# Rational Decumulation

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Comments Welcome!

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## ABSTRACT

We focus on the decumulation decision that faces an individual upon entering retirement, and seek a rational set of choices for an individual who receives a lump-sum settlement from retirement savings programs, together with accumulated private savings and Social Security credits. In the spirit of Merton (1969, 1971) and Richard (1975), we develop a continuous-time model to study the asset allocation choices, where life annuities are included along with fixed income and equity as the asset classes, and the inflation-protected life annuity is the riskless asset in an intertemporal context with an uncertain lifetime.

Unlike previous continuous-time models of annuities, wherein the existence of "actuarial notes" or "instantaneous term annuities" is posited and individual behavior relative to these hypothetical annuities is examined, our model accommodates more realistically the principal features and structure of actual annuities that are available – i.e., we consider irrevocable life annuities. Individual behavior differs markedly from earlier studies under a variety of economic conditions. In particular, high levels of annuitization are shown to be rational under a wide range of risk aversion levels, even when stock market returns and annuity price loadings are assumed to be much greater than is generally the case.

Ours is also the first study to model individual behavior under the possibility of default by the insurer issuing annuities. We find that even a little default risk can have a very large impact on annuity purchase decisions. We further find that state insolvency guaranty programs can have a big impact upon the levels of rational life annuity purchases – particularly annuities of large size. This occurs even if the guaranty limits are relatively low. Higher guaranty limits have a much smaller incremental impact on annuity purchases. Insurers with lower credit ratings may benefit relatively more from such programs.



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## INTRODUCTION

Retirement security is the central policy concern of our time.[1] This concern has been heightened with the gradual demise of defined benefit pensions, which in many countries have been substituted by defined contribution retirement savings plans. For example, defined benefit pensions in the U.S. decreased from 175,000 plans in 1983 to less than 25,000 today, according to the Employee Benefit Research Institute. While initially, many of the closed plans were small or medium size in terms of assets and number of employees covered, some were quite large, and over the past two years several of the largest plans have been frozen or discontinued.[2] By way of contrast, defined contribution plans in the U.S. have grown from roughly 17,000 in 1984 to a number approaching 1 million today.

In such plans, the risk of outliving one's accumulated assets has also been shifted from the employer to employee and, equally important, the investment risk has also been shifted to the individual. Zvi Bodie (2003) has observed: "From a social welfare perspective, this development might actually be a step backward. Risk is being transferred to those who are least qualified to manage it… Like surgery, asset allocation is a complex procedure requiring much knowledge and years of training. No one would imagine that you or I could perform surgery to remove our own appendix after reading an explanation in a brochure published by a surgical equipment company. Yet, we seem to expect people to choose an appropriate mix of stocks, bonds, and cash after reading a brochure published by an investment company. Some people are likely to make serious mistakes."

At the same time, many employers who have transferred the major retirement risk of outliving one's income to employees have not equipped them with the tools and options to intelligently deal with hedging against this risk. While the reasons are complex, and in large measure deal with legal liability,[3] we feel that annuitization has been put on a shelf while other options are more widely pushed upon retirees by employers, mutual funds, and financial consultants. There will be a heavy price to pay for this down the road.

## REVIEW OF THE LITERATURE

The academic literature on life annuities, which pay an income throughout one's lifetime, has noted the many advantages of annuitization as a method of managing investment risks and the risk of outliving one's income. In 1965, Menahem Yaari demonstrated under a restrictive set of assumptions that full annuitization was the optimal asset allocation for retirement savings. For full annuitization to be optimal, he assumed that consumers were expected utility maximizers, with intertemporally separable utility. Their only uncertainty was time of death. Then, if consumers had no bequest motive and could pur-

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[1] See Blitzstein, Mitchell and Utkus (2006).
[2] See Vanderhei (2003). Recent examples include Verizon, Hewlett Packard, General Motors, Lockheed-Martin, Bethlehem Steel, United Airlines, Sprint, Motorola, and IBM.
[3] Perun (2004) analyzes the legal liability of corporations that offer life annuities to retirees receiving their lump sum distributions.

chase life annuities at actuarially fair prices, they would rationally seek to annuitize fully all of their savings.

Since that time, scholars have investigated in greater detail the annuitization alternative, and have refined the initial results of Yaari under a more relaxed set of conditions. Most recently, Davidoff, Brown and Diamond (2005) have shown in an elegant proof that the original results of Yaari hold true under a significantly less restrictive set of assumptions. In particular, they proved that in a complete market setting, full annuitization is optimal without assuming exponential discounting, without relying on the expected utility axioms and intertemporal separability. Their result holds even if annuities are not purchased at actuarially fair prices, as long as the consumers have no bequest motive and annuities pay a rate of return greater than conventional assets of matching financial risk. They show further that in some incomplete market settings, where a desired consumption path via annuities is unavailable through full annuitization, it is still optimal to annuitize a substantial amount of one's wealth. This remains true even if the consumer has a strong bequest motive.

In contrast to the prescriptions of economic theory, observed levels of annuitization are generally far below those considered optimal by most economists. Scholars have put forward a series of considerations, both perceived and real, that they feel may have inhibited significant investment in life annuities. These possible explanations include:

1. bequest motive
2. health shocks
3. self-insurance through family or other networks
4. social security crowding out private annuitization
5. premiums above actuarially fair prices
6. adverse selection
7. regulatory barriers for retirement plan sponsors
8. high-profile failures of insurance companies
9. irrevocability – illiquidity
10. imperfect information
11. erosion of purchasing power due to inflation

Kotlikoff and Summers (1981) argued that intergenerational transfers played a much larger role in capital formation than previously thought. Friedman and Warshawksy (1990) and Bernheim (1991) found that the bequest motive which fueled intergenerational transfers, when combined with the perception of actuarially unfair premiums, was sufficient to significantly limit annuitization. In countries such as the United Kingdom that have mandatory annuitization, the bequest motive does not – cannot – inhibit purchases; instead, it generates a political backlash because there is no choice but to relinquish that capital to the state, rather than to heirs, upon death (Orszag, 2000). Other researchers (e.g., Hurd, 1987, 1989) contradict these assertions, concluding that the bequest motive had only marginal impact on annuity purchases. Using models that separate married couples and unmarried consumers, Brown and Poterba (2000) and Vidal-Meliá and Lejárraga-Garcia (2005) concur with previous assertions that the bequest motive is not a significant factor in consumers' decisions to forego annuitization.

Another obstacle to full annuitization is the need for liquidity to handle unexpected and catastrophic health care expenses (Sinclair and Smetters, 2004). While popular for tax-efficient growth, annuities do not provide readily accessible funds in the case of "health shocks." Health insurance covers many expenses, at least initially, in the case of an accident or other acute medical problem. Long-term care following the initial problem, however, is usually not covered. Sinclair and Smetters' model for health shocks parallels the arguments of Kotlikoff and Spivak (1981) and Brown and Poterba (2000) that indicate family networks are more likely sources of funding in the case of dire circumstances, whether due to health emergencies or outliving income. The perception among consumers is that once the money is put into an annuity, it cannot be tapped for such emergencies. Informal self-insurance appears less risky by comparison.

Besides family networks or other forms of self-insurance, many consumers feel that Social Security will provide a sufficient safety net. This leads to a crowding out of private annuitization. Perun (2004) and Ameriks and Yakaboski (2003) argue that the perception of unfair pricing in the annuity market combines with this reliance on Social Security to inhibit annuity purchases. Despite increasing life spans and demographic trends that are straining social welfare systems, consumers still trust these systems to bear the risk of greater longevity. This baseline of protection, according to Bernheim (1991) and Vidal-Meliá and Lejárraga-Garcia (2005), is the most significant deterrent to annuity purchases.

Even for consumers with complete information who also discount the potential of Social Security to handle the growing burden of retirees, the knowledge and desire to enter the annuities market are often decreased due to the perceived high price and front-loaded cost of annuities. The debate about annuity pricing among scholars focuses on the determination of whether or not premiums are actuarially fair. For consumers, it is the mere perception of overly high premiums, regardless of what the true situation is, that dampens the purchasing of annuities. This perception causes consumers to seek other products for two reasons: they cannot afford annuities (Lopes, 2003) or they think they can get a better return elsewhere on their investment (Milevsky, 2001).

On the affordability question, Lopes (2003) argues that the disparity between available assets and annuity costs precludes otherwise willing consumers from purchasing them. The income threshold for entry varies widely by country, with strong pension plans and low inflation being correlated with lower rates of annuitization, and vice versa. In the United Kingdom, for example, the requirement that retirees purchase annuities by age 75 further exacerbates the perception of high cost. Consumers feel that annuities are "poor value for money" and that the insurance companies offering annuities can arbitrarily raise prices because of the mandatory purchase requirement (Orszag, 2000). Another component of the perception of overpricing is the slow rate at which insurers adjust annuity pricing when mortality and/or interest rates change (Murthi, Orszag, and Orszag, 1999). Milevsky (2001) shows that, due to premium loading, in some cases consumers may be able to generate better returns with other investments until about age 75, depending upon annuity pricing and market conditions. This conclusion rests on the ability to annuitize later at revised prices that do not reduce any gains accrued through delayed annuitization.[4]

On the other hand, Mitchell, Poterba, Warshawsky, and Brown (1999) find that annuities offered in 1995 were reasonably priced and that the transaction costs of purchasing an annuity have declined rapidly. In particular, they found that the expected present values of fixed nominal annuities reflected a markup in the range of 6% - 10%, when evaluated relative to the mortality tables of those who actually purchase annuities. Yet we find that today, eleven years later, the markups on nominal annuities have dropped to about half those levels and that fixed real annuities reflect a markup of only 2%.[5] Orszag (2000) emphasizes that for consumers whose preferences fit Yaari's original assumptions, annuities are affordable because any transaction costs are more than compensated by the higher rate of return.

Depending on the consumers' specific preferences or socioeconomic situations, annuities can be very affordable or prohibitively expensive. Much of the disagreement on pricing is rooted in the adverse selection observed in annuity purchases. The measured presence of adverse selection not only inhibits consumer demand, but it also decreases incentives for providers to offer annuities. On the demand side, Finkelstein and Poterba (2004) show that annuity purchasers outlive non-purchasers. When purchases are further differentiated by contract size and type, the effects of adverse selection are even more pronounced (Brunner and Pech, 2005). Lopes (2003) outlines the distinction between active and passive selection, arguing that selection in the annuity market is primarily passive due to the negative correlation between mortality rates and wealth accumulation.

Private information about longevity that leads to annuity purchases – family health history, lifestyle, etc. – also deters insurers from offering these products (Blake, 1999). Though Finkelstein and Poterba did not find moral hazard – annuity purchasers extending life through various means in order to continue receiving annuity payments – to be a significant factor, providers increasingly fear exposure to growing longevity risk produced by continuing improvements in health care and life expectancy (Piggott, et al., 2005). Providers seek to protect themselves by charging higher premiums; whether actuarially fair or not, these premiums, as noted above, then deter purchasers. This self-reinforcing cycle of adverse selection, low purchases, and increasing longevity and premiums drives down purchases and creates significant entry barriers for both providers and consumers.

While providers worry about larger payouts due to increasing longevity, retirement plan sponsors are also shying away from including annuities in their plans. In the United States, as opposed to the UK's statutorily mandated annuitization, plan sponsors are not

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[4] It should be noted here that if a person at age 65 decided to wait to annuitize until age 75, if in fact the individual lived to age 75, in retrospect, he or she would have done better to buy the annuity at age 65 unless his or her investment portfolio outperformed the returns on life annuities, which generally exceed fixed income interest rates. This, of course, assumes no bequest motive, and modest annuity price loadings.

[5] This calculation, current as of March, 2006, assumes that the mortality rates of annuity purchasers are consistent with the unloaded IAM Basic Mortality Table, which is designed to reflect the mortality rates of people who actually purchase annuities. If the mortality tables are adjusted to reflect the mortality improvement commonly assumed of 0.5% to 1.0% per year, markups for real and nominal life annuities are even lower and in some cases approach zero when promised benefits are discounted by Treasury rates.

required to include any annuities. Indeed, recent regulatory changes and legal precedents have significantly skewed the incentives to employers against inclusion of annuities (Perun, 2004). Most significantly for the annuity market, these changes broadened the definition of fiduciary responsibility for plan sponsors that offer annuities. When plans simply disburse funds as a lump sum, they do not make any investment decisions, thereby avoiding fiduciary responsibility. But when a plan purchases an annuity for a participant, that investment decision then becomes subject to the fiduciary standards established by the Employee Retirement Income Security Act (ERISA). Consequently, plan sponsors pragmatically decline to offer annuities rather than deal with the administrative hurdles and the risk of liability based on performance. Low demand, coupled with these burdens, limit enthusiasm for bucking this trend.

Many of these regulatory changes stemmed from the failure of insurers in the 1990s, including Executive Life Insurance Company of California and Mutual Benefit Life of New Jersey. As Perun (2004) notes, these failures ended both consumer enthusiasm and regulatory laxity just when the shift from defined benefit to defined contributions plans was accelerating. At what should have been the ideal time for the increasing numbers of consumers, with rapidly growing assets, to invest millions of dollars into annuities, the market environment was deteriorating. And the dollars flowed elsewhere. With diversification becoming the mantra for portfolio design, consumers are reluctant to depend on one product from one provider for any substantial portion of their retirement income. (This is quite ironic when many of the same people who do not trust an annuity do trust Social Security.[6])

Some writers have noted the high psychological threshold that individuals face when making an irrevocable decision (Bodie, 2003; Stanford, Drew, and Stanhope, 2003), along with the illiquidity that annuities entail (Browne, Milevsky, and Salisbury, 2003). Although it may be fully rational to annuitize a substantial portion of one's wealth at the onset of retirement, or even earlier, this psychological barrier is a real one for many people.

Another issue that has limited annuitization is imperfect information. We mentioned earlier the adverse selection that faces the insurer and consumer with regard to health status, which leads to prices that tend to reflect the fact that annuitants live about 10 percent longer than non-annuitants. But the imperfect information has other dimensions. Many consumers know very little about them, their features, and their availability. Because plan sponsors typically do not include them as an option, there is often little education offered to retiring employees by their employers (Perun, 2004).

[6] Robert Arnott (2005, p. 16) discusses a principle established four hundred years ago in English common law that may have a bearing on whether the promises of Social Security will be kept. During the era of debtor prisons, English common law established that debts are not inherited by the next generation, except to the extent that they can be covered by assets inherited by that next generation. The same principle applies today in international law. He suggests that the unfunded pension obligations of the Social Security System may fit the definition of "odious debt," established by this principle. If true, then future generations would be within their rights to refuse to honor, in part or in full, the promises made to ourselves which we chose not to fund fully.

Finally, some people resist annuitization because they are fearful of the recurrence of inflation. For fixed life annuities, this can be a real concern, although in recent years inflation has greatly moderated. Moreover, there are scaled annuities that offer increasing payments over time, as well as inflation-indexed annuities that protect against the effects of general inflation (Bodie, 2003). There remains the risk that the rate of inflation in the cost of living which faces the retiree may diverge markedly from that which faces other consumers, because existing real annuities are indexed to general inflation rates, such as the consumer price index.

In light of these considerations, some have suggested a delay in the annuitization decision so that the inflation risk inherent in a fixed nominal life annuity is at least partly mitigated. There are three problems with such delays. First, the accumulated assets need to be invested in something during the interim while awaiting the time to purchase a payout annuity for one's remaining lifetime. If invested in traditional vehicles, such as fixed income and equities, the value erosion that is likely to be engendered by rising inflation and interest rates may offset part or all of the gain that one hopes to garner by delaying the annuitization decision. Second, if life expectancy improves beyond the rate of improvement assumed in current pricing, the prices of the annuities themselves will climb. We calculated for our base case that a 1% annual improvement in mortality is associated with roughly a 5% increase in the price of an annuity, or a 5% reduction in monthly payouts. This decline in monthly annuity payouts may be offset if the interest rate embedded in annuity pricing also increases, but it needs to increase sufficiently to offset any reduction caused by an unanticipated improvement in mortality as well as the probable reduction in accumulated asset values occasioned by inflationary forces during the delay period. Third, the awaited inflation and rising interest rates may not occur; indeed, the interest rates embedded in annuity pricing may remain stable or decline, leaving the annuitant with lower monthly payments. If interest rates and mortality rates decline together, these reductions could be substantial.[7]

The fact that an individual's needs often do not coincide with the flat monthly nominal or real payments typical of traditional life annuity contracts is not insurmountable. In today's market one can create a portfolio of annuity products that generates a payout pattern to satisfy most desires. There are life annuities with period-certain payout guarantees of anywhere from two to 30 years. Inflation protection can be achieved by electing to have monthly payments increase each year by 1 to 6 percent, or by the full percentage change in the consumer price index.[8] Income will be lower initially (compared to a level annuity) but will rise by the pre-determined amount or inline with inflation.

[7] See footnote 10 for more detail on this point.

[8] One popular inflation-indexed life annuity provides a floor of 0% and a cap of 10%, which are binding only if the inflation rate falls outside these bounds; otherwise, the policy is fully indexed for inflation. A popular variable annuity invests in a portfolio consisting almost entirely of inflation-linked Treasury and corporate bonds. However, unlike the inflation-indexed life annuity, the variable annuity is really just an accumulation vehicle whose payout will ultimately depend on the performance of the unit-linked fund, which may diverge from the actual inflation rate due to changes in the real rate of interest, liquidity, bond defaults, market conditions, and so forth.

By paying a higher price (of roughly 30% - 40% for men and women, respectively) at the time of annuity purchase, an individual can include a provision to have his or her monthly payments increase by as much as 400 percent beyond some age, such as 85. Alternatively, the annuity may be designed to have monthly payments reduced beyond a certain age. By combining the purchase of immediate life annuities, both real and nominal, with deferred annuities, an even wider range of payout opportunities can be engineered. Another annuity feature available with some life annuities is the option to receive an advance of up to six months of payments twice during the payout period of the annuity, to meet lumpy financial needs such as health care or new auto purchase.

Recently, much attention has been given to alternatives to annuitization. In particular, "self annuitization" approaches known as phased withdrawal plans have been investigated. In such approaches, the retiree allocates his or her retirement savings across a variety of asset categories, and follows some sort of disciplined procedure of withdrawals over time to meet consumption needs.

Milevsky and Robinson (2000), and Dus, Maurer, and Mitchell (2005) are good examples of this research. They investigate several phased withdrawal plans and find that none offers the security of a life annuity. In some of the phased withdrawal plans, the probability and/or size of a shortfall is substantial. There remains some risk that an individual will outlive his own assets, or that the investments will generate a return so small that lifestyle, or even life, cannot be supported. Life annuities, by contrast, pool longevity risk across a population of annuitants, and provide guaranteed investment returns throughout life, thereby eliminating this risk. To achieve a similar riskless guarantee of income throughout one's uncertain lifetime without life annuities would cost between 25% and 40% more.

## MODEL DEVELOPMENT

We offer two principal contributions to the literature. First, by carefully modeling the essential features of a life annuity, we can generate more meaningful indicators of optimal annuitization levels under a wide variety of circumstances. Second, we design our model to capture the effects of default risk on the part of the annuity provider, as well as guaranty funds that serve to mitigate the effects of insurer insolvency. These two contributions have some fascinating implications for the rational levels of annuitization.

### Term annuities vs. life annuities

Continuous-time formulations of economic models are powerful tools that enable economists to derive more refined empirical hypotheses and more precise theoretical solutions than can otherwise be obtained from their discrete-time counterparts (Merton, 1990). They are particularly suited to the study of intertemporal problems over long spans of time such as asset allocation and annuitization choices over the period of retirement.

In previous continuous-time models of intertemporal consumption, asset allocation and annuitization, the kind of annuity examined has been an instantaneous term annuity. Such a hypothetical annuity was cleverly designed for its mathematical tractability and ability to elicit analytical, closed-form solutions to optimal annuitization levels, not because it had any close counterpart in the real world. In the Yaari (1965), Richard (1975), Purcal and Piggott (2005), and other continuous-time models, the annuities modeled typically last only for an instant in time, or are renegotiated continuously. These types of annuities behave like the mirror image of instantaneous term life insurance.

In term insurance of the instantaneous variety, which these continuous-time models also posit, the consumer pays an insurance premium for term life insurance that pays a benefit to his or her beneficiaries if life ceases during that instant. Otherwise, the premium stays with the insurer and no benefit is paid. Then, if insurance is desired again over the ensuing moment, a new term policy is purchased at a new price reflective of the high age and its associated increased mortality rate. This activity can be repeated continuously and thereby, term life insurance protection may be obtained throughout life. Perpetual insurability is assumed, and no changes in the mortality rate tables are considered, nor in the time vector of interest rates that also can influence future term insurance premiums. The drawbacks of such modeling are well understood, because in practice, term life insurance may be unavailable, or insurability not assured beyond some ages, or if health declines, and the basis risk associated with changing interest rates and evolving mortality tables over time is high.

In the case of instantaneous term annuities, the individual has an opportunity at each moment in time to "purchase" a term annuity that lasts for an instant, and pays a rate of return higher than the market interest rate, if he or she survives the instant, but pays nothing if he or she dies.[9] At the next moment in time, the individual may elect to purchase a new annuity that will pay an even higher rate of return (owing to an increased probability of mortality as the individual ages) if he or she survives to the next instant, or nothing otherwise. The process of rescinding or renegotiating the levels and prices of these successive annuities continues until death occurs.

Although few have focused on it, this kind of annuity carries with it severe drawbacks along with its advantages. Among the advantages, the annuity purchase can be reversed at each moment in time, in whole or in part, at the option of the annuitant. In other words, unlike life annuities commonly available, which generally pay a specified stream of real or nominal income throughout life, and which when purchased are irrevocable, an instantaneous term annuity may be purchased in any amount for one moment, and in another amount or not at all at the next instant. However, as typically modeled, the annuity does not allow for changing interest rates nor evolving mortality tables, both of which can combine to produce extreme basis risk.[10] If mortality rates continue to decline, further

[9] To be technically correct, the annuitant in these models does not actually purchase the momentary annuity; rather, he or she sells instantaneous term life insurance on his or her own life to the insurer in return for an insurance premium. The annuitant retains his or her assets and together with the insurance premium received, consumes and/or invests them, if alive until the next moment, but the estate forfeits the value of assets associated with the life insurance sale if the annuitant dies.

[10] While we cannot provide dimensions of this risk based on nonexistent instantaneous term annuities, the basis risk also exists for delayed annuitization decisions. For example, when annuity providers recently switched to using the 2000 annuity table from the earlier mortality table, along with lower assumed interest rates, the annuity payments available to a 65-year old and a 70-year old consumer were reduced by over 11% and 16%, respectively, from what they were offered only one year earlier through one large annuity

reductions in annuity income offered could occur, particularly for unexpected improvements in life expectancy. For an individual who is attempting to optimize consumption and bequest over an uncertain lifetime, such basis risk can severely reduce an individual's expected utility. But of course, such instantaneous term annuities are not available in practice.

By contrast, a typical life annuity that pays a specified stream (fixed in real or nominal terms, or following another preferred time path) does not face that kind of basis risk over time, as its cost is locked in from the outset.[11] On the other hand, there could be a case of "buyer's remorse" with these more commonly available annuities if interest rates and mortality rates combine in the future to allow for higher streams of future consumption. But in the face of lengthening lifespans, locking in the mortality rates embedded in today's annuity pricing could be a good move (Philipson and Becker, 1998).

### Insurer Insolvency

Insurer financial solvency is monitored by state insurance departments. When a company fails and is declared insolvent by a state commissioner of insurance, the state courts are asked to give the commissioner authority to seize the company's assets and manage the company until it can be rehabilitated or liquidated. The commissioner, in conjunction with the state insurance guaranty association, makes a determination whether the insurer can be rehabilitated or if it should be liquidated. In the case of company liquidation, a deputy receiver may be appointed to carry out the tasks associated with selling off the assets and satisfying the valid claims against the insurer, following the payment priorities established by state law. The insolvent insurer's annuity and insurance policies may be transferred to a financially stable insurer that will become responsible to provide continuing coverage and pay claims. Alternatively, the guaranty association may provide direct coverage and either continue the insolvent insurer's policies or issue replacement policies.

If an annuitant's remaining policy value at the onset of insurer insolvency exceeds the guaranty association coverage limits, the value in excess of guaranty limits may be submitted as a creditor claim against the estate of the failed insurance company. The contract holder then receives distributions as the failed company's assets are monetized by the receiver.

The National Organization of Life and Health Guaranty Associations (NOLHGA) works together with the state guaranty associations to provide continued protection for annuitants in the case of multi-state insurance insolvencies. Although many insolvencies are handled with the help of NOLHGA, it does not necessarily make any assessment against

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company. Of course, if interest rates and mortality rates had gone in the other direction, the stream of payments available through annuities might have improved. Our point is that there is a great deal of uncertainty interjected into the utility maximization problem when basis risk is included in instantaneous term annuities.

[11] We compared the expected utility associated with a sequence of instantaneous term annuities vs. the commonly available life annuity and found the fictional form to produce higher utility, owing to its flexibility. However, our calculation did not take into account the extreme basis risk associated with the fictional annuity, which would have substantially reduced its utility.

the member associations, because oftentimes sufficient funds to cover the liabilities can be raised through judicious management and liquidation of the failed company's assets. However, since 1988 NOLHGA has assessed its member associations to facilitate the continuing annuity coverage of 87 failed insurers.[12] In Table 1, we list those insurers along with the NOLHGA allocated annuity assessments. The overall assessments can continue for several years. (Our numbers do not include the unallocated annuity assessments for group retirement plans, nor the life and health assessments.)

Some of the largest and most active annuity providers have found themselves among the list of insolvent insurers, although the NOLHGA assessments are generally relatively modest, as the vast majority of policy values were able to be funded by the sale of insurer assets and transfer of its policies to another insurer. Moreover, several prominent annuity providers that became insolvent are not even shown in our list, as their claims were satisfied without resorting to NOLHGA assessments.

## Theoretical model

Robert Merton (1969, 1971) derives optimal consumption and portfolio allocation rules over time for an investor with a fixed life span and a bequest motive. In Merton's model, the investor's preferences are represented by a HARA utility function, lifespan is finite and of certain length, and the investment choice is restricted to a risk-free asset and a single risky asset. Scott Richard (1975) extends the Merton model by considering a random life span. In addition, Richard introduces an instantaneous term insurance contract that can be either bought as life insurance or sold as an instantaneous term annuity. The purchase or sales price can be either actuarially fair or loaded. While Richard is able to solve, in closed form under certain assumptions, for optimal consumption, investment and life insurance or annuity decisions, the instantaneous term annuity does not exist in practice.[13]

With HARA class utility functions, including CRRA preferences, economists have recognized that consuming and investing in such a way as to leave any chance of not providing for one's survival level or minimum threshold level of consumption throughout one's lifetime is associated with infinite disutility in those states, and hence, an expected infinite disutility across all states. Hence, Richard (1975) and Purcal and Piggott (2005) explicitly, and Yaari (1965) implicitly are modeling the intertemporal consumption and investment decisions, with and without a bequest motive, for that portion of wealth which exceeds the amount required to provide fully for this minimum threshold level. They all recognize that the only way to provide for that level of minimal consumption is through a default-free annuity which continues throughout one's lifetime. Essentially, they focus on consumption beyond a survival level $U(C_T - \psi) = U(C)$, where $C_T$ is total consumption

[12] NOLHGA lists assessments for 209 life/health insurer insolvencies during that period, but their assessments for allocated (i.e., individual) annuities involved only 87 of those companies. The data do not identify whether those 87 companies wrote immediate or deferred annuities, or both, but all deferred annuities can enter into an annuitization phase at the option of the policyowner.

[13] For the reader less familiar with the financial literature on annuities, an excellent treatise from the financial perspective on the use of actuarial models and underlying pricing of life-contingent annuities is given by Milevsky (2006).

and $\psi$ is the minimum threshold of consumption tolerable to the individual making the consumption/asset allocation decisions. Bodie (2003) has suggested that an even higher threshold be utilized: "Habit formation provides a strong rationale for financial products that guarantee that future consumption will not fall below a level established by prior consumption as a minimum acceptable standard of living." Our model is robust to either interpretation.

Therefore, $U(C)$ is interpreted as the utility of this additional consumption, and any purchase of annuities is actually supplemental to fund this additional consumption. In essence, the continuous-time models take into account the priority an individual has to meet his or her own consumption needs before allocating remaining wealth to accommodate additional consumption and bequests.

In our treatment below, we follow Richard and others in modeling the allocation of wealth in excess of that which is necessary to provide for the minimum acceptable level of consumption. When we refer to wealth henceforth, we will always mean excess wealth in that sense; consumption will mean additional consumption; and annuitization will connote annuitization supplemental to that necessary to provide for the minimal acceptable stream of consumption.

Our model will begin as an extension of Merton's model to include an uncertain life span. We will then follow Richard's lead in recognizing that a stream of future mortality-dependant income can be incorporated into the decision maker's framework. This will allow us to consider annuities that are more consistent with products available in the marketplace. Thus, the annuities that we consider will be based on level nominal or real lifetime income.[14]

We will model the investor's decision on the date of retirement. The investor will have an initial excess wealth level at the start of retirement, time $\tau$, equal to $X(\tau)$. He will choose to keep some fraction of his wealth liquid to be invested in a combination of the risk-free and risky assets and annuitize the balance of his wealth. Accordingly, the investor will hold investable wealth, $W(\tau)$, and purchase an annuity at a price of $A(\tau)$ that pays $Y$ per period. Thus, $W(\tau) = X(\tau) - A(\tau)$.

We begin with an objective function that incorporates both utility of consumption and utility of bequest. For a given choice of $A(\tau)$, the individual's objective function is

$$\max_{C,\pi,Z} E\left[\int_{\tau}^{T} U\big(C(s),s\big)ds + B(Z(T),T) \;\middle|\; A(\tau)\right], \tag{1}$$

where $T$ is his uncertain time of death, $U$ is the his utility of consumption, $C$, and $B$ is the investor's utility of bequest, $Z$. The investor is able to divide the balance of his investable wealth between two securities: a risk-free and a risky security with the price of the risky security following a geometric Brownian motion of the form

[14] We illustrate in this paper inflation-indexed life annuities providing a level real income stream throughout an annuitant's remaining life. However, our model is easily generalized to incorporate a wide variety of real or nominal income payout patterns.

$$\frac{dQ(t)}{Q(t)} = \alpha dt + \sigma dq(t) \ , \tag{2}$$

where $\alpha$ is the expected return on the risky security and $dq(t)$ is the Wiener increment.

Following Purcal and Piggott (2005), we will represent mortality risk with fair probabilities based on annuity mortality tables using an adaptation of Richard's expense-loaded probability structure. Let the investor's age at death, $T$, have a cumulative distribution function given by $F(T)$ and a probability density function of $f(T)$ for a newborn at birth. The survival function, $S(T) = 1 - F(T)$, yields the probability that the investor lives to age $T$. The probability of a person age $t$ surviving to age $T$ is then $S(T)/S(t)$. Further, let $\mu(T)$ denote the instantaneous conditional probability of death at time $T$ given survival to that age. This is known as the force of mortality by demographers and actuaries and is calculated as

$$\mu(T) = \frac{f(T)}{S(T)} \ . \tag{3}$$

The fair value of the annuity income stream, $a(t)$ can be calculated as

$$a(t) = \int_t^{\omega} Y \frac{S(\theta)}{S(t)} e^{-r(\theta - t)} d\theta \ , \tag{4}$$

where $\omega$ is the maximum age attainable in the mortality distribution. The price of the annuity is $A(t) = \eta a(t)$ where $\eta$ is the loading factor. When $\eta = 1$, the annuity is sold at an actuarially fair price. When $\eta > 1$, then the annuity is sold at a loaded price.

The investable wealth will evolve according to the stochastic differential equation

$$dW(t) = \left[-C(t) + Y + rW(t) + \pi W(t)(\alpha - r)\right]dt + \sigma dq(t) \ , \tag{5}$$

where $r$ is the risk-free rate, and $\pi$ is the proportion of wealth invested in the risky asset. The investor will then have adjusted wealth of $W(t) + a(t)$ each period.

Following Merton and Richard, we will use stochastic dynamic programming to derive the optimal controls. Define

$$J(W,\tau) \equiv \underset{C,\pi,Z}{Max}\, E\left\{ \int_{\tau}^{\omega} \frac{S(T)}{S(\tau)} \mu(T) \int_{\tau}^{T} U(C,s)ds + B(Z,T)dT \;\middle|\; A(\tau) \right\} \ . \tag{6}$$

Next, assume

$$U\big(C(t),t\big) = \left(\frac{h(t)}{\gamma}\right) C(t)^{\gamma} \ . \tag{7}$$

Similarly, let

$$B\big(Z(t),t\big) = \left(\frac{m(t)}{\gamma}\right) Z(t)^{\gamma} \ , \tag{8}$$

where $h(t)$ and $m(t)$ are, respectively, consumption and bequest discount functions and $\gamma$ is the risk aversion parameter. For simplicity we will assume that

$$h(t) = m(t) = e^{-\rho t} \ .$$

Given the form of the utility and bequest functions in (7) and (8), optimal consumption and bequest are given by

$$C^* = \left(\frac{h}{J_W}\right)^{1/\delta}, \quad \gamma < 1,\ h > 0,\ C > 0 \tag{9}$$

and

$$Z^* = \left(\frac{m}{J_W}\right)^{1/\delta}, \quad \gamma < 1,\ m > 0,\ Z > 0\ , \tag{10}$$

where $\delta = 1 - \gamma$.

Based upon (7) - (10) and following Merton and Richard, $J(W,t)$ is a solution to the Hamilton-Jacobi-Bellman equation

$$0 = k(t)\left(\frac{\delta}{\gamma}\right)J_W^{-\gamma/\delta} + J_t - \mu J + \left(rW + Y\right)J_W - \frac{1}{2}\frac{J_W^2}{J_{WW}}\left(\frac{\alpha - r}{\sigma}\right)^2\ , \tag{11}$$

where[15]

$$k(t) = \left(\frac{\mu(t)}{\delta}\right)m(t)^{1/\delta} + h(t)^{1/\delta}\ , \tag{12}$$

subject to

$$J(W,T) = B\big(Z(T),T\big) = \left(\frac{m(T)}{\gamma}\right)Z(T)^{\gamma}\ . \tag{13}$$

A solution to (11) is

$$J(W,t) = \left(\frac{b(t)}{\gamma}\right)\big[W(t) + a(t)\big]^{\gamma}\ , \tag{14}$$

where $b(t)$ is given by (5) and

$$b(t) = \left\{\int_t^{\omega} k(\theta)\frac{S(\theta)}{S(t)}\exp\left[\frac{\gamma}{\delta}\left(\frac{(\alpha - r)^2}{2\delta\sigma^2} + r\right)(\theta - t)\right]d\theta\right\}^{\delta}\ . \tag{15}$$

Based upon the guess of $J(W,t)$ in (14), we find that the optimal controls are

[15] Our equation (12) differs from the specification of $k(t)$ in Richard's equation (39) because we are considering a lifetime real annuity with level payments of $Y$ rather than the instantaneous term annuity considered by Richard.

$$C^*(W,t) = \left(\frac{h(t)}{b(t)}\right)^{1/\delta} \left[W(t) + a(t)\right] , \tag{16}$$

$$Z^*(W,t) = \left(\frac{m(t)}{b(t)}\right)^{1/\delta} \left[W(t) + a(t)\right] \text{, and} \tag{17}$$

$$\pi^*(W,t)W(t) = \left(\frac{\alpha - r}{\delta\sigma^2}\right)\left[W(t) + a(t)\right] . \tag{18}$$

This is the familiar result that the optimal controls are linear in adjusted wealth. From (16) - (18), it is apparent that the decision maker considers the actuarially fair present value of future annuity payments when choosing consumption, bequest and investment.
Thus far we have considered the utility maximization problem for a given choice of annuity purchase, $A(\tau)$. The model yields analytic solutions for the optimal controls when the annuity purchase is taken as given. Now we must consider the optimal annuity purchase. Unfortunately, this will require numeric solutions of the larger optimization problem. The objective function becomes

$$\max_{A,C,\pi,Z} E\left[\int_{\tau}^{T} U\big(C(s),s\big)ds + B(Z(T),T) \;\middle|\; A(\tau)\right] . \tag{1′}$$

To find the optimal annuity purchase we simulate (1′) for varying levels of $A(\tau)$. We generate paths for wealth using a discrete version of (5). Then, we apply (16) - (18) to those paths and numerically calculate lifetime expected utility using (1′) where the expectation is taken by averaging across all of the simulated paths. Reasonably stable solutions are found using 5,000 paths but we used 10,000 paths to verify the numerical results except in the case where we consider default risk. Because of the low probabilities associated with default, we used 50,000 paths to estimate the impact of default risk on the annuitization decision.

There is one other constraint that must be imposed to find the optimal annuitization level. When large portions of retirement wealth are annuitized, remaining investable wealth is low. That is, for large $A(t)$, $W(t)$ is small. This does not create a problem for optimal consumption in (16) because investable wealth plus income from the annuity will adequately fund desired consumption. But, when a large fraction of wealth is committed to an annuity there may not be sufficient funds to allow all of the decision maker's desired investment in the risky asset.[16] Thus, we impose a liquidity constraint in the simulation such that

$$\pi^*(W,t)W(t) = Min\left\{\left(\frac{\alpha - r}{\delta\sigma^2}\right)\left[W(t) + a(t)\right] \;,\; W(t) + Y - C(t)\right\} . \tag{18′}$$

[16] This is a consequence of our model capturing the irrevocability of an annuity purchase. Had we modeled instantaneous term annuities, together with borrowing and instantaneous term life insurance, this constraint would not have been necessary. However, as previously stated, that more flexible alternative is not possible in practice and would inject substantial basis risk into the optimization simulation.

The implication of this constraint is that the decision maker will consume at his desired level. However, he will invest only up to his available liquid funds in the risky asset. If his investable funds are adequate to cover his desired investment, he will have an optimal allocation between risky and risk-free assets.[17] If, however, his investable funds are insufficient to cover his desired allocation to the risky asset, then the decision maker is liquidity constrained and will have a sub-optimal asset allocation.

Taken together, the model being simulated incorporates several interesting concerns that must be considered when purchasing a lifetime annuity. These concerns exist when we leave behind the instantaneous term annuity of Richard's and Yaari's models and consider an irrevocable lifetime annuity.

The first concern is that bequest must funded out of liquid assets. We have purposefully omitted term insurance because of its lack of availability at some higher ages. The impact of the bequest consisting of remaining liquid assets at the time of death is that the decision maker must consider expected returns and risk in the risky and risk-free assets and allocate sufficient funds to equate the marginal expected utility of bequest with the marginal expected utility of lifetime consumption.

The second concern is that the funds committed to the annuity are no longer available to allocate to the risky asset. Thus, as the investor chooses higher annuitization levels he is constrained in his ability to allocate funds to the risky asset in some states and earns a lower expected return than he would optimally desire on his investable funds in those states. The lower expected return on a constrained asset portfolio must be and was balanced against the benefit of reduced uncertainty surrounding future consumption provided by the annuity.

Finally, because we incorporate default risk into the simulation, the decision maker must be concerned with the possibility that some or all of the promised income stream from the annuity may suddenly disappear during his lifetime. Therefore, when there is default risk the investor must hold back sufficient assets to survive the possible disappearance of the annuity on an expected utility basis. This need is mitigated to some extent by the existence of state insurance guaranty programs and NOLHGA.

## CALIBRATION

### Real rate of interest

Our estimate for the real rate of interest reflects the real yield to maturity on the longest maturity U.S. Treasury inflation-indexed bonds currently being traded. Our base case uses a 2% real yield, and we test the effects of real yields ranging from 1% up to 3%, in 0.5% increments.

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[17] It is important to note that investment in the default-free life annuity is a substitute for risk-free asset investments in the asset allocation decision; indeed, because it provides protection against the major risk of lifetime uncertainty, it *is* the risk-free asset in an intertemporal context, and the default-free bond becomes a risky asset. Rubinstein (1976) provides the economic rationale for this result.

## Rate of time preference

Our model incorporates parameters for the rate of time preference or impatience on the part of the retired individual. For purposes of our numerical simulations, we set the impatience level in our base case at 2%, which was equal to the level chosen for our base case real rate of interest, but we let it range from 1% to 3% across differing levels of real rates of interest.

## Equity risk premium

An excellent review and update of the more than 320 articles published on the equity risk premium over the past 20 years is given by Siegel (2006). He has projected an equity premium of 2-3 percent over the future. While past rates going back to 1872 have been as high as 6 percent, most financial economists are looking for a much lower equity risk premium going forward.[18] Siegel's projections are within the general range projected by Arnott and Bernstein (2002), Claus and Thomas (2001), and many others. Accordingly, our base case equity risk premium was 3%, but we show the effects of equity risk premiums ranging from 2% up to 7%, across different levels of volatility.

## Equity volatility

Jones and Wilson (2006) find that inflation adjustments have little impact on the estimates of return variability. Their inflation-adjusted geometric standard deviation of returns is 19.66%, which differs only slightly from the nominal geometric standard deviation of 19.44% over the 1871-2004 period. In our simulations, our base case volatility was 20%, but we varied the volatility of equity returns from 15% to 25% across an array of equity risk premiums. The highest volatility number was used to show the effects of a poorly diversified equity portfolio, while the 15% and 20% numbers were used to bridge the volatility typically manifest over long periods of time in broad equity market indices.

## Risk aversion parameters

We examine the usual relative risk aversion coefficients that have been studied in the economic literature with regard to pensions and annuities. We also include a risk aversion coefficient of 0 in our analysis, as it has been much studied in connection with the turnpike portfolio literature. Although the amount of risk aversion is minimal with a coefficient of 0, consistent with a generalized logarithmic utility function, it has been shown to be consistent with the highest growth strategies over extended periods of time.[19] We test our consumption and asset allocation choices over a range of risk aversion levels extending from 0 to –5, with the base case focusing on a risk aversion parameter of –2. Our model accommodates separate risk aversion parameters for consumption and bequest, but for illustrative purposes we use consistent preferences.

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[18] There are several studies that call into question whether the past equity risk premium measures were upward biased, most recently Jacquier, Kane and Marcus (2005).

[19] See, for example, Mossin (1968), Hakansson (1974), Ross (1974), Leland (1972), and Grauer and Hakansson (1987).

## Annuity price loading factors

Mitchell, Poterba, Warshawsky, and Brown (1999) find that annuities are reasonably priced, especially for consumers without bequest motives, and that the transaction costs of purchasing an annuity have declined rapidly. These findings have been buttressed by more recent studies. When evaluated according to the mortality tables that reflect those who actually purchase annuities, as opposed to the general population mortality tables that reflect approximately 10% higher mortality rates, the current loadings (March of 2006) ranged from 3% to 5% on nominal annuity prices above actuarially fair values, and 2% loadings on real annuities. This one-time charge compares favorably to the front-end or deferred loads of many mutual funds, together with their *annually* recurring gross expense ratios, which according to Morningstar averaged 1.67% on assets across all mutual funds in February of 2006. Bogle (2005) shows that over a period of 40 years, such charges together with market tracking lags can extract a substantial fraction of the appreciation in the equity market.

In our base case, we conservatively assume a 10% markup on annuity prices, but examine prices ranging from actuarially fair prices up to 40% markups.

## Mortality probabilities

Milevsky (2001) and others have suggested that in continuous-time applications, a realistic force-of-mortality approximation to a discrete mortality table can be based on the Gompertz law. The distributional specification is

$$\lambda\left(x \middle| c, d\right) = \frac{1}{d} e^{\frac{x-c}{d}} , \tag{19}$$

where $c$ is the mode and $d$ is the scale measure of the probability distribution. Under this distribution, the conditional probability of survival has been shown to be

$$ {}_t p_x = e^{-\int_0^t \lambda(x+s)ds} = \frac{e^{e^{-c/d}(1-e^{x+t/d})}}{e^{e^{-c/d}(1-e^{x/d})}} . \tag{20}$$

The parameters of this distribution were estimated to fit the tail (above 65 years of age) of the basic annuity 2000 period mortality table for males.[20] The fitted values were $c$ = 87.9830 and $d$ = 11.1879.

## Estimation of Default Rates

To model the possibility of default by the annuity provider, we employed the concept of a bond mortality table introduced by Altman (1989). Our table was calibrated to be consistent with cohort average cumulative default rates between 1920 and 2005, as reported by

[20] The *basic* table is based on industry experience of those who annuitize and does not contain margins for conservatism.

Moody's (2006).[21] The Moody's data report cumulative bond defaults by original ranking and cohort.

Insurers are not rated by Moody's using the same nomenclature that applies to typical corporate bond issuers; rather, a "claims paying ability" rating is assigned. Nonetheless, the criteria for assigning claims paying ratings are analogous to those used in corporate bond ratings, so we use the data on bond defaults as a proxy for the default probabilities associated with insurer claims paying ability. We extracted a vector of estimated default probabilities across time, based on our Gompertz function fit to the Moody's cumulative default table for Aaa-rated corporate bonds. We used the same equational form and fitting procedure as discussed previously in the human force-of-mortality approximations. We discretize this smooth fitted form in monthly increments for simulation purposes, and then alter the resulting vector of default probabilities to reflect claims paying ratings of Aa, and finally A. We do not model the default probabilities of insurers with lower ratings than these. We examine how the individual annuity purchases would change in the face of such varying default probabilities, and compare these with the base case of no possibility of default.

Because of the paucity of defaults over time among those bonds receiving the highest initial rankings, the cumulative default rate schedule appears to be somewhat bumpy as time horizon beyond issue is extended. Therefore, similar to our modeling of human mortality rates, we fit a Gompertz function to the bond cumulative default rates to more closely approximate the expectations of their "mortality rates." A close fit was achieved for the first 20 years of bond seasoning by setting $c$ = 119.6882, 104.2381, and 104.2129, and $d$ = 32.1028, 47.8295, and 60.2025 for bonds with initial rankings of Aaa, Aa, and A, respectively. Unfortunately, we had to extrapolate beyond 20 years because the Moody's cumulative bond default data do not extend beyond 20 years. The maximum cumulative default rate deviations we observed for any year between our fitted function and the Moody's discrete cumulative probabilities were .0023, .0050, and .0037 for the Aaa, Aa, and A ratings, respectively, owing to the lumpiness of observed defaults.

## Summary of parameters for the base case scenario

| Variable | Base Case | Ranges Tested | Increments |
|---|---|---|---|
| Real rate of risk-free interest, $r$ | .02 | .01 - .03 | .005 |
| Rate of annual time preference, $\rho$ | .02 | .01 - .03 | .01 |
| Equity market risk premium, $\alpha - r$ | .03 | .02 - .07 | .01 |

[21] Moody's provides these data in its Exhibit 33. We could have chosen to use cumulative default rates that reflect only the period since 1970, as shown in its Exhibit 34. However, we felt that the longer period was more representative of the kinds of default experience that can occur over the long time periods that annuities may last. Nonetheless, some notion of rational consumer behavior based upon the more recent period can be extracted from our analysis, as the 1.912% 20-year cumulative default rate of Aa-rated bonds during the shorter period since 1970 corresponds very closely to the 1.871% 20-year cumulative default rate of Aaa bonds during the longer period since 1920, although they follow different paths to get to those levels.

| | | | |
|---|---|---|---|
| Equity market portfolio volatility, $\sigma$ | .20 | .15 - .25 | .05 |
| Coefficient of risk aversion, $\gamma$ | –2 | –5 - 0 | 1 |
| Annuity loading factor, $\eta$ | 1.1 | 1.0 - 1.4 | .1 |
| Default risk of annuity | Riskless | Riskless - A-rated | Aaa, Aa, A |

Maximum age used in simulation, $\omega$ = 110 years

## RESULTS

In Figure 1, we present the optimal levels of annuitization across different values of the risk-free rate of interest and the retiree's time preference rate, or impatience for deferring consumption. We find that under normal equity market risk premium levels and volatility, and with a 10% price markup on annuities, a moderately risk-averse individual with an ordinary bequest motive would annuitize a substantial portion of his excess wealth. As the risk-free rate of interest embedded in annuity rates rises, the percent of excess wealth that goes toward annuitization climbs from the mid-70% range up to the high-70% range, with the most patient individuals reaching above 80% annuitization.

In Figure 2, we illustrate optimal annuitization levels for different values of equity risk premiums and stock market volatility. As expected, when stock market volatility increases, the percent of excess wealth that is annuitized increases across all levels of equity risk premium. On the other hand, lower stock market volatility decreases annuitization levels. We also observe that increasing the equity risk premium from the base case of 3% reduces sharply the percent of excess wealth that is rational to invest in life annuities, especially at low levels of equity volatility. For instance, optimal annuitization of excess wealth drops from 77% at an equity risk premium level of 3%, our base case, down to 45.5% if the equity risk premium more than doubles to 7%, while maintaining equity volatility constant at 20%. This is consistent with Milevsky's (2001) finding that annuitization is decreased or postponed when returns outside the annuity increase.

In Figure 3, we show the effects of different price loadings on the individual's optimal annuitization levels, across varying degrees of risk aversion. We observe that when the risk aversion parameter reaches –5, reflecting strong risk aversion, the amount of annuitization declines only slightly, from 79.5% to 78%, as markups increase from 0% to 40%. On the other hand, the least risk-averse individual's optimal annuitization levels drops precipitously, from 66.5% at actuarially fair prices to as low as 7.5% if price markups reach 40%. Our base case individual, with a moderate risk aversion coefficient of –2, does not show much sensitivity toward price loadings as long as they stay below 30%. This result is consistent with the findings of Purcal and Piggott (2005), although they examined an instantaneous term annuity. The percentage of excess wealth annuitized was 77% at actuarially fair prices and 75.5% when prices were marked up by 30%.

In our base case, as well as the comparative statics that stemmed from it in Figures 1, 2, and 3, we assumed that an annuity provider would remain solvent throughout the life of the annuitant. In Figure 4, we examine individual behavior when the insolvency of the annuity provider is possible. We present three new cases, comparing them against the base case of no defaults.

### Case 1: Annuity provider defaults, no recovery

People who purchase life annuities come from a wide spectrum of the population, and many are not knowledgeable at the time of purchase about state programs for protecting policyholders in the event of insolvency and the recoveries that may be expected. Indeed, under the State Guaranty Associations laws, no insurer is permitted to use the existence of State Guaranty Associations as part of its marketing effort for annuity sales. Nonetheless, most states require that policyowners be informed of the State Guaranty Association coverage *after* the policy has been purchased. Therefore, we begin by assuming that the annuity purchaser believes that there will be no recovery if the annuity provider becomes insolvent.[22]

We can see in Figure 4 that the level of rational annuity purchases declines sharply as the claims paying ability of the insurer is lowered. For example, in our base case scenario, but with no loadings on price, an individual with $1,000,000 of accumulated excess wealth at retirement age 65 will purchase a life annuity with an expected present value of $770,000 from an A-rated insurer if the annuity is viewed as default free (see top line of the chart). However, when there is a possibility of default consistent with A-rated corporate bonds, the life annuity purchase will drop to around $180,000, if no recovery upon default is expected. In other words, the individual will reduce the size of an A-rated life annuity purchase by over 76% of the original level. In the face of default rates consistent with an Aa-rated insurer, the annuity purchase would drop from $770,000 to $200,000, a 74% decline, and with an Aaa-rated insurer, from $770,000 to $450,000, a 42% drop. The prospect of default can be especially frightening to an aged annuitant, because he of she is often beyond an age where it would be easy to return to the work force. Therefore, this sensitivity to insurer quality is to be expected.

### Case 2: Annuity provider defaults, individual recovers up to 10% of initial excess wealth

We next substitute the recovery rates associated with the minimum guaranteed benefits supported by the National Organization of Life and Health Guaranty Associations (NOLHGA) and its member states. We examine the case where the annuitant is covered by the state insolvency guaranty program up to a limit of $100,000, measured as the present value of expected remaining benefits. In practice, 21 of the state guaranty programs provide default coverage for much more, and in past insolvencies, annuitants have often recovered much or all of their remaining contract values, even if their remaining policy values exceeded the guaranty limits.[23]

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[22] Merton (1990, pp. 146-147) looks at the case where the risk-free asset is not riskless because there is a Poisson event of default that it will go to zero (no recovery), which is similar to a risky life annuity in the case where the stated interest rate is constant, *r*.

[23] See Table 2 to review the guaranty provided by each state, and backed by NOLHGA. While most state guaranty programs cover $100,000, one program covers $200,000, 15 programs cover $300,000 of the remaining present value of annuity contracts, one program covers $360,000, and four cover $500,000. With

Notwithstanding the levels of these guarantees, at least in the beginning years of NOLHGA, there were occasional delays before annuity benefits were continued, owing in part to the necessity for NOLHGA to wait until an insolvency had been officially declared, and responsibility for payment had been shifted from the receiver or rehabilitator to NOLHGA. In addition, if under state statute it is deemed that the insolvent insurer was offering embedded annuity interest rates that were higher than justified, there is sometimes applied a "haircut" or "faircut" in the payout levels to reflect embedded interest rates that are considered fair and reasonable.

To capture the effect of guaranty provisions in our simulations, we compute the present value of expected remaining annuity benefits as of the beginning of each month and compare it to the guaranty limit. The monthly annuity payments are then adjusted downward in the event of a default to reflect a $100,000 remaining present value, if it was binding, or left the same as before, if not binding. We incorporate no delays or haircuts in the ensuing guaranty payments.

When the individual is aware of the protection afforded by a state guaranty program, his or her behavior will be different. For instance, if $100,000 of the remaining annuity value (in present value terms at the time of default, which amounts to 10% of initial excess wealth) is covered by the state guaranty program, the individual will purchase an annuity with an initial value worth up to 59% of his or her excess wealth at retirement, or $590,000, from an A-rated insurer, and as much as $615,000 from a Aa-rated insurer, and $730,000 from a Aaa-rated insurer. These data show that the insurance coverage offered by guaranty programs is highly discriminatory in favor of lower rated insurers, who are able to sell much more in annuities than would otherwise be the case. In a sense, a subsidy is being offered to insurers who are riskier.[24]

These are rather astonishing increases in the size of annuities willing to be purchased devolving from the insurance guaranty associations, especially in light of the assumption that only 10% of an initial excess wealth of $1 million would be covered by the program. Upon further analysis, it is easy to understand why. Four reasons contribute to the sizable increases in the propensity to annuitize with even modest levels of guaranty. First, a $100,000 guaranty amounts to 13% initial protection for an annuity of $770,000, not a 10% initial protection, and an even higher percentage of value protection for smaller an-

respect to guaranty association coverage of variable annuities, generally only the amounts invested in the Fixed Account of variable products, or in any investment option that is invested in the general account (such as Dollar Cost Averaging Accounts) is covered by the guaranty association. A death benefit for a variable product would also generally be covered. Amounts invested in the Separate Account are not covered by state guaranty associations.

[24] Insurers are not assessed contributions by the state guaranty programs based on their riskiness, but usually based on their volume of business in the state. Except in New York, where a pre-funded fund actually exists, these assessments are levied *after* an insolvency occurs. Because these insolvency programs do not charge a risk-based contribution, higher quality insurers find it unfair for less creditworthy insurers to advertise their NOLHGA protection, which would only serve to further exploit the subsidy they are already effectively receiving at the expense of higher quality insurers, and in some states, the taxpayers themselves, as these contributions are often directly credited against any state taxes owed, or are tax-deductible expenses.

nuities. Second, if a default is going to occur, it occurs on average about 25 years after the annuity purchase from an insurer with a claims paying rating of single-A at the outset. By that time, an annuity worth $590,000 at initial purchase (i.e., 59% of initial wealth) will have only about $198,000 of present value of payments remaining, so the guaranty of $100,000 looms large. And if the default occurs even later, up to 100% of the remaining annuity value may be covered by a guaranty level of $100,000. In essence, the older people are even better protected than the younger ones by the fixed guaranty program limits. Third, given how the CRRA utility functions bend sharply at low levels of additional consumption and bequest, and then flatten out rather quickly, a steep rise in expected utility occurs even with low guaranty levels. Finally, distant periods when default is more likely to occur are discounted more heavily by mortality and compounded interest rates.

### Case 3: Annuity provider defaults, individual recovers up to 25% of initial excess wealth

Figure 4 shows that when the guaranty association will insure an annuity up to 25% of initial excess wealth, the percent of excess wealth annuitized under our base case parameters will hardly drop at all as the claims paying rating of the annuity provider drops from Aaa to A. If initial accumulated excess wealth at retirement were again $1 million, this protection would be consistent with a guaranty level of $250,000. This would amount to almost 100% coverage of an annuity of $730,000 if were to take 25 years (the average) for a default to occur, if it occurs at all. Accordingly, the individual behaves almost as if it is a default-free annuity.

It is clear from Figure 4 that while sales by insurers of all claims paying ratings benefit from the insurance insolvency guaranty programs of their state and NOLHGA, the lower-rated companies have the most to gain. Also, higher guaranty levels such as the $500,000 limits that are provided by New York, New Jersey, Connecticut and Washington, make all but the highest value annuities fully covered.

Finally, an annuitant who desires coverage for higher limits may purchase annuities from a number of providers, rather than place all of his or her funds with a single insurer.[25] This, in turn, has three effects. First, in the event of the default of one provider, only a portion of the total annuity coverage is at risk. Second, it is unlikely that two or more of the providers will also default. Third, and most importantly, the smaller individual policies are likely to be fully covered by the State Insolvency Guaranty program.

[25] Coverage for annuitants under State Guaranty Associations is generally per person and not per policy. Accordingly, an annuitant with several annuities from the same insurer is subject to the same aggregate coverage limit as if he held a single policy. However, if the annuities are purchased from several insurers, each policy may benefit from the guaranty program coverage limits. This is very similar to the FDIC coverage policies for bank deposits. Under FDIC provisions, all deposit accounts held by a depositor at different branches or offices of the same bank are added together and insured up to the FDIC limit. However, all deposit accounts maintained by a depositor at one bank are insured separately from accounts that the same depositor maintains at a different, separately chartered bank.

### Case 4: Indifference annuity yields

In Figure 5a, we present the indifference annuity yield spreads necessary to compensate a risk-averse investor for exposure to the risk of annuity issuer default. We take as a starting point the base case parameters presented in the summary of parameters. In the base case with no default risk, the utility maximizer chose to annuitize 77% of initial excess wealth. All else equal, if the purchased annuity generates higher income for a given purchase price, expected utility increases. Similarly, all else equal, if default risk increases then expected utility will decline. At some annuity yield above the risk-free income level, the higher promised income, accompanied by greater default risk, will generate the same expected utility as if there were no default risk. This income level, as a percent of annuity purchase price, we denote the "indifference annuity yield." To calculate the indifference annuity yield, we varied annuity income while holding annuitization at 77% of initial excess wealth and all other parameters as in the base case to find the income level that produced the same expected utility as the base case did without default risk.

There is a slight modification to equation (4) required to calculate expected utility when the purchased annuity is subject to default risk. If the risk-free actuarially fair annuity pays income of $Y$ then the annuity with default risk will have to pay $Y_d > Y$ to induce investors to purchase it. Since we are holding the initial purchase of the annuity constant at 77% of initial wealth, we also hold the annuity purchase price constant while increasing promised income. Thus, the discount rate on the risky annuity increases from $r$ to $r_d$. With these changes we substitute the following equation,

$$a(t) = \int_t^{\omega} Y_d \frac{S(\theta)}{S(t)} e^{-r_d(\theta - t)} d\theta \ , \tag{4$'$}$$

for equation (4) when calculating expected utility with a risky annuity.

Figure 5a reveals that the spreads necessary to maintain consumer indifference are much higher when the guaranty limits are lowest. For example, when life annuities have a guaranty of $100,000, an individual will be indifferent between the credit quality of real annuity providers if one rated Aa provides 53 extra basis points of yield over the Aaa insurer, and that an insurer rated A provides 87 basis points above an insurer rated Aaa.[26] These figures are based on the purchase of an annuity having an initial value of $770,000, or 77% of initial excess wealth. Lower valued annuities would have greater proportional guaranty coverage, and the incremental yield required to make the consumer indifferent would be less.

In Figure 5b, we include the case where there is no guaranty program and a zero recovery is assumed under insurer insolvency. The annuity yield spreads are extraordinarily high, owing to the utility penalty a consumer places on products that can produce no recovery, in which situations the consumer hovers perilously close to his or her survival income

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[26] We are assuming here that there will be no recoveries from the insolvent insurer beyond the guaranty limits. Also, we are using the base case risk aversion parameter of –2. Higher risk aversion levels will require greater yield spreads in order to maintain indifference across the various credit qualities of providers.

level, below which there is infinite disutility. If consumers could purchase tiny amounts of annuities from many insurers (which is impractical), or if they assumed substantial recoveries even under no guaranty programs, they would not demand such high annuity yield spreads to reach their level of indifference between risky and riskless annuities.

### Asset allocation

In Figure 6, we display the overall rational asset allocations at the start of retirement for an individual faced with fixed income, equity, and annuity choices. It is apparent that under a wide variety of risk aversion levels, the purchase of life annuities will occupy a dominant role in asset allocation, even under conditions of a bequest motive, a 10% price loading, and with normal real rates of interest, time preferences, equity risk premium levels, and market volatility. We see annuitization levels ranging from 43.5% of excess wealth, in the case of minimal risk aversion and maximum growth orientation, to 83% annuitization under stronger risk aversion. It should also be remembered that these figures do not reflect total annuitization, because as discussed earlier, it is rational to annuitize the entire threshold level of minimum income in order to secure an amount of consumption that will allow survival. Of course, some or all of this can be done with government annuities, such as that provided by Social Security.[27] The expected present value of future income for a 65-year-old person today receiving median Social Security benefits is roughly $200,000, and maximum benefit levels are currently around $410,000 in expected present value terms. If these benefits are more than necessary to satisfy the threshold level of consumption required, the additional amounts can offset some of the annuitization suggested under the supplemental annuitization analysis given here. If, on the other hand, the Social Security program will not generate sufficient income to satisfy minimal consumption needs, then it should be supplemented with the purchase of high-grade private annuities.

## EPILOGUE

As Robert Merton (2003) has stated with respect to the inclusion of real annuities and other hedges in our optimal investment policy recommendations, “Executing these proposals efficiently is no small task. That said, I see this issue as a tough engineering problem, not one of new science. We know how to approach it in principle, and what we need to model, but actually doing it is the challenge.” With this study, we take two small steps in that direction.

We see future inroads can be made by better modeling of equity risk. We assumed that equity market behavior could be approximated by geometric Brownian motion, but equities exhibit greater skewness and kurtosis than what is accommodated by that process, which tends to understate true equity market risk. We chose to model the behavior with geometric Brownian motion because it allowed us to get closed-form solutions for a portion of the complex asset allocation problem.

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[27] Alicia Munnell (2006) provides an analysis of the declining role of Social Security that suggests it is unlikely to meet basic needs in the future.

We did not include the effect of existing private pensions, because our focus was motivated by their waning importance in the economy and their substitution by savings plans. As the model stands now, their present value could be included in the estimation of wealth and excess wealth, and an offset could be provided to reflect the fact that some annuitization already exists in the form of Social Security and private pensions, the latter of which may be backed by the Pension Benefit Guaranty Board.

Another improvement could be in the modeling of risk-free interest rates. Again, in our quest to achieve an algebraic solution to the asset allocation problem, we opted for the simplest behavior of interest rates. We could readily incorporate a vector of fixed interest rates reflecting a term structure of interest, but the real advance will come when interest rates are modeled as a separate stochastic process.

We have given little attention to the bequest treatment and spousal effects. These have been carefully modeled by Brown and Poterba (2000), Purcal and Piggott (2005), and Vidal-Melia and Lejarraga-Garcia (2005). We have not included tax effects. We have not examined the asset allocation decisions of retirees over time, but focused only on their initial allocations. Our model is sufficiently robust to allow for this to be included, but our focus was narrow for presentation purposes, and others have done an excellent job already in examining these aspects of the annuitization decisions (e.g., Milevsky (2001), Milevsky and Young (2002), and Dushi and Webb (2004)).

In the face of these limitations, we feel that the essential elements have been captured to allow a useful analysis of the effect of life annuity irrevocability, along with effects arising from the possibility of insurer insolvency and the stability provided by state guaranty associations. If our results are robust, they have important implications regarding the substantial levels of annuitization that are rational and the key role of guaranty associations or other arrangements to reduce the risk and/or impact of insurer defaults in the future.

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**Table 1**

## Nationwide Assessment Activity by Insolvency

(1988-2004, as of 12/16/2005)

Source: National Organization of Life and Health Insurance Guaranty Associations

| Company - Allocated Annuity | Assessed ($000's) | Years |
|---|---|---|
| Alabama Life Insurance Company | 568 | 1994 |
| Allied Bankers Life Insurance Company | 7 | 1990-1999 |
| Amalgamated Labor Life Insurance Company | 100 | 1989-2000 |
| American Educators Life Insurance Company | 285 | 1994-2003 |
| American Equitable Life Insurance Company | 5,687 | 1990-1998 |
| American Independence Life Insurance Company | 22,018 | 1990-1996 |
| American Protectors Insurance Company | 1,182 | 1990-1997 |
| American Security Life Assurance Company of Florida | 2,100 | 1991 |
| American Security Life Assurance Company of North Carolina | 30,100 | 1992-2001 |
| American Standard Life & Accident Insurance Company | 960 | 1990-2004 |
| American Teachers Life Insurance Company | 596 | 1990-1999 |
| American Trustee Life Corporation | 27 | 1988-1990 |
| American Trustee Life Insurance Company of Minnesota | 1,550 | 1988-1996 |
| AMS Life Insurance Company | 68,492 | 1993-2003 |
| Andrew Jackson Life Insurance Company | 10,978 | 1992-2003 |
| Associated Life Insurance company | 800 | 1989-2001 |
| Baldwin United | 3,823 | 1988-1992 |
| Benefit Life Insurance Company | 28 | 2001 |
| Centennial Life Insurance Company | 100 | 1998-2004 |
| Coastal Insurance Company | 170 | 2990-2001 |
| Coastal States Life Insurance Company | 18,159 | 1996-2003 |
| Columbia Life Insurance Company | 5,147 | 1991-1992 |
| Confederation Life Insurance Company | 45,034 | 1995-2003 |
| Consolidated National Life Insurance Company | 1,401 | 1994-2000 |
| Consumers United Insurance Company | 5,495 | 1993-2002 |
| Continental Security Life Insurance Company | 2,500 | 1989-1996 |
| Continental Trust Life Insurance Company | 2,494 | 1988-1990 |
| Corporate Life Insurance Company | 74,080 | 1994-1996 |
| Credit Life Corporation of America | 90 | 1994 |
| Diamond Benefits Life Insurance Company | 5,958 | 1989-2004 |
| Eagle Life Insurance Company | 80 | 1997-2004 |
| Excalibur Life Insurance Company | 5,000 | 1991-2001 |
| Executive Life Insurance Company | 1,390,989 | 1991-2004 |
| Family Guaranty Life Insurance Company | 4,951 | 1999 |
| Farm & Home Life Insurance Company | 18,736 | 1990-1996 |
| Farmers & Ranchers Life Insurance Company | 885 | 1999-2000 |
| Fidelity Bankers Life Insurance Company | 2,123 | 1995-2004 |
| First Capital Life Insurance Company | 1,917 | 1992-2000 |
| First Columbia Life Insurance Company | 1,929 | 1988-1996 |
| First National Life Insurance Company of America | 18,560 | 1999-2004 |
| Franklin American Life Insurance Company | 89 | 2000-2003 |
| Franklin Protective Life Insurance Company | 2,083 | 1999-2001 |
| Galaxia Life Insurance Company | 42 | 1990-2000 |
| George Washington Life Insurance Company | 215 | 1991-2004 |
| Gibson National Life Insurance Company | 2,983 | 1990-1991 |
| Great Lakes American Life Insurance Company | 48 | 1990-2003 |
| Great Southwest Life Insurance Company | 432 | 1990-1996 |
| Guarantee Security Life Insurance Company | 183,795 | 1991-2004 |
| Inter-American Insurance Company of Illinois | 36,918 | 1992-2004 |
| International Financial Services Life Insurance Company | 278 | 1999-2003 |
| International Security Life Insurance company | 84 | 1993-1995 |
| Investment Life Insurance Company of America | 21,370 | 1993-2003 |
| Investors Equity Life Insurance Company of Hawaii, LTD | 22,525 | 1994-1998 |
| Kentucky Central Life Insurance Company | 10,844 | 1994-2004 |
| Knickerbocker Life Insurance Company | 9,399 | 1989-2000 |
| Legacy Life Insurance Company | 451 | 1991-2000 |
| Life Assurance Company of Pennsylvania | 25,521 | 1991-2001 |
| Life Insurance Company of America | 6,680 | 1988 |
| Life of Indiana Insurance Company | 2,495 | 1990-2000 |
| Life of Montana Insurance Company | 12,663 | 1988-1992 |
| Lincolnwood National Life Insurance Company | 10,439 | 1991-2002 |
| London Pacific Life & Annuity Company | 53,475 | 2004 |
| Louisiana Security Life Insurance Company | 85 | 1995-1997 |
| Midwest International Life Insurance Company | 946 | 1993 |
| Midwest Life Insurance Company | 77,999 | 1990-2002 |
| Mutual Benefit Life Insurance Company | 21,067 | 1992-2004 |
| Mutual Security Life Insurance Company | 118,786 | 1991-2003 |
| National Affiliated Investors Life Insurance Company | 35 | 2000-2001 |
| National American Life Insurance Company of Pennsylvania | 26,823 | 1996-2003 |
| National Annuity Life Insurance Company | 172 | 1995-1996 |
| National Heritage Life Insurance Company | 282,250 | 1995-2004 |
| National Investors Life Insurance Company | 8,456 | 1988 |
| New Jersey Life Insurance Company | 21 | 1993-2003 |
| Old Colony Life Insurance Company | 15,092 | 1993-2003 |
| Old Faithful Life Insurance Company | 3,448 | 1992-1994 |
| Pacific Standard Life Insurance Company | 16,962 | 1993-2003 |
| Senior Security Life Insurance Company | 198 | 1994-1996 |
| Sierra Life Insurance Company | 542 | 1991-1998 |
| Southern National Life Insurance Company | 18 | 1990-2000 |
| Summit National Life Insurance Company | 32,418 | 1994-2003 |
| Twentieth Century Life Insurance Company | 32,050 | 1991-1996 |
| U.S. Annuity Life Insurance Company | 1,399 | 1989-1997 |
| Underwriters Life Insurance Company | 514 | 1990-2002 |
| Unison International Life Insurance Company | 10,009 | 1993-2004 |
| United Life of North America | 6 | 1993-1996 |
| Universal Security Life Insurance Company | 6,300 | 1989 |
| World Life & Health Insurance company of Pennsylvania | 8,328 | 1991 |
| **Grand Total:** | 2,821,852 | |

**Table 2**

## National Organization of Life & Health Guaranty Associations

Coverage Limits for Annuities in Payout Status, as of December 31, 2005

| | | | |
|---|---|---|---|
| Alabama | $300,000* | Montana | $100,000 |
| Alaska | $100,000 | Nebraska | $100,000 |
| Arizona | $100,000 | Nevada | $100,000 |
| Arkansas | $300,000 | New Hampshire | $100,000 |
| California | $100,000** | New Jersey | $500,000 |
| Colorado | $100,000 | New Mexico | $300,000 |
| Connecticut | $500,000 | New York | $500,000 |
| Delaware | $100,000 | North Carolina | $300,000 |
| Washington, D.C. | $300,000 | North Dakota | $100,000 |
| Florida | $300,000* | Ohio | $100,000 |
| Georgia | $300,000 | Oklahoma | $300,000 |
| Hawaii | $100,000 | Oregon | $100,000 |
| Idaho | $300,000 | Pennsylvania | $300,000 |
| Illinois | $100,000 | Puerto Rico | $100,000 |
| Indiana | $300,000 | Rhode Island | $100,000 |
| Iowa | $300,000 | South Carolina | $300,000 |
| Kansas | $100,000 | South Dakota | $300,000 |
| Kentucky | $100,000 | Tennessee | $100,000 |
| Louisiana | $100,000 | Texas | $100,000 |
| Maine | $100,000 | Utah | $200,000 |
| Maryland | $100,000 | Vermont | $100,000 |
| Massachusetts | $100,000 | Virginia | $100,000 |
| Michigan | $100,000 | Washington | $500,000 |
| Minnesota | $360,000*** | West Virginia | $100,000 |
| Mississippi | $100,000 | Wisconsin | $300,000 |
| Missouri | $100,000 | Wyoming | $100,000 |

Source: NOLHGA. Note that these limits apply to the state of residency on the date of the liquidation order, and not those applicable in the state of residency on the date of purchase.
*Guaranty association provides $300,000 coverage if annuity is in payout status; if in deferral status, the cash value limit is $100,000.
**Benefits for annuity policies in California are covered at 80% of the contractual obligation, subject to the statutory limit.
***The Minnesota association's benefit limit listed reflect adjustment for inflation in accordance with a US Department of Commerce index as described in §61B.19 Subd. 6. The $360,000 limit is for annuities in payout status; for annuities in deferral status, there is currently a $120,000 limit.

**Figure 1**
**Optimal Annuitization for Different Values of Rf and rho**

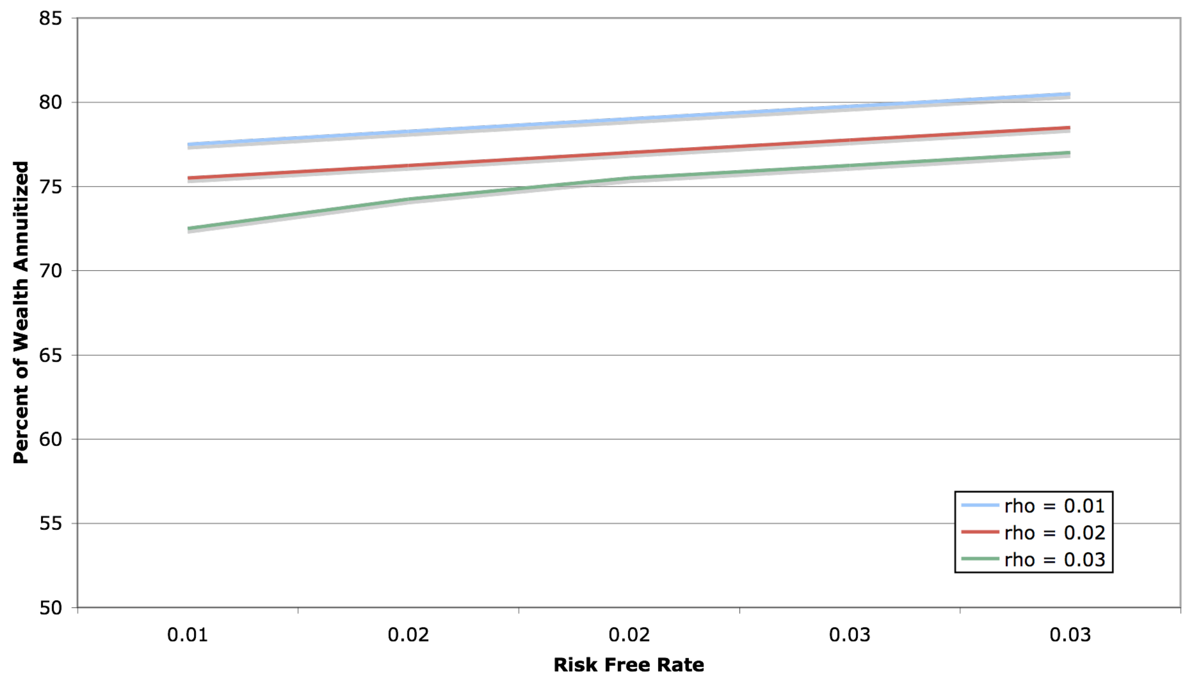



**Figure 2**
**Optimal Annuitization for Different Values of Sigma and Risk Premium**

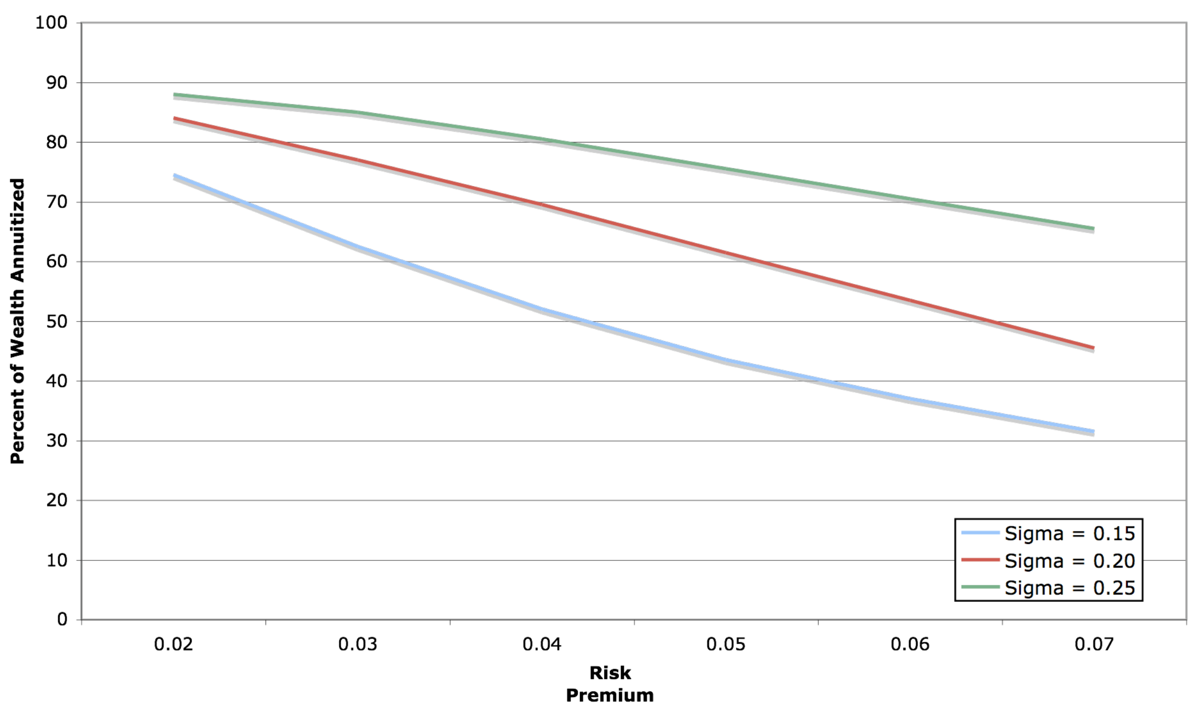


**Figure 3**
**Optimal Annuitization for Different Values of Gamma and Markup**

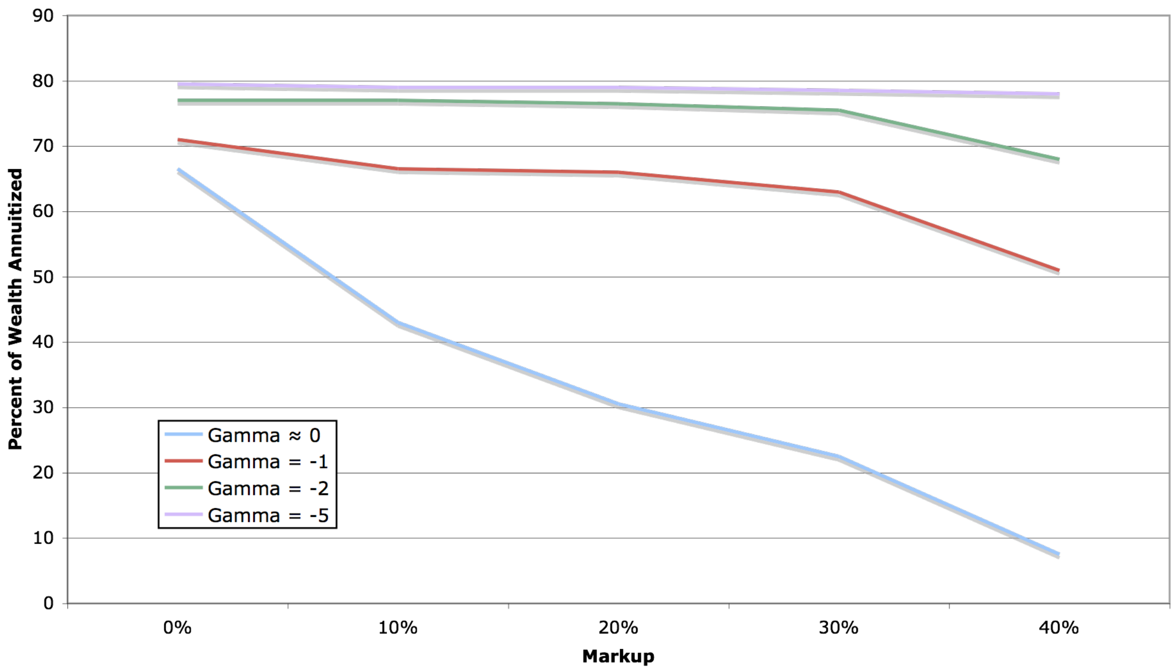



**Figure 4**
**Optimal Annuitization for Different Levels of Default Risk**

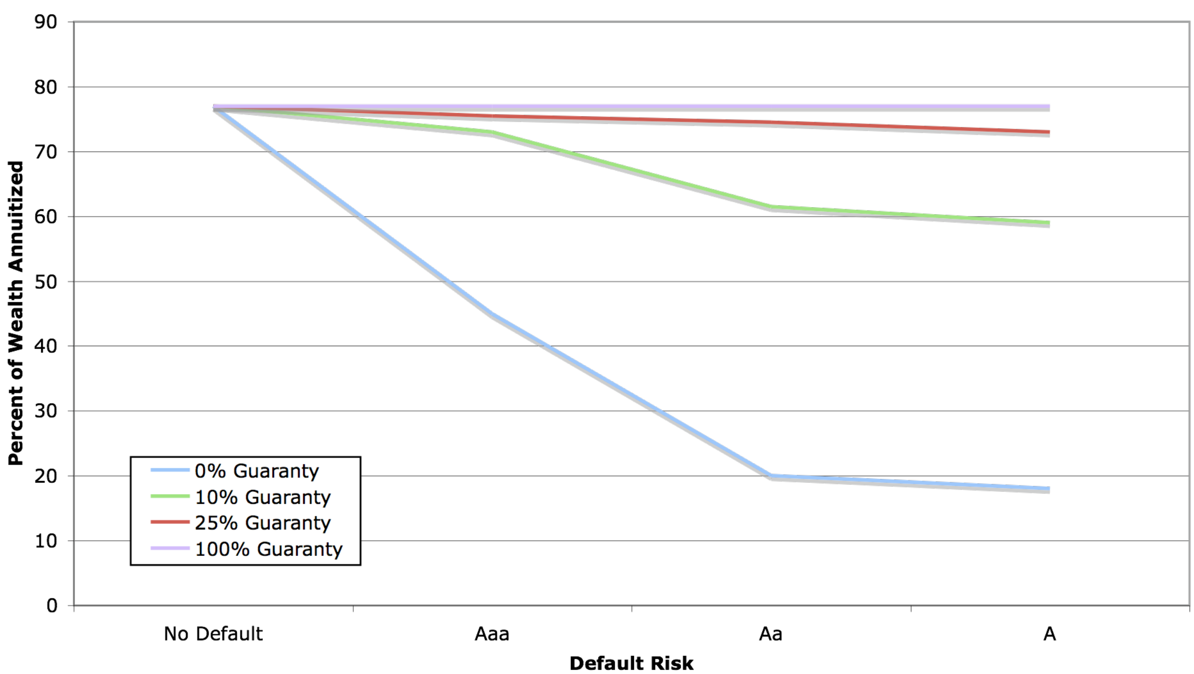
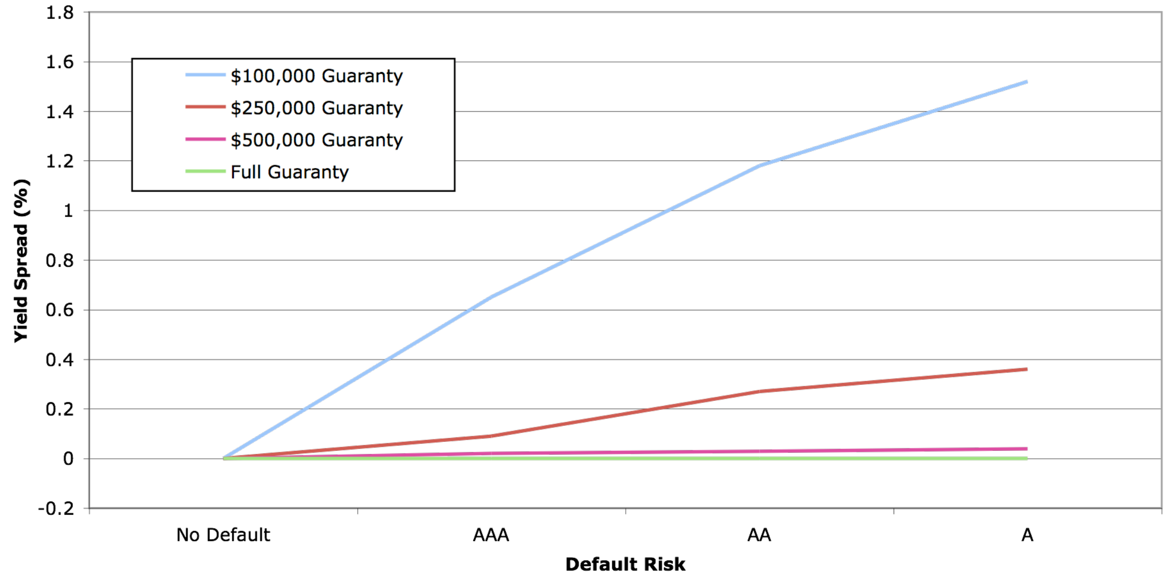
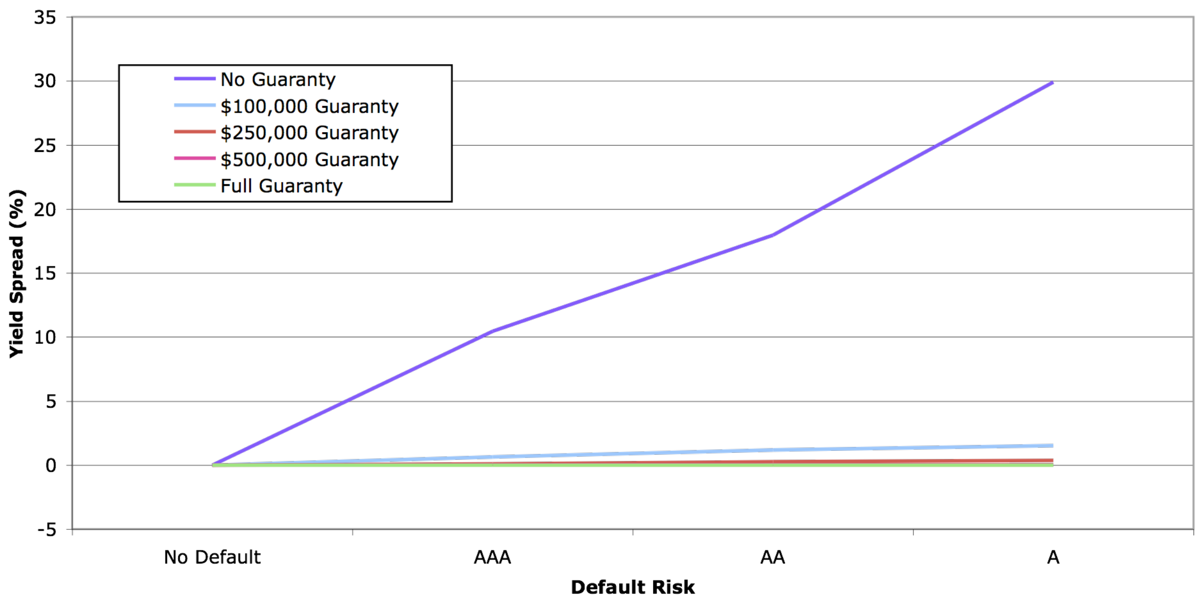


**Figure 5a**
**Indifference Annuity Yield for Different Levels of Default Risk**



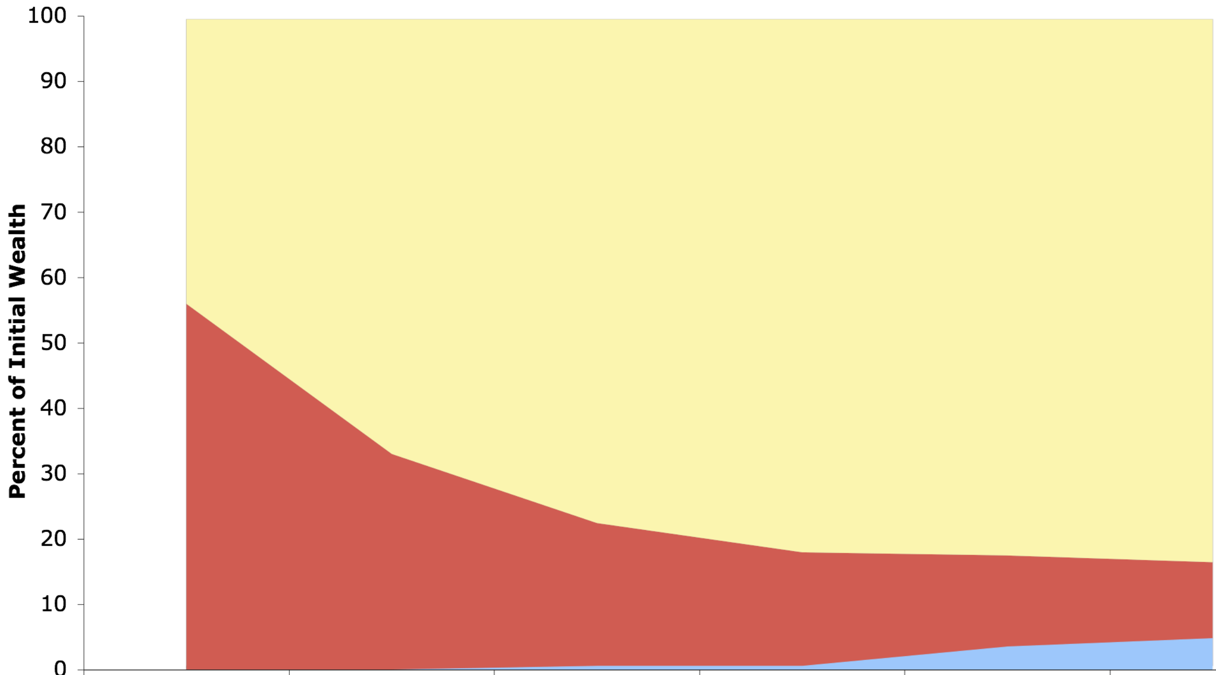

Note that the annuity yields shown are based on the purchase of a $770,000 annuity, or 77% of $1,000,000 initial wealth. For smaller annuities, the requisite annuity yields would be less because a greater proportion of the annuities would be coered by the guaranty, and for larger annuities, a higher spread would be needed to make the annuitant indifferent among annuity providers.

**Figure 5b**
**Indifference Annuity Yield for Different Levels of Default Risk**




The scale is changed in Panel 5b to accommodate the extraordinary yield spreads required to attract purchasers when zero recovery is assumed in the event of insurer insolvency. The lines depicting requisite spreads under the $250,000 and $500,000 guaranty programs overlap under this scale due to compression. The different spreads such programs would elicit are revealed better under the scale used in Panel 5a, where zero recovery is omitted.

**Figure 6**
**Initial Asset Allocation for Varying Risk Aversion**




| | 0 | -1 | -2 | -3 | -4 | -5 |
|---|---|---|---|---|---|---|
| Annuity | 43.5 | 66.5 | 77 | 81.5 | 82 | 83 |
| Stock | 55.9 | 32.9 | 21.9 | 17.4 | 13.9 | 11.6 |
| Bond | 0.11 | 0.11 | 0.61 | 0.61 | 3.61 | 4.91 |

**Gamma**