jk} m_k p_k + \sum_{j=1}^{M} q_j p_j}{2 \sum_{j=1}^{M} q_j \eta_j \sum_{k=1}^{M} \delta_{jk} p_k} \tag{B7}$$

The profit-maximizing price increase for the HM is given by equation (B7) with $N = 0$.

If one assumes that the HC would raise only the price of product j (and would leave the other prices unchanged), then the profit-maximizing price increase would be equal to (using equation (B5) with $m_k + X = m_k$, and then equation (B1)):

$$X_j^* = \frac{1}{2}\left(-m_j + \frac{1}{\eta_j} + \sum_{k \neq j}^{M+N} \delta_{jk} m_k \frac{p_k}{p_j}\right) = \frac{1}{2} \sum_{k \neq j}^{M+N} (1 - \omega_{jk})\delta_{jk} m_k \frac{p_k}{p_j} \tag{B8}$$

For the HM, one sets $N = 0$ in equation (B5) and obtains:

$$X_j^* = \frac{1}{2}\left(\sum_{k \neq j}^{M} \delta_{jk} m_k \frac{p_k}{p_j} - \sum_{k \neq j}^{M+N} \omega_{jk} \delta_{jk} m_k \frac{p_k}{p_j}\right) \tag{B8$'$}$$

These expressions show that the profit-maximizing (single-product) price increase for either the HC or the HM tends to be lower when firms sell multiple products inside the candidate market than when they sell a single product inside the candidate market *ceteris paribus*. However, when firms also sell products outside the candidate market, that fact tends to increase (reduce) the profit-maximizing price increase for the HC if those products are substitutes (complements) for the products in the candidate market, while the opposite is true for the HM.

3. Analysis of the Symmetric Case

In the situation where there is symmetry across products and across firms, equations (B1) and (B6) reduce to:

$$(1 - \delta_F)m = \frac{1}{\eta} \tag{B9}$$

$$XMpq(1 - \eta(1 - \delta_M)(m + X) + \eta\delta_N m) \tag{B10}$$

where δ_F denotes the total diversion ratio from any product in the candidate market to all the other products sold by the same firm (inside as well as outside the candidate market), δ_M denotes the aggregate diversion ratio from any product in the candidate market to all the other products in the candidate market, and δ_N denotes the aggregate diversion ratio from any product in the candidate market to all the products outside the candidate market owned by the HC. For the HMT, one simply sets $\delta_N = 0$ in equation (B10).

Using equation (B9) to substitute for η, equation (B10) is positive (and thus the price increase X is profitable for the HC) if and only if:

$$(1 - \delta_F)m > (1 - \delta_M)(m + X) - \delta_N m \tag{B11}$$

In the special case of single-product firms (i.e., $\delta_F = \delta_N = 0$), equation (B11) can be written as $\delta_M > X/(m + X)$. This is the standard formula for the profitability of a *uniform SSNIP* discussed in Section 2.b of this chapter (see equation (2)).

The more general formula implied by equation (B11) is:

$$\delta_M > \frac{X + (\delta_F - \delta_N)m}{X + m} \tag{B12}$$

Thus, when there are multi-product firms (i.e., $\delta_F > 0$) that also sell products outside the candidate market (i.e., $\delta_N \neq 0$), a uniform price increase of X percent is less (more) likely to be profitable *ceteris paribus*–and thus the candidate market is less (more) likely to be a relevant market—if the diversion ratio to the products within a firm is higher (lower) than the diversion ratio to the products outside the candidate market (owned by the HC), i.e., if $\delta_F > (<)\delta_N$. This also can be seen from the HC's profit-maximizing price increase, i.e., Equation (B7), which in the symmetric case reduces to:

$$X^* = \frac{(\delta_M + \delta_N - \delta_F)m}{2(1 - \delta_M)} \tag{B13}$$

For the HMT one sets $\delta_N = 0$ in equations B(12) and B(13), as noted earlier.

For the single-product profit-maximizing price increase, suppose that margins and prices are symmetric across all products and firms, but sales volumes, diversion ratios, and own-price elasticities are not symmetric. Equations (B1) and (B5) then reduce to:

$$\left(1 - \delta_{jF}\right)m = \frac{1}{\eta_j} \tag{B14}$$

$$X_j \eta_j p q_j \left(-m - X_j + (\delta_{jM} + \delta_{jN})m + 1/\eta_j\right) \tag{B15}$$

where δ_{jF} denotes the total diversion ratio from product j to all the other products sold by the same firm (inside as well as outside the candidate market), δ_{jM} denotes the aggregate diversion ratio from product j to all the other products in the candidate market, and δ_{jN} denotes the aggregate diversion ratio from product j to all the products outside the candidate market (owned by the HC).

Using equation (B14) to substitute for η_j, equation (B15) is positive (and thus the price increase X_j is profitable for the HC) if and only if:

$$X_j < (\delta_{jM} + \delta_{jN} - \delta_{jF})m \qquad \text{(B16)}$$

In the special case with single-product firms (i.e., $\delta_{jF} = \delta_{jN} = 0$), equation (B16) can be written as $\delta_{jM} > X_j/m$. This is the standard formula for the profitability of a *single-product SSNIP* discussed in Section 2.b of this chapter (see equation (3)).

The profit-maximizing single-product price increase is given by:

$$X_j^* = \frac{(\delta_{jM} + \delta_{jN} - \delta_{jF})m}{2} \qquad \text{(B17)}$$