Unilateral Effects of Mergers with General Linear Demand

Jerry Hausman[a], Serge Moresi[b] and Mark Rainey[c]
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Abstract

We derive the formula for the unilateral price effects of mergers of two products with linear demand in the general asymmetric situation. These predicted price changes use the same information on pre-merger prices, marginal costs, quantities, and diversion ratios that is required to calculate upward pricing pressure in the 2010 Horizontal Merger Guidelines. We also show that in many cases only one diversion ratio needs to be estimated and provide the formula for the price effects in that situation.

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[a] Corresponding author. Department of Economics, MIT, 50 Memorial Drive, Cambridge MA 02142. Tel: +1 617 253 3644; fax: +1 617 253 1330. Email: jhausman@mit.edu

[b] Charles River Associates, 1201 F Street NW, Suite 700, Washington DC 20004. Email: smoresi@crai.com. I thank Steve Salop and Carl Shapiro for helpful comments and discussions.

[c] Greylock McKinnon Associates, One Memorial Drive, Suite 1410, Cambridge MA 02142. Email: mrainey@gma-us.com

The 2010 Department of Justice and Federal Trade Commission Horizontal Merger Guidelines apply a measure of "upward pricing pressure" to consider the effect of a merger in a differentiated products industry with two merging firms, each of which produces a single product.[1] The upward pricing pressure measure leads to a "gross upward pricing pressure index" (GUPPI) which is defined for product 1 as:[2]

$$GUPPI_1 = D_{12} \frac{p_2 - c_2}{p_1} \tag{1}$$

where p_1 and p_2 are the pre-merger prices of the two merging products, D_{12} is the diversion ratio from product 1 to product 2 when the price of product 1 increases,[3] and c_2 is the marginal cost of product 2.[4] Thus, GUPPI measures "the value of diverted sales … in proportion to the lost revenues attributable to the reduction in unit sales" when the price of product 1 increases.[5]

Given the estimates of the cross price elasticities and the own price elasticities (and hence of the diversion ratios), predicted price changes follow under a Bertrand-Nash assumption and an assumed shape of the demand curves. Indeed, it is easy to demonstrate that, with linear demand and constant marginal costs, in the "symmetric case" of equal diversion ratios ($D_{12} = D_{21} = D$)

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[1]   Mergers are "horizontal" when the merging firms sell products that are substitutes for one another.

[2]   Salop and Moresi (2009a); Farrell and Shapiro (2010a).

[3]   It is straightforward to demonstrate that the diversion ratio from product 1 to product 2 is equal to the ratio of the cross price elasticity of product 2 (with respect to the price of product 1) divided by the own price elasticity of product 1 multiplied by the ratio of unit sales of product 2 divided by the unit sales of product 1:

$$D_{12} \equiv -\frac{\partial Q_2 / \partial p_1}{\partial Q_1 / \partial p_1} = -\frac{e_{21}}{e_{11}} \frac{Q_2}{Q_1}$$

Under the assumption of a single product firm, as used in the Merger Guidelines, the own price elasticity is equal to the negative inverse of the price cost margin, i.e., $m_1 = -1/e_{11}$ where $m_1 = (p_1 - c_1)/p_1$ is the price-cost margin and e_{11} is the own price elasticity of demand for product 1. Thus, an estimate of the diversion ratio implies an estimate of the cross price elasticity, which is the fundamental economic measure of competition between two products.

[4]   Marginal cost may change post-merger if efficiencies lead to a lower marginal cost. In the Merger Guidelines, efficiencies are analyzed separately from the GUPPI, and thus we ignore efficiencies in this note. It is straightforward to extend the analysis and account for post-merger marginal cost reductions.

[5]   U.S. Department of Justice and Federal Trade Commission (2010, p. 21).

and equal marginal costs, prices and shares, the unilateral[6] profit-maximizing price increase post-merger is equal to $0.5 * GUPPI /(1-D)$.[7] However, the assumptions of the symmetric case are often unrealistic for the differentiated products situation. We demonstrate how to calculate the price increases for a linear demand system using the same information required to calculate GUPPI for the two products being analyzed.

Proposition 1. Under linear demand and constant marginal cost, the unilateral profit-maximizing price increases of the two merging products (holding the prices and characteristics of the other products constant), are given by

$$\frac{\Delta p_1}{p_1} = \frac{2D_{12}\frac{p_2-c_2}{p_1} + D_{12}D_{21}\frac{p_1-c_1}{p_1} + \frac{(p_1-c_1)^2}{(p_2-c_2)p_1}\frac{Q_2}{Q_1}(D_{21})^2}{4 - 2D_{12}D_{21} - \frac{p_2-c_2}{p_1-c_1}\frac{Q_1}{Q_2}(D_{12})^2 - \frac{p_1-c_1}{p_2-c_2}\frac{Q_2}{Q_1}(D_{21})^2} \tag{2}$$

and an analogous equation for the percentage increase in the price of product 2. Here, Q_i is the volume of product i and all variables are evaluated at the pre-merger equilibrium.

Proof. In the pre-merger equilibrium, each firm i chooses its price p_i to maximize $(p_i - c_i)Q_i(p_i, P_{-i})$, where $Q_i(p_i, P_{-i})$ is the demand function for product i and P_{-i} denotes the prices of firm i's competitors. The first-order condition characterizing the pre-merger equilibrium is[8]

$$\frac{\partial Q_i(p_i^0, P_{-i}^0)}{\partial p_i} = -\frac{Q_i(p_i^0, P_{-i}^0)}{p_i^0 - c_i} \equiv -\frac{Q_i^0}{p_i^0 - c_i} \tag{3}$$

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[6]   Unilateral effects analysis evaluates the merging firms' incentives to raise price post-merger assuming that the non-merging firms do not adjust their prices or reposition their products. This assumption has offsetting effects on the calculated price changes. The assumption of no price response will tend to understate the price changes of the merging firms, but the assumption of no product repositioning will tend to overstate the price changes of the merging firms. These assumptions are the assumptions used in the Merger Guidelines to calculate GUPPI.

[7]   Salop and Moresi (2009b).

[8]   We use a superscript "0" to denote the pre-merger value of a variable.

The merged firm sets p_1 and p_2 to maximize $(p_1 - c_1)Q_1 + (p_2 - c_2)Q_2$. The first-order condition with respect to p_1 is

$$(p_1 - c_1)\frac{\partial Q_1}{\partial p_1} + (p_2 - c_2)\frac{\partial Q_2}{\partial p_1} = -Q_1 \tag{4}$$

Using the definition of the diversion ratio (see footnote 4), equation (4) can be rewritten

$$p_1 - c_1 - (p_2 - c_2)D_{12} = -\frac{Q_1}{\partial Q_1 / \partial p_1} \tag{5}$$

Since demand is linear, $\partial Q_1 / \partial p_1$ and D_{12} are constants that do not depend on price. Thus, using equation (3) we can replace $\partial Q_1 / \partial p_1$ with $-Q_1^0 / (p_1^0 - c_1)$. Making this substitution and decomposing p_i into $p_i^0 + \Delta p_i$, equation (5) can be rewritten

$$p_1^0 - c_1 + \Delta p_1 - (p_2^0 - c_2 + \Delta p_2)D_{12} = \frac{(p_1^0 - c_1)Q_1}{Q_1^0} \tag{6}$$

Dividing both sides of equation (6) by p_1^0 leads to

$$\frac{\Delta p_1}{p_1^0} - \frac{p_2^0 - c_2 + \Delta p_2}{p_1^0}D_{12} = \frac{p_1^0 - c_1}{p_1^0 Q_1^0}(Q_1 - Q_1^0) \tag{7}$$

Due to linearity, the bracketed term on the right-hand side is equal to

$$Q_1 - Q_1^0 = \frac{\partial Q_1}{\partial p_1}\Delta p_1 + \frac{\partial Q_1}{\partial p_2}\Delta p_2 \tag{8}$$

Using equation (3) and the definition of the diversion ratio, we rewrite equation (8) as

$$Q_1 - Q_1^0 = -\frac{p_1^0 Q_1^0}{p_1^0 - c_1}\frac{\Delta p_1}{p_1^0} + \frac{p_2^0 Q_2^0}{p_2^0 - c_2}D_{21}\frac{\Delta p_2}{p_2^0} \tag{9}$$

Substituting into equation (7) and rearranging terms, we obtain

$$2\frac{\Delta p_1}{p_1^0} - \left(\frac{p_2^0}{p_1^0}D_{12} + \frac{p_1^0 - c_1}{p_2^0 - c_2}\frac{p_2^0 Q_2^0}{p_1^0 Q_1^0}D_{21}\right)\frac{\Delta p_2}{p_2^0} = \frac{p_2^0 - c_2}{p_1^0}D_{12} \tag{10}$$

Combining this equation with the analogous first-order condition with respect to p_2, Cramer's rule leads to equation (2). ∎

Proposition 1 holds under general linear demand. To calculate the profit-maximizing price increases given by equation (2), one needs information about the diversion ratios, the prices, the marginal costs and the volumes of the two merging products. This is essentially the same information as that needed to calculate the GUPPIs.

It is useful to note that, in many cases, only one diversion ratio needs to be estimated because it is reasonable to assume that the cross-price derivatives of the demand functions are equal or approximately equal (i.e., $\partial Q_2 / \partial p_1 \cong \partial Q_1 / \partial p_2$). First, consider the case where the merging products are intermediate goods used as inputs by downstream firms. Cost minimization implies that the cross-price derivatives of the conditional factor demands are equal. Thus, the cross-price derivatives of the unconditional factor demands will be equal if the downstream firms have constant marginal costs.[9] Even without constant marginal costs, the cross-price derivatives will be approximately equal if the inputs are a small proportion of variable costs. Second, for consumer goods, Slutsky symmetry implies that the same relationship holds apart from income effects, which are typically (but not always) small for differentiated consumer products involved in a merger analysis. In many situations, therefore, the numerators of the two diversion ratios can be assumed to be equal. Furthermore, as shown in equation (3), the denominators of the two diversion ratios are known functions of quantities and margins under the assumption of profit maximization by single-product firms pre-merger. Thus, if D_{12} is known in these situations, then D_{21} is equal to $D_{12}(\partial Q_1 / \partial p_1)/(\partial Q_2 / \partial p_2)$.

Proposition 2. If $\partial Q_2 / \partial p_1 = \partial Q_1 / \partial p_2$, then Proposition 1 implies

$$\frac{\Delta p_1}{p_1} = \frac{D_{12}(p_2 - c_2) + D_{12}D_{21}(p_1 - c_1)}{2(1 - D_{12}D_{21})p_1}$$

(11)

$$= \frac{GUPPI_1}{2} \times \frac{1 + \dfrac{p_1 - c_1}{p_2 - c_2}D_{21}}{1 - D_{12}D_{21}}$$

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[9]   This result follows from the fact that with constant marginal cost the cost function takes the form $C(w,q) = h(w)q$, where w is the vector of input prices (including the inputs supplied by the merging firms) and q is the customer's output in the downstream market. The result can be extended to increasing marginal costs if the underlying production function is homothetic so that $C(w,q) = h(w)g(q)$.

Proof. Omitted.

Proposition 2 shows that the formula for the unilateral profit-maximizing price increase can also be expressed in terms of the GUPPI. The first term (i.e., $0.5*\text{GUPPI}_1$) is the profit-maximizing price increase of product 1 holding the price of product 2 constant. Equation (11) implies a higher price increase because it accounts for feedback effects between the two price increases.

Note that in the symmetric case (i.e., the two products have equal pre-merger prices, quantities, margins, and diversion ratios) equations (2) and (11) both reduce to[10]

$$\frac{\Delta p}{p} = \frac{D}{2p}\frac{p-c}{1-D} \tag{12}$$

In a recent article, Farrell and Shapiro (2010b, p. 4) have reported a formula for the price change in the asymmetric case. Their formula is (implicitly) based on the assumption that the own-price derivatives of the demand functions are equal (i.e., $\partial Q_1 / \partial P_1 = \partial Q_2 / \partial P_2$) and thus does not apply in the general asymmetric case.[11] Further, this condition is unlikely to hold, even approximately, in the differentiated products situation.

To summarize, we have derived the formula for the unilateral price effects of mergers with linear demand and two products in the general asymmetric situation. These predicted price changes use the same information which is required to calculate the GUPPI measure of the new Merger Guidelines. However, the predicted price changes are more informative since they measure the variables which are at the core of merger analysis, potential price changes, as well as the variables required to estimate the effect on consumer welfare and economic efficiency that arise from the merger.

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[10] This formula for the symmetric case was reported in Shapiro (1996).

[11] See Shapiro (2009).

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