ECONOMIC VALUATION MODELS FOR INSURERS

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ABSTRACT

Recently much attention has been given to the approaches insurers undertake in valuing their liabilities and assets. For example, in 1994 the American Academy of Actuaries created a Fair Valuation of Liabilities Task Force to address the issue [see Doll et al. (1998), Reitano (1997), Babbel (1997, 1998b), Babbel and Merrill (1996), and Merrill (1997)]. In 1997, the Academy established a Valuation Law Task Force and a Valuation Tools Working Group to investigate the various valuation approaches extant and to recommend the models best suited to the task.

Much of the literature on valuation has focused on the strengths and shortcomings of the various models. Some of the work has addressed larger questions, but in our view it is useful and necessary to provide a taxonomy of approaches and evaluate them in a systematic way in accordance with how well they achieve their aims.

In this paper we focus primarily on the economic valuation of insurance liabilities, although we do address some valuation issues for assets. We begin by defining insurance liabilities in Section I. Next, in Section II we discuss the criteria for a good economic valuation model and provide a taxonomy of valuation models in Section III. In Section IV, we examine insurance liabilities in the context of this taxonomy and identify the minimum requirements of an economic valuation approach that purports to value them adequately. An illustration of the application of a modern valuation model is given in Section V. We conclude in Section VI by discussing some limitations of our analysis and offer some recommendations for implementation.

I. INSURANCE LIABILITIES

We have argued elsewhere that a useful starting point for considering the economic valuation of insurance liabilities is their present value [see Babbel (1998b, 1997) and Merrill (1997)]. It turns out that this is not only a good starting point, but perhaps also a good ending point insofar as liability valuation is concerned; this can be seen by considering the sources of uncertainty underlying insurance liabilities. We can divide uncertainty into three categories: actuarial, market, and nonmarket systematic risks.

A. Actuarial Risks

Actuarial risks include such things as casualty, liability, morbidity, and mortality risks. These risks are the insurance company's bread and butter. Exposure to these risks is managed through diversification and by writing large numbers of similar policies. Because an insurance company can reduce actuarial risks to an arbitrarily small level through risk pooling and diversification, we need not account for them, beyond their expected costs, in valuing liabilities.[1] Any residual exposure can be addressed in the process of determining surplus requirements.

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[1]We are speaking here, of course, of relative risks and not absolute risks. If the insurer elects not to reduce these risks, investors in insurance stocks can easily do so by diversifying their own portfolios of insurance and other stocks. Hence, they will refuse to pay a premium for the stock of a company that does not reduce its actuarial risks.

Our approach differs from most traditional actuarial *reserving* methods in that such methods typically adjust for actuarial risk (and sometimes even for indirect expenses) by reducing the discount rates to reflect a risk charge.[2] By entangling the actuarial risk charge with the reserves, the procedure produces conservative estimates of reserves. In essence, reserves so stated are a measure of liabilities together with a portion of the economic surplus deemed necessary to maintain solvency.[3] In contrast to the traditional approach, we suggest separating the value of liabilities from any element of economic surplus. This allows a closer, more explicit examination of the liabilities' economic value and behavior over time and across various scenarios and enables a more precise analysis of the amount of economic surplus needed to maintain solvency.

B. Market Risks

Market risks include fluctuating interest rates, inflation rates, and exchange rates. Insurance companies are exposed to these market risk factors. Their exposure can be measured and priced because of the existence of an active market for securities that are exposed to the same sources of uncertainty. Then their risk exposure can be managed using Treasury instruments and their derivatives, and forward exchange contracts where necessary.

C. Nonmarket Systematic Risks

Nonmarket systematic risks include changes in the legal environment, tax laws, or regulatory requirements. It is not generally possible to hedge these risks. Such nonmarket systematic risks are not easily handled in valuation models. However, relative to market risks they generally have a much less significant impact on the present value of insurance liabilities.

The lack of explicit models of these nonmarket systematic risks has not hindered the development of valuation tools such as mortgage-backed-security pricing

models. The chance of a tax code change removing mortgage interest rate deductibility may be no less remote than a change in the legal environment surrounding insurance claim settlements, yet these potential changes are not explicitly incorporated into any of the valuation models in use today. They may, however, be reflected in the spread over Treasuries that these securities elicit or, more appropriately, a deduction from the expected inflows that the securities will generate to their investors.

D. Valuing Liabilities

In our opinion, insurance valuation models should focus on the present value of liabilities, including their market risk sensitivities. The present value of liabilities tells us how much money an insurer would need today to satisfy, on a probabilistic basis across various economic states of the world, the obligations imposed on it through the insurance policies it has written. Here, "economic states" are defined broadly to incorporate market factors such as interest rates and inflation that can be hedged with financial securities. Treasury securities and their derivatives are the natural hedge instruments for such obligations, because they do not introduce credit risk or other extraneous risks.

An insurer may hope to satisfy its liabilities with fewer assets by making interest rate, equity, or low-credit-quality bets, but hope should not be confused with expectation. The present value, properly computed via Treasury-rate-based lattices, simulations, or closed-form solutions to stochastic differential equations, takes into account any interest and inflation rate sensitivities in the cash flows. To the extent that lapse, surrender, or other factors cause mortality, morbidity, and accident frequency and severity to be related to interest and inflation rates, they are reflected directly in the modeling of cash flows.[4] Economic surplus needed to cushion variations from these interest-sensitive projections is not included in the valuation of liabilities; however, a good estimate of the present value of liabilities[5] is necessary for determining the amount of economic surplus needed.

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[2]Indeed, our approach is not intended to be a *reserving* method at all. It is more closely aligned, in spirit, with producing an actuarial value and is embraced by that definition. The major difference in what we are proposing from the typical *implementation* of estimating an actuarial value is that we take into account explicitly the impact of interest rate sensitivities of expected cash flows in deriving their economic value.

[3]"Economic surplus" is sometimes referred to as "net tangible value." It is derived by subtracting the present value of liabilities from the market value of tangible assets. For a thorough discussion of the importance of this concept, see Babbel (1998a).

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[4]An early example of this approach was first published in 1989 and republished in Asay et al. (1993).

[5]It is important to keep in mind that our estimate of the economic value of liabilities must not be misconstrued as an estimate of policy reserves. Were reserves to be set at the level of the liabilities' economic value, they would prove to be inadequate over the short run half of the time and inadequate over the long run with certainty [see Buhlmann (1970)].

II. CRITERIA FOR A VIABLE ECONOMIC VALUATION MODEL

Several attributes are essential for a viable economic valuation model. We propose the following seven criteria:

Relevant Values. A model that purports to be an economic valuation model should produce values that are consistent with the definition of economic value that is being used so that results will be relevant and meaningful. This criterion is undoubtedly the most important one and a precursor for the others. Another type of model should not be substituted for an economic valuation model simply because the former is easier to apply; if its focus is on the wrong thing or if it simply is not designed to adequately address the dimension of economic valuation, then it should not be used for economic valuation. When practitioners attempt to use an accounting model or other such dubious method to estimate economic value, it reminds us of the legendary intoxicated man who was searching under a street light for his lost keys. An approaching police officer inquired of the hapless man what he was doing, and the man explained that he had lost his keys near a parking meter some distance away, but that it was easier to search for them under the bright light than around the locality at which they were dropped.

Practicality of Implementation. A viable model should be implementable. It almost goes without saying that the reach of a valuation model must not exceed its (and our) grasp. The generality of the model must be tempered by our ability to implement it. While it is common among academicians in the field of financial economics to develop theoretical models that are far removed from any current application, the actuarial profession cannot afford that luxury. Actuaries require models that can be developed readily and implemented with available computer technology. A model that fails on this front may offer future promise but cannot help with the valuation work that needs to be done today.

Consistent Prices. A viable model should produce consistent prices across all assets and liabilities. Many valuation models in use today are specialized for only one class of assets or liabilities: for example, an option-pricing model may be used to price options; an equity valuation model may be used to price common stock; a bond-pricing model may be used for bonds; and an actuarial model may be used for insurance liabilities.[6] The only consistency obtained in using such disparate models is that they render a value in dollar terms. It would be far better to have a general valuation model capable of valuing most, if not all, classes of assets and liabilities. This would encourage consistency of assumptions and is necessary for generating future economic scenarios for solvency testing and business planning. Parsimony should not be mistaken for efficiency: while it is correct that a particular financial instrument, such as a short-term default-free noncallable bond, may require only a single random factor to generate an appropriate value, that does not imply that such economy in modeling should be the guiding principle in model selection. A more inclusive model may incorporate factors that are not immediately germane to the valuation of a particular instrument but that allow the valuation of not only the financial instrument in question but a host of other instruments as well, such as callable corporate bonds, in a consistent fashion.

Calibration to Prices. A viable model should calibrate closely to observable prices. Economic value is the same as market value when the financial instrument in question is tradable in an active market. The standard for a good fit, which has been widely accepted in recent years, is that the model should closely match observed Treasury prices, particularly "on-the-run" Treasury bills, notes and bonds, and the prices of Treasury derivatives, including options on Treasury futures. These prices are important because the instruments are widely traded and very liquid. We have confidence that reported prices are not stale and that there is sufficient trading volume subject to the discipline of arbitrage forces to ensure that prices are at appropriate levels. The model should also calibrate reasonably well to the market prices of other financial

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[6]Practitioners on Wall Street sometimes clumsily attempt to apply the same model for pricing Treasury and corporate bonds. Because a model that prices Treasuries well does not fit well the observed market prices of corporate bonds, practitioners often assume a different (lower) volatility of short-term Treasury rates than that used for pricing the Treasuries themselves. While this *ad hoc* procedure may produce model price estimates that are closer to corporate bond market prices, it clearly violates the consistency standards of accepted valuation principles.

instruments, but insofar as valuing insurance liabilities is concerned, calibrating to other financial instruments is unnecessary.[7]

Noncontroversial Principles. A viable model should be noncontroversial. It should be based on well-established principles and accepted valuation practices. It would be unwise for any body of professionals to adopt a valuation model for reporting or managerial purposes that is based on fanciful theory or unproven technology.

Specificity and Auditability. A viable model should be specifiable and auditable. By ''specifiable,'' we mean that regulators and managers ought to be able to specify a set of economic parameters to be used in the valuation for studying a company. Of course, the best values of the parameters for valuation are those that are inferred from observed market prices and the behavior over time of interest rates and other economic factors that influence value. But a viable model should also be open to a set of parameter specifications from parties who have an interest in promoting solvency and managing a company. Once the parameters are estimated or otherwise specified, the output of the model should be amenable to auditing, to ensure that it is producing scenarios and values consistent with those parameters.

Value Additivity. A viable model should feature value additivity. By this, we mean that the values estimated for various parts of a portfolio of assets or liabilities should be equal to the value of the entire portfolio when summed together.[8] In a world that does not foster costless arbitrage, the sum of the parts may not be exactly equal to the value of the whole, but should fall within some bounds related to transactions costs.

Having discussed seven criteria needed in a viable valuation model, we next proceed to a discussion of the menu of available models, their relation to one another, and their inclusiveness of the factors that determine economic value.

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[7]The reason it is unnecessary to have the model calibrate closely to market prices of other financial instruments is that only the default-free rates of interest are used in deriving the economic value of insurance liabilities; see Babbel (1994, 1997, 1998b).

[8]As explained in Section V, while the values must be consistent with the value additivity principle, the risks need not be additive.

III. A TAXONOMY OF VALUATION MODELS

There are many seemingly different valuation models in the literature. Some examples include general equilibrium models, partial equilibrium models, contingent claims models, the Black and Scholes option-pricing model, arbitrage pricing theory (APT), the capital-asset-pricing model (CAPM) and its variations, and a wide variety of models for pricing interest rate-contingent claims. In this section we introduce a simple taxonomy for understanding the relationships between valuation models.

In any valuation model there will be cash flows, with their associated probabilities of occurrence, and discount rates. A key feature of financial valuation models is how they account for risk in the cash flows. There are three common methods for handling risk: modifying the cash flows, the discount rate, or the probabilities.

Use of a Certainty Equivalent Cash Flow. The certainty equivalent cash flow is the one that would make the investor indifferent between a known outcome and facing a random draw on a set of risky cash flow outcomes (Robichek and Myers 1965). This approach depends on the specification of a utility function. Given an assumed utility function, individuals' risk aversion may be ranked by comparing their certainty equivalent cash flows. This approach is not often used in security valuation models, but has been profitably applied to capital budgeting problems.

Risk Premium Approach. The second approach is to embed a risk premium in the discount rate to allow for the excess return that investors seek as compensation for bearing risk (Robichek and Myers 1965, Rubinstein 1976). In practice, valuation models for fundamental securities like stocks, bonds, or projects within a firm tend to use the risk premium approach. The CAPM and APT are well-known models that have been developed specifically to estimate risk premiums.

Risk-Neutral Option Pricing. In risk-neutral pricing, the probability distribution is typically adjusted to compensate for risk so that cash flows can be discounted at the risk-free rates as if investors were risk neutral. Risk-neutral valuation is most commonly applied to derivative securities. It is important to keep in mind that with risk-neutral valuation, the price is determined by the absence of arbitrage—the existence of the risk-neutral measure is synonymous with

the absence of arbitrage.[9] Thus, investors with widely varying levels of risk aversion can agree on the price yielded by a risk-neutral model because it represents the price that allows no arbitrage opportunities. However, there need be no inconsistencies between the models; properly implemented, all approaches will lead to the same valuation.[10]

Our framework for considering valuation models can be represented with a two-by-two matrix depicting two dimensions along which a model may be extended or simplified (Table 1). The first column of the matrix represents models with deterministic, or known, cash flows. The second column represents models with stochastic cash flows. Similarly, the rows represent the deterministic or stochastic nature of interest rates as used in a model. Therefore, moving to the right or down in the matrix represents an increase in the complexity of the valuation model. Cell D would be the most general model, featuring stochastic cash flows and interest rates. We present valuation equations for each of the four cells. Each valuation equation is in terms of a single cash flow; securities with multiple cash flows are treated as portfolios of single cash flows.

Table 1
Matrix of Valuation Model Complexity

	Deterministic Cash Flows	Stochastic Cash Flows
Deterministic Interest Rates	A	B
Stochastic Interest Rates	C	D

In our matrix, cell A represents the simplest class of models. Both the sequence of future interest rates and the future cash flows are known with certainty today, at time zero. Let x_t be a cash flow at time t. Let $r_t(n)$ represent an n-period spot rate of interest at time t. Define the short rate to be the spot rate that applies to the shortest time increment. For example, in discrete time the one-period spot rate, $r_t \equiv r_t(1)$, would be the short rate of interest. In continuous time, the instantaneous spot rate, $r_t \equiv r_t(\Delta t)$, would be the short rate. In the equations below, we render the continuous-time formulations.

Given these definitions, our valuation equation for cell A is

$$V_0 = x_t \exp\left(-\int_0^t r_s \, ds\right). \quad (1)$$

This equation includes the simple present value formula from introductory finance, in which interest rates are assumed constant as a special case; that is,

$$V_0 = x_t \exp[-r_0(t)t].$$

In cell B we assume that the cash flow is stochastic and possibly affected or determined by probabilistic factors. The cash flow is denoted by $x_t(\mathbf{y}_t)$, where $\mathbf{y}_t$ is a vector of sources of randomness. For example, payments to an auto insurance policyholder are determined by the random losses that occur during a period of coverage. The loss distribution is usually described by frequency and severity distributions. In this case the vector, $\mathbf{y}_t$, might be composed of two components: a random variable for frequency and a random variable for severity. The actual payment, $x_t(\mathbf{y}_t)$, would depend on realized losses as well as on policy provisions such as deductibles and limits of liability.

Another example can be found in option pricing. Consider an equity option-pricing model. In this case $\mathbf{y}_t$ would be a scalar representing the stock price at time t. The option's cash flows are deterministically linked to the price of the stock. Thus, $x_t(\mathbf{y}_t)$ would represent the cash flow to the option.

The valuation equation for cell B is

$$V_0 = \mathrm{E}^*[x_t(\mathbf{y}_t)] \exp\left(-\int_0^t r_s \, ds\right), \quad (2)$$

where the asterisk denotes that the expectation is being taken over the risk-neutral probability measure of the stochastic factors that determine the cash flow. If we can exactly replicate the cash flows of a security using a portfolio of other securities, then the security of interest and its replicating portfolio should have the same price. Otherwise, there would be an arbitrage opportunity. As we stated above, Harrison and Pliska (1979) show that if there are no arbitrage opportunities, then we can construct a probability measure under which we can price risky cash flows as if investors were risk neutral. This is the risk-neutral, or equivalent martingale, measure. The risk-neutral measure is the probability measure that sets Equation (2) equal to the price of the replicating portfolio. A risk-neutral measure can be used in place of the realistic (or ''true'') probability distribution, because with perfect replication of cash flows, the replicating portfolio

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[9]Jarrow and Turnbull (1996) state this proposition on page 163 in the context of a binomial pricing model. A more general proof is given in Harrison and Pliska (1979).

[10]See Singleton (1989) and Babbel and Merrill (1996, pp. 40–44) for a more thorough discussion of the relationship between risk-neutral pricing and pricing based on a risk premium.

and the security being valued must have the same price or else there would be an arbitrage opportunity. Thus, all investors regardless of their level of risk aversion will agree on the price.

Equation (2) says that the value of a stochastic future cash flow is the present value of the risk-neutral expected cash flow, discounted at the risk-free rate. Alternatively, we could specify this valuation equation as

$$V_0 = \mathrm{E}[x_t(\mathbf{y}_t)] \, \exp\left(-\int_0^t [r_s + \lambda(s)]ds\right), \quad (2')$$

where the expectation is over the true probability distribution and $\lambda(t)$ is the (possibly time-varying) risk premium. In complete markets in which the components of $\mathbf{y}_t$ are actively traded, Equations (2) and (2′) will yield identical valuations: for example, options could be valued using either equation. However, when the underlying source of uncertainty is not actively traded, Equation (2′) is more likely to be used. Thus, when valuing auto insurance where $\mathbf{y}_t$ would be composed of the random variables representing frequency and severity, Equation (2′) is used.

The market price of risk must be estimated; this is usually accomplished by specifying a utility function for a representative investor. The risk aversion associated with the assumed utility function determines the functional form of $\lambda(t)$. The market price of risk can then be determined empirically by calibrating the model to market data—that is, estimating $\lambda(t)$ to best equate model prices to market prices.

Examples of models that fall within cell B are the CAPM, APT, Black-Scholes option-pricing model, and many traditional insurance-pricing models. In each case, some sort of distribution of future cash flows is assumed and the expectations of those cash flows are discounted at deterministic, and often constant, interest rates. While the CAPM and APT are referred to as pricing models, they actually determine an appropriate discount rate to compensate for risk when discounting risky cash flows. Thus, they fall within the category of risk premium models. The Black-Scholes model, on the other hand, is the classic example of Equation (2). Their pricing model is derived using the absence of arbitrage between the option and a dynamically rebalanced portfolio that exactly replicates the option's potential cash flows.

In cell C we assume that cash flows are known in advance but that interest rates are stochastic. This is the approach taken in fixed-income valuation models, such as those of Cox, Ingersoll, and Ross (1985), Brennan and Schwartz (1977), or Hull and White (1990).

The valuation equation in this case is

$$V_0 = x_t \, \mathrm{E}^*\left[\exp\left(-\int_0^t r_s \, ds\right)\right], \quad (3)$$

where the asterisk once again represents the risk-neutral expectation operator. If the expectation is taken over the true probability distribution and we incorporate a risk premium, then the valuation equation would be

$$V_0 = x_t \, \mathrm{E}\left[\exp\left(-\int_0^t [r_s + \lambda(s)]ds\right)\right]. \quad (3')$$

The various theories of the term structure of interest rates put forth possible formulations for $\lambda(t)$.[11] Note that the local expectations hypothesis suggests that $\lambda(t) = 0$. This means that the expected return over the next period on all interest rate-contingent securities is the risk-free rate. If this hypothesis holds, then Equations (3) and (3′) are identical.

Finally, cell D is the most general case. Here, both cash flows and interest rates are treated as stochastic. In addition, cash flows may depend on or be influenced by interest rates. The valuation equation for cell D is

$$V_0 = \mathrm{E}_r^*\left[\exp\left(-\int_0^t r_s \, ds\right) \mathrm{E}_y^* [x_t(\mathbf{r}_t, \mathbf{y}_t)]\right], \quad (4)$$

where $\mathbf{r}_t$ denotes a vector of short rates that have occurred from time 0 to time t and the subscript on the risk-neutral valuation operator indicates over which variable the expectation is being taken.

This model is the most general in the matrix. Examples of potential applications of this formula include options on bonds, interest rate caps and floors, mortgage-backed securities, catastrophe bonds used to fund reinsurance, and insurance liabilities. The analog to Equation (4) that incorporates risk premiums in the discount rate is

$$V_0 = \mathrm{E}_r\left[\exp\left(-\int_0^t [r_s + \lambda_r(s) + \lambda_y(s)]ds\right)\right.$$
$$\left. \times \mathrm{E}_y [x_t(\mathbf{r}_t, \mathbf{y}_t)]\right], \quad (4')$$

for which the subscript on λ denotes the risk premium relative to interest rates or the factors contained in $\mathbf{y}_t$.

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[11]See Santomero and Babbel (1997, pp. 88–101) for a discussion of the various term structure hypotheses.

In reality, cells A, B, and C are just special cases of cell D; therefore, we can treat Equation (4) as the valuation model in almost all cases. We can think of Equation (4) as a model with optional features that can be turned on or off as needed. When valuing deterministic cash flows that do not require the complexity of Equation (4), as in the case of pricing a simple bond, we can "toggle off" the stochastic cash flow portion of the model and use only what is left in cell C, or Equation (3). On the other hand, when valuing short-lived equity options, the important feature is the stochastic cash flow. Modeling stochastic interest rates does not affect estimated value enough to justify injecting the added complexity, so we can "toggle off" the stochastic interest rate portion of Equation (4) and use cell B, or Equation (2). Finally, when we are teaching present value, the stochastic portion of Equation (4) can become quite confusing. Thus, we use cell A or equations similar to (1) to teach discounting and capital budgeting. At the core, though, a very wide variety of valuation models are encompassed in Equation (4).

The valuation formulas presented above can be generalized to apply to multifactor models. In most multifactor interest rate models, the additional factors relate to components of the stochastic evolution of the short rate of interest. For example, Fong and Vasicek (1991) assume that the volatility of the short rate is itself stochastic. Thus, the two factors, or sources of uncertainty, are random shocks to the short rate itself and random changes in the volatility of the short rate. Similarly, most models of the evolution of the short rate of interest incorporate mean reversion. Hull and White (1994) assume that the level to which rates revert is stochastic. In this case, the two sources of uncertainty are random shocks to the short rate and random shocks on the level to which the short rate reverts. Table 2 provides a classification of many of the models in popular use today.

One last comment on discrete-time versus continuous-time models is in order. The impact on the valuation equations given above is in the probability distributions and the integral over short rate paths. For a discrete-time model, the expectation would be over a discrete distribution and the interest rates would be for discrete periods of time. Therefore, the integral in the exponential function would have to be changed to a summation.

Table 2
Examples of Models in Each Cell of the Matrix of Valuation Model Complexity

	Deterministic Cash Flows	Stochastic Cash Flows
Deterministic Interest Rates	Gordon Growth Model (Gordon and Shapiro 1956) APV, Adjusted Present Value (Myers 1974) Weighted Average Cost of Capital (Miles and Ezzell 1980)	Black Model (Black 1976) Black-Scholes Model (Black and Scholes 1973) Merton Model (Merton 1973) Quantos Model (Babbel and Eisenberg 1993) Binomial Option Pricing (Cox, Ross and Rubinstein 1979) Mean-Variance, CAPM (Markowitz 1959, Sharpe 1964) Arbitrage Pricing Theory (Ross 1976) Stulz Model (Stulz 1982) Margrabe Model (Margrabe 1978)
Stochastic Interest Rates	CIR Model (Cox, Ingersoll and Ross 1985) Vasicek Model (Vasicek 1977) Longstaff-Schwartz Model (Longstaff and Schwartz 1992) Random Volatility Model (Fong and Vasicek 1991)	CIR Model Extension Vasicek Model Extension Longstaff-Schwartz Model Extension Random Volatility Model Extension GAT Model (Ho and Lee 1986) Derivatives Solutions Model—BDT (Black, Derman, and Toy 1990) Tillinghast Model (1979) SS&C Model (Beaglehole and Tenney 1991) BMW CFS Valuation Engine (Babbel and Merrill 1997)

IV. THE VALUATION MODEL NEEDED FOR INSURANCE LIABILITIES

In reviewing the taxonomy of economic valuation models, it appears that we must opt in favor of a model from Class D. Only models of this class are capable of producing viable estimates of economic values for financial instruments that feature stochastic cash flows influenced by stochastic interest rates.

We have argued elsewhere that virtually all insurance company liabilities exhibit these characteristics (Babbel 1990). Accordingly, if an economic valuation of insurance liabilities is our main objective and we wish to satisfy criterion (1), we must use models from Class D to obtain viable values.

While it is possible to enlist a valuation model from another class for valuing insurance liabilities, the manipulation of model parameters necessary to approximate a proper economic value is itself inconsistent with financial valuation principles. Such models are subject to imprecision, mischief, and controversy and yield a dubious result. Only by accident could they meaningfully address the question, ''How much money would an insurer need today to satisfy, on a probabilistic basis across various economic states of the world, the obligations imposed on it through the insurance policies it has written?'' Moreover, one could not determine whether the ''values'' produced by such models were close approximations to true economic values without first computing them with a Class D model.

We are fortunate indeed if we opt for an economic valuation model from that class, because it is also the best model for valuing the myriad of disparate financial assets held by most insurers. While it is true that models from Classes A, B, and C could produce viable estimates of value for some asset categories, only Class D models embrace the valuation technology that can be used with virtually all asset categories. Moreover, Class D models produce output that is relevant and meaningful. The number produced serves as a threshold above which an insurer must operate if it is to stay in business long. This is also useful to regulators who need to know how much it should take to fully defease the liabilities the insurer has underwritten. It is a number that is easily compared among insurers and meaningfully related to the market value of assets supporting the liabilities. It also is a number that serves as a sound starting point for the analysis of the amount of economic surplus needed to maintain solvency.

Now let us revisit the remaining six criteria for a viable economic valuation model to see how well Class D models may satisfy them. The criterion of practicality is met because these models are clearly implementable. Wall Street has been using Class D models for nearly a decade in the valuation of massive amounts of mortgage-backed securities. Like insurance liabilities, mortgage-backed securities are subject to a considerable amount of cash flow uncertainty, most of which devolves from fluctuating prepayment rates occasioned by changing interest rates. In the case of insurance liabilities, some of the uncertainty, such as the incidence of lapse and surrender, devolves from the vacillation of future interest rate levels and paths. This uncertainty can be modeled in a fashion similar to mortgage-backed-security pre-

payments. To the extent that uncertainty stemming from mortality, morbidity, accident experience, and some base levels of lapses and surrender is not related directly to interest rates, it can be reflected directly in the expected cash flows input into the valuation model.

With respect to the third criterion, Class D models are capable of producing consistent prices across all assets and liabilities. Of course, to achieve full consistency, we must use the same general valuation model for both assets and liabilities and not merely one or another member of the class.[12] Perhaps the greatest benefit to practitioners of using a single Class D model is in the generation of future scenarios and distributions of values. By having all liabilities and assets valued using a single model, we can be assured of consistency when it comes to modeling the effect of a change in one or more parameters on the economic well-being of the company.

Class D models calibrate well to observable market prices, where available, satisfying this criterion. Not all Class D models are created equal, however: some calibrate more closely than others. For example, single-factor models typically calibrate poorly to observable prices without relying on deterministic time-varying model parameters, which are difficult to justify theoretically (see, for example, Black, Derman and Toy 1990). Additionally, they often imply perfect correlation of movements between short-term and long-term interest rates over time, which gives a very misleading picture of interest rate risk exposure. Finally, if the single-factor models do not feature time-varying parameters, they typically require substantial distortion of parameter values in order to achieve a good fit to observed prices. However, multifactor models can achieve a far better fit to observed prices and observed economic phenomena without imposing perfect correlation in movements across the term structure and without distorting the parameters to unrealistic levels. Thus, while they are more complex and painstaking to run, their valuation capabilities and scope make them far more useful for financial institutions such as insurers. Indeed, we believe that anything less than a two-factor model is inadequate for modeling most insurance liabilities.

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[12]This is not to say that one cannot produce good estimates of economic values using different valuation models; rather, that it is preferable to use a unified model capable of valuing types of financial instruments under consideration, both assets and liabilities, in order to ensure consistency.

Class D models have been in use since 1979 and are now so ubiquitous that their use is noncontroversial from a practitioner's viewpoint. From a theoretical point of view, there are currently no valuation models that are more favorably received, despite their drawbacks. In fact, today it is far more controversial to enlist a model from another class for valuing an insurer's assets and liabilities, because it would not capture the economic importance and valuation impact of the interplay between stochastic cash flows and stochastic interest rates. Accordingly, we can say that class D models fulfill the criterion of noncontroversial principles.

Another attractive aspect of Class D models is that they are specifiable and auditable. After a particular model has been found with a suitable set of attributes, its parameters may be estimated or otherwise specified, and the model's output is ready for audit. Once a software implementation of the model has been certified for accuracy, most of the auditing attention can be turned toward the model's inputs and assumptions. Because there is so little that is subjective in Class D models, they meet this criterion well. And those elements that are subjective (for example, surrender rates as a function of interest rates) can be the focus of additional scrutiny and sensitivity testing.

Finally, we consider the value additivity principle. The way in which Class D models compute value for a single financial instrument is based strictly on the value additivity of its components. Moreover, if the same general valuation model is used for a portfolio of assets or block of liabilities, value additivity is automatically assured.

V. AN ILLUSTRATION

To concretely show the application of Class D models to insurance liabilities, we provide two charts. In Figure 1, we show a lattice of short-term interest rates (''short rates'') as they evolve in discrete steps over the time period from t to $t + 12$. Each jump in interest rates is either 10% above or below its prior level. Below the lattice we show the probability distribution of interest rates at time $t + 12$, with the scale on the horizontal axis being ordinal only. At the bottom of the figure, using a cardinal scale we show how our simple multiplicative stochastic interest rate process, when smoothed, converges to a lognormal distribution.

Figure 2 continues with our illustration to demonstrate how to interface the insurance cash flows with the lattice of short rates. At two of the nodes, we have shown how the cash flows can be modeled using an arctangent lapse function fitted to lapse data. Depending on where along the arctangent curve the interest rate spreads lie, we could get a different distribution of lapse rates. Nonetheless, the expected cash flows given by the arctangent function associated with each node would be used as inputs to the valuation process, and their value would be determined by applying the sequences of short rates that could give rise to those cash flows. In some models, a change of probability measure is enlisted and applied to each node to achieve arbitrage-free pricing.

What these figures show is how cash flow uncertainty arises from several sources. First, there is the uncertainty surrounding future interest rates. The economic valuation of liabilities would reflect this interest rate risk (as captured by the arctangent function). Second, there is additional uncertainty associated with the lapse rates at each node. This residual lapse risk is not priced explicitly, but would need to be reflected in the level of economic surplus needed to maintain solvency.[13]

These and other risks may not be additive because of their correlations with one another; accordingly, the overall uncertainty should be assessed to determine appropriate surplus levels. Furthermore, additional uncertainty stems from the use of the model itself and the assumed distributions of interest rates and cash flows. Such uncertainty would increase the amount of surplus deemed to be prudent. Finally, by construction, the model assumes that the asset managers are fully aware of the attributes of the liabilities and have taken every effort to hedge their market risks. If, instead, the investment department has strayed from matching its assets to the requisite liabilities, additional surplus would be needed to accommodate its choices. Yet the liability valuation model provides a useful benchmark that helps determine the level of surplus adequacy.

VI. LIMITATIONS AND RECOMMENDATIONS

In the final analysis, we can say that Class D models meet all seven listed criteria for a viable insurance valuation model. Two questions remain: what are the drawbacks of using such models, and which models are most suitable for insurers?

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[13]The economic surplus elicited will manifest itself, in an accounting sense, in the insurance reserves and statutory surplus that the insurer holds on its books.

Figure 1
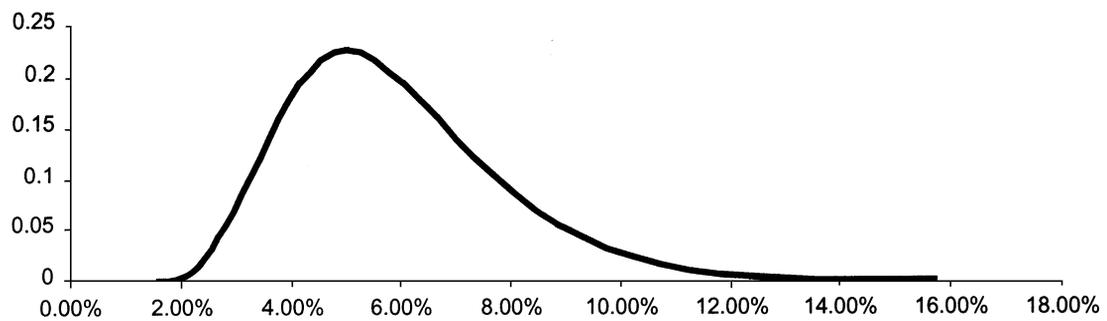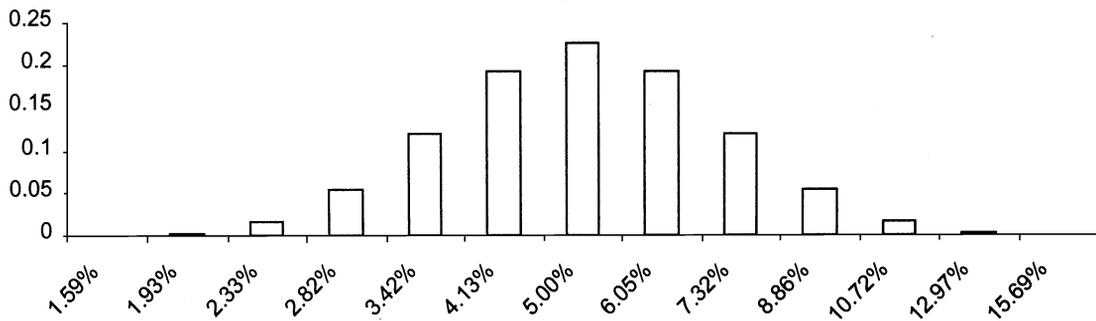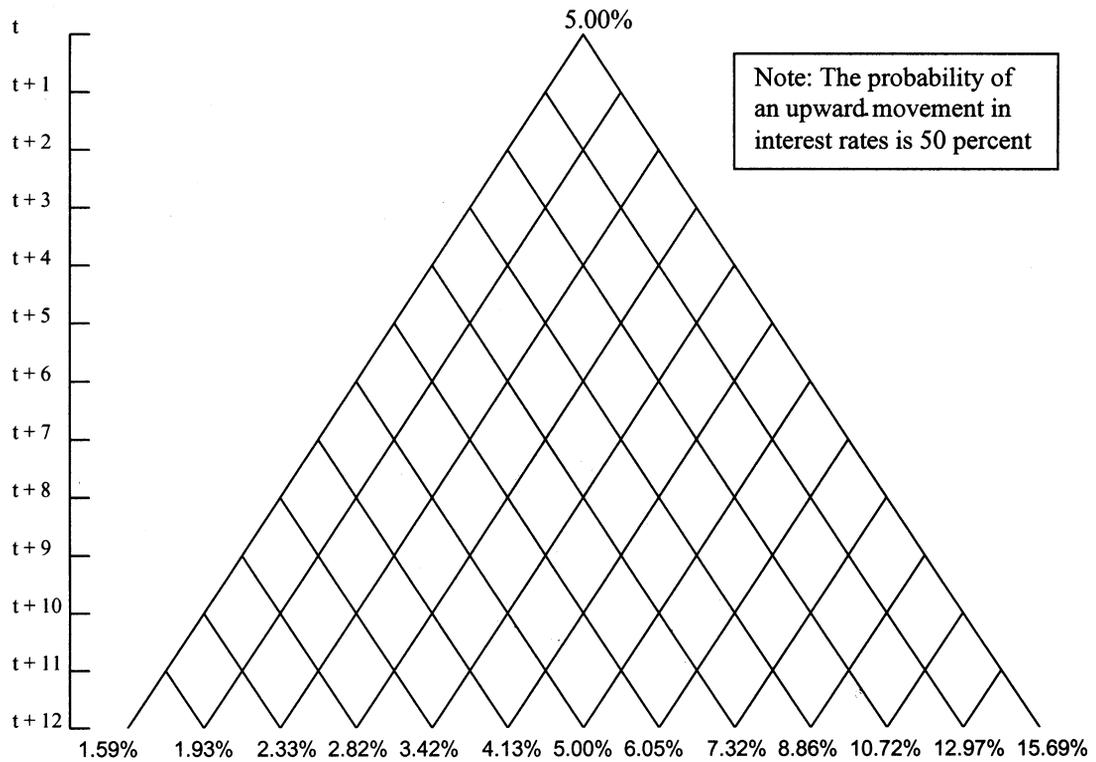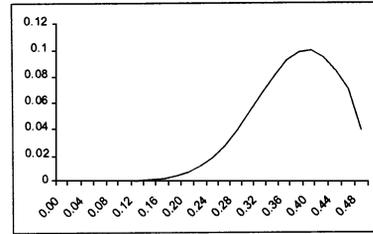**The Distribution of Interest Rates Generated by a Binomial Lattice**

Note: The probability of an upward movement in interest rates is 50 percent
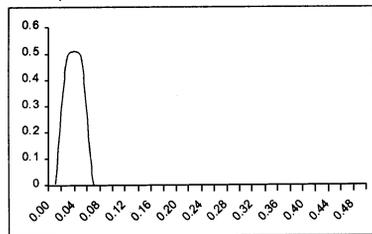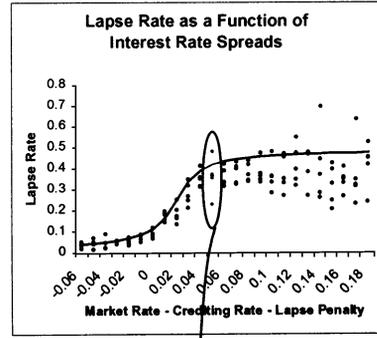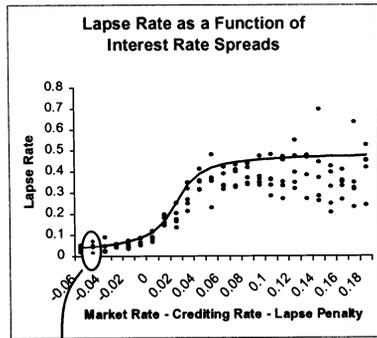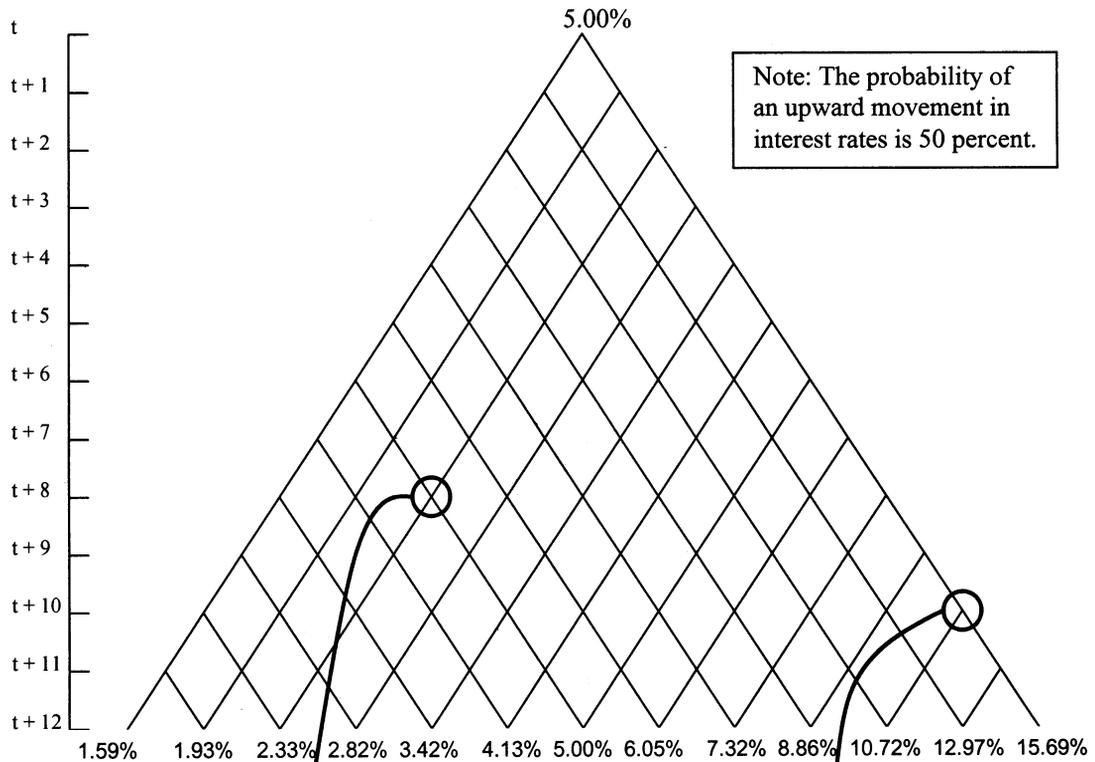
Figure 2
Uncertainty Surrounding Interest Rate-Contingent Cash Flows

The major drawbacks in using such models for insurance liability valuation purposes are manifold:

Analytic Complexity. Class D models require a higher level of analytic capability than is found at some insurance companies. However, most actuaries who are not already conversant with these models could adopt them with some directed training, because their curriculum already equips them to undertake tasks of equal complexity.

Input Availability. Class D models elicit valuation inputs that may not be readily available, such as lapse functions and crediting rate algorithms. But these inputs are also necessary with far less sophisticated models. The only nuance is to model these inputs as (perhaps fuzzy) functions of the underlying factors (such as interest rates and inflation) that drive the valuation.

Data-Processing Capacity. The computer requirements for data analysis using stochastic methods are more extensive than those for simpler accounting models. Because most insurers already have tremendous data-processing capacity, we do not see this as a binding constraint.

Acceptance by Regulators. Regulators may not embrace the output of these models until they become more familiar with them and may therefore require additional scenario testing using more primitive models. Over time, however, as regulators' comfort level increases, the advantages and insights that can be garnered through Class D models will surely impel regulators to use them. When traveling from Boston to Hartford, most people would opt for a Mercedes over roller skates, especially if the skates were never designed to go that far.

Short-Run Costs. There are additional short-run costs when going from what has traditionally been an accounting focus to an economic focus—software costs, data assembly and modeling costs, and training costs. Yet we suggest that in a competitive environment, the insurance companies that delay adopting the economic focus will in the end incur greater costs due to mispricing of policies and asset/liability imbalances.

For the selection of a suitable class D model, we are reluctant to discuss the competing ''brands'' of available products and products under development. Therefore, we restrict our comments to some general observations that may help guide an insurer in its search for a suitable model for valuing its liabilities.

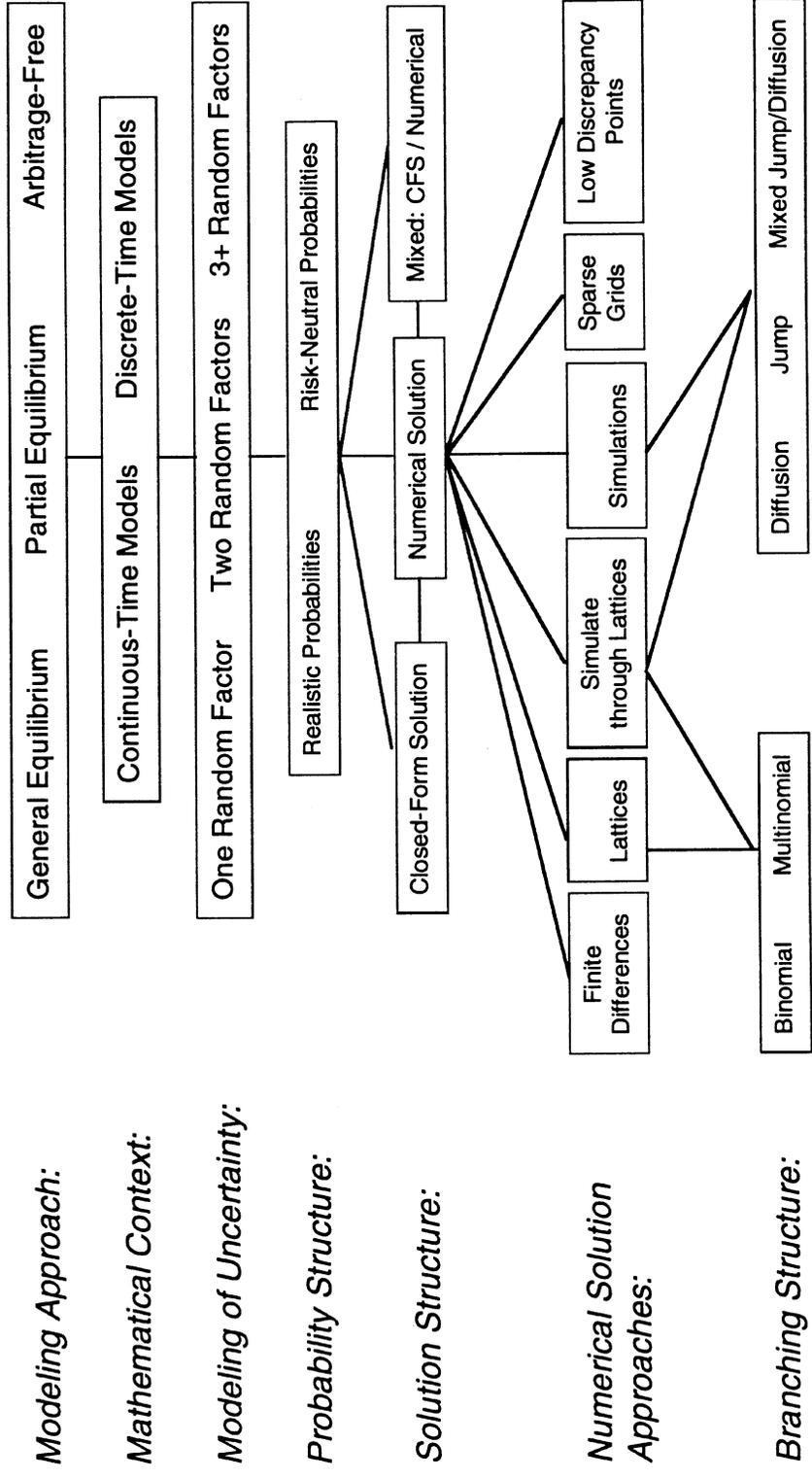Numerous variants of Class D models exist. They can be distinguished one from another by the choices they make in deriving the valuation models. In Figure 3 we list the issues that confront the insurer and the choices available. The first decision that needs to be made is whether to use an equilibrium or an arbitrage-free pricing approach. The equilibrium approach derives implications for the behavior of the term structure of interest rates from assumptions and observations about the general economy. This approach is suitable for scenario testing and ''broad brush stroke'' valuation and is especially adept at producing reasonable future scenarios for use in the management of insurance financial risk. An arbitrage-free pricing approach is more suitable for daily trading. However, it is less likely to produce helpful future economic scenarios for solvency testing, because the evolution of the term structure of interest over time under this kind of model often does not reflect certain stable economic relationships that are observed in practice.

The model chosen must also be congruent with valuation of all classes of liabilities. For insurers whose predominant asset holdings are publicly traded securities, having a model to value the assets becomes less important, because such securities have readily observable market values. What is important, however, is that the liability valuation model be consistent with observed prices on Treasury securities and their derivatives. This feature enables assets and liabilities to be simulated harmoniously over time based on random movements in the factors that determine Treasury prices. Models that tend to accommodate these needs are most often based on discrete-time approaches.

We think that the minimum configuration for a suitable liability valuation model should include at least two random factors. Which two factors to model explicitly depends on the nature of the business. For example, companies with long-tailed property/casualty lines or defined-benefit pension business should probably opt for a model whose focus is on inflation and a reference real or nominal rate of interest. This is because elements of the liabilities are strongly influenced by inflation and interest rates. (Such models would also work for other types of insurance companies, but the inflation factor would be a less direct way to model nominal interest rates.) For a life insurer without pension business, it would be sufficient for both factors to focus on nominal interest rates. Available models have factors related to short-term and long-term nominal interest rates, short-term rates and the spread between short-term and long-term rates, short-term rates and random volatility, short-term rates and a random level for mean reversion, and so

Figure 3
Choices Available for Implementing Class D Models

Choices:

Modeling Approach:
General Equilibrium | Partial Equilibrium | Arbitrage-Free

Mathematical Context:
Continuous-Time Models | Discrete-Time Models

Modeling of Uncertainty:
One Random Factor | Two Random Factors | 3+ Random Factors

Probability Structure:
Realistic Probabilities | Risk-Neutral Probabilities

Solution Structure:
Closed-Form Solution | Numerical Solution | Mixed: CFS / Numerical

Numerical Solution Approaches:
Finite Differences | Lattices | Simulate through Lattices | Simulations | Sparse Grids | Low Discrepancy Points

Branching Structure:
Binomial | Multinomial | Diffusion | Jump | Mixed Jump/Diffusion

forth. Models incorporating more than two factors are also available.

Another criterion to guide in the selection of a model is whether it features closed-form solutions for Treasury bonds. For models with such capabilities, the economies in modeling and computer intensity are simply enormous, especially for scenario analyses. While all Class D models can produce present values, either through simulation, lattice, or closed-form solutions, the advantage of a model with closed-form solutions is that it allows for the modeling of future values with little extra effort. Such models compute the entire term structure of interest rates at each future point of time under each simulation scenario without incurring any additional simulation runs. Without this feature, a model would need to incur many tens of millions of additional simulation runs to achieve a similar level of richness and to provide for a rigorous and consistent depiction of asset/liability management.

A final useful element for a valuation model is a completely open architecture. New asset and liability instruments are continually being introduced, and no valuation program without an open architecture would be useful for very long in today's dynamic economic environment.

ACKNOWLEDGMENTS

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Discussion

Jacques F. Carriere[*]

The purpose of this discussion is to demonstrate that model "D," or Equation (4), cannot be used to value a simple whole life insurance policy. Thus, it is not the most general case. The reason for this failure is that Equation (4) can be used only to value cash flows where the time of the cash flow is known with certainty.

In the ensuing discussion, I draw extensively on the general theory of non-arbitrage pricing, as given in Duffie (1992). Let r_t denote the *instantaneous short rate* at time $t \geq 0$. This is a stochastic process that is adapted to the history (σ-algebra) $\mathcal{H}_t$ with $\mathcal{H}_0$ as the trivial algebra. Next, let S_t denote a security price with D_t as the corresponding dividend (cash-flow) process. Again, assume that both processes are adapted to the history $\mathcal{H}_t$. Under non-arbitrage pricing,

$$S_t = \mathrm{E}^* \left[\exp\left\{ -\int_t^\mathfrak{I} r_s \, ds \right\} S_\mathfrak{I} + \int_t^\mathfrak{I} \exp\left\{ -\int_t^s r_u \, du \right\} dD_s \,\middle|\, \mathcal{H}_t \right],$$

where the expectation, E^*, is taken with respect to the equivalent martingale measure, denoted as Q. Insurance products are not priced in an efficient market like traded securities and so this equation is really a pricing axiom.

Next, let $T(20)$ denote the time of death of a life that is currently aged 20. In the ensuing discussion, let us assume that $T(20) < 100$. Consider the dividend process for a continuous whole life insurance of $1. This is equal to:

$$D_s = 1[s \geq T(20)].$$

Let us value this cash flow at $t = 0$ while assuming that $S_\mathfrak{I} = 0$ at $\mathfrak{I} = 100$. The pricing axiom yields:

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$$S_0 = E^* \left[\int_0^{100} \exp\left\{ -\int_0^s r_u \, du \right\} dD_s \,\Big|\, \mathcal{H}_0 \right]$$

$$= E^* \left[\exp\left\{ -\int_0^{T(20)} r_u \, du \right\} \,\Big|\, \mathcal{H}_0 \right].$$

The article would suggest that

$$S_0 = E^* \left[\exp\left\{ -\int_0^t r_u \, du \right\} \,\Big|\, \mathcal{H}_0 \right]$$

for some fixed t. The only way that this can happen is when $Pr[T(20) = t] = 1$, an unreasonable assumption.

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AUTHORS' REPLY

DAVID F. BABBEL AND CRAIG MERRILL

In his discussion Jacques Carriere addresses issues that are crucial to our suggestions on valuation. He correctly points out the difficulty in valuing a cash flow with a random arrival date. He also states that the pricing relationships in our paper must be taken as axioms rather than as implications of the absence of arbitrage. Implicit in this assertion is the observation that the valuation models are fashioned to be arbitrage-free, yet the insurance liabilities cannot be subject to the forces of arbitrage, because of both the lack of trading in secondary markets and the randomness surrounding claims and lapses. We comment on the arbitrage pricing aspects first and then address the mathematical issues.

In applying no-arbitrage pricing results to insurance liabilities, we are reminded that it was not long ago that the mortgage-backed securities market emerged. Pricing was not subject to the forces of arbitrage due to the uncertainty surrounding the prepayment speeds, and there were no close comparables traded in the market. Nonetheless, satisfactory pricing algorithms were eventually developed based on two-factor stochastic interest rate models. Consequently, option-adjusted spreads, correctly calculated,[1] have narrowed

considerably and all but disappeared in some segments of that market. This suggests that even though these valuation models cannot rely on the forces of riskless arbitrage, they can still closely approximate value.

This brings us to the other point made by Dr. Carriere. Our suggestions relate to the valuation of blocks of liabilities. Thus, the focus of our discussion is on valuing (potentially uncertain) cash flows on a known date. Dr. Carriere is exploring the pricing of a simple life insurance policy with a known cash flow arriving on an uncertain date. His discussion is correct except for the application of his observation to our paper.

It is important to remember that Equation (4) of our paper applies to an expected cash flow at a known point in time, as is the case when we are attempting to value the cash flow for a block of liabilities at time t. We can make this more apparent by rewriting Equation (4) from the paper as

$$V_0^t = E_{r,y}^* \left[\exp\left(-\int_0^t r_s \, ds \right) E_{y|r}^* [x_t(\mathbf{r}_t, \mathbf{y}_t)] \right],$$

where the superscript on V emphasizes that this is the value of a cash flow arriving at time t. Also, note that the interior expectation is taken over the marginal distribution of $\mathbf{y}$ given $\mathbf{r}$, while the exterior expectation is taken over the joint distribution of $\mathbf{r}$ and $\mathbf{y}$. This is the same as in our paper, but the notation here clarifies this observation. Thus, the term

$$E_{y|r}^*[x_t(\mathbf{r}_t, \mathbf{y}_t)]$$

is the expected cash flow *given* a particular path of the short rate of interest at time t. Then, the expectation over the joint distribution of $\mathbf{r}$ and $\mathbf{y}$ gives the time 0 value of the (potentially interest-sensitive) expected cash flow at time t.

We can extend this equation to value a block of liabilities by integrating over t:

$$V_0 \int_0^\infty V_0^t \, dt.$$

We are suggesting that one should first take an expectation with respect to the variables representing sources of uncertainty other than interest rates. Then, the interest-sensitive expected cash flows for each point in time can be valued by integrating over the

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[1] An option-adjusted spread is a "fudge factor" of sorts to capture in the interest rate lattices or paths the mispricing of the security based on the model price versus the market price [see D. Babbel

and S. Zenios (1992)]. It is common to calculate option-adjusted spreads based on some fixed level of volatility for purposes of historical charting, but when pricing the securities, the volatility assumed should relate to the future period over which the mortgages will be repaid.

interest rate paths. Finally, if the liability has potential cash flows at multiple points in time, we can integrate, or sum as appropriate, over time.

Using his notation, Carriere suggests that our paper implies

$$S_0 = E^* \left\{ \exp\left(-\int_0^t r_u \, du \right) \right\}$$

for some fixed t, that is, that we are attempting to value a cash flow with a random arrival time as if the arrival time were known.[2] That is not the case. In fact, what we are actually suggesting (again using Dr. Carriere's notation) is

$$S_0^t = E_{r,y}^* \left\{ \exp\left(-\int_0^t r_u \, du \right) E_{y|r}^* (D_t - D_{t^-}) \right\},$$

where $t^- = \lim_{h \downarrow 0} t - h$ and $S_0 = \int^{100} S_0^t dt$.

Recall Dr. Carriere's definition of the dividend process. Graphically, it would be presented as
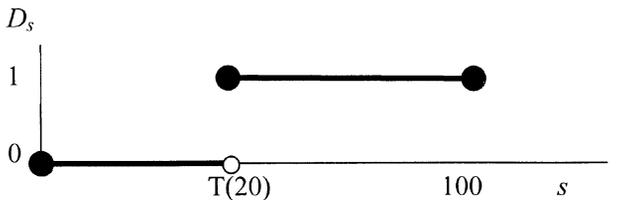
The term $D_t - D_{t^-}$ represents the change in the dividend process over the increment of time from just before time t to time t. Thus, while dD_t is always zero, the difference $D_t - D_{t^-}$ will capture the cash flow to the policy.

The distinction is subtle, but important. Dr. Carriere's expression for S_0 implies that the cash flow arrival date is known, which as he shows is an unreasonable assumption. We are actually suggesting that interest rate-contingent valuation be done for expected cash flows. In the case of the simple life insurance policy, the value of the policy is the integral, over time, of the value of expected cash flows for each point in time.

The equations in the paper are for expected cash flows at a single point in time. Dr. Carriere's comment allows us to make explicit that the value of a block of business, or even a simple life insurance policy, is the integral over time of the expected cash flows at each point in time.

ACKNOWLEDGMENT

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Additional discussions on this paper can be submitted until January 1, 1999. The author reserves the right to reply to any discussion. See the "Submission Guidelines for Authors" for detailed instructions on the submission of discussions.

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[2]Note that the sigma algebra is omitted from Carriere's notation for the sake of simplicity.