

## CORRELATION AND REGRESSION ANALYSIS IN ANTITRUST CLASS CERTIFICATION

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In antitrust litigation, the question of whether a class of differentiated products sold to various direct purchasers at a wide variety of prices should be certified often has been cast in terms of whether there is an identifiable structure underlying those disparate prices. If such a structure exists, it is argued that impact (i.e., causational harm) on class members can be determined through a common method or with common evidence.<sup>1</sup>

Correlation analysis, or an analysis of whether disparate prices are correlated, has been used by economists to support the notion of such a “pricing structure,”<sup>2</sup> while regression analysis has been used to support the claim that a

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<sup>1</sup> See Roger D. Blair & Christine Piette Durrance, *Economic Pitfalls in Antitrust Class Certification*, ANTITRUST, Summer 2007, at 69, 69–71; John H. Johnson & Gregory K. Leonard, *Economics and the Rigorous Analysis of Class Certification in Antitrust Cases*, 3 J. COMPETITION L. & ECON. 341, 348 (2007) [hereinafter *Class Certification*]; James F. Nieberding & Robin A. Cantor, *Price Dispersion and Class Certification in Antitrust Cases: An Economic Analysis*, J. LEGAL ECON., Sept. 2007, at 61.

<sup>2</sup> In *In re Urethane Antitrust Litigation*, 237 F.R.D. 440 (D. Kan. 2006), the plaintiffs’ expert economist performed a correlation analysis in order to support his opinion that a pricing structure existed. In *In re Rubber Chemicals Antitrust Litigation*, 232 F.R.D. 346, 353 (N.D. Cal. 2005), the plaintiffs’ expert economist performed a correlation analysis “as a method for demonstrating common impact . . . . Based on his review of numerous documents produced by Defendants, [the plaintiffs’ expert economist] found high price correlations between buyers and across products.” Correlations appear to have been used to support the determination of a pricing structure by plaintiffs’ expert economist in *Winoff Industries, Inc. v. Stone Container Co. (In re Linerboard Antitrust Litigation)*, 305 F.3d 145, 153 (3d Cir. 2002): “Significantly, [the plaintiffs’ expert economist] . . . stated that he found that linerboard and corrugated box prices were closely correlated. He concluded that linerboard transaction prices, as well as corrugated containerboard prices, behaved similarly over time across different regions of the country and across different types of linerboard.” In *In re Dynamic Random Access Memory (DRAM) Antitrust Litigation*,

model can explain the underlying sources of variation in those prices.<sup>3</sup> How-

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No. M 02-1486 PJH, 2006 U.S. Dist. LEXIS 39841, at \*42–44 (N.D. Cal. June 5, 2006), *aff'd*, 546 F.3d 981 (9th Cir. 2008), the plaintiffs' expert economist performed a correlation analysis: "[A]ll prices for DRAM products were linked and closely correlated during the class period in question, regardless of the type of DRAM purchased, customer category to whom DRAM was sold, or manner in which DRAM was purchased." The defendants' expert economist challenged this finding: "[T]hese differences, when properly taken into account, present a far more differentiated picture of the DRAM market—one in which price is *not* correlated across product and customer class." *Id.* at \*44. In *In re Static Random Access (SRAM) Antitrust Litigation*, No. C 07-01819 CW, 2008 U.S. Dist. LEXIS 107523, at \*46–47 (N.D. Cal. Sept. 29, 2008), the plaintiffs' expert economist relied on correlation analysis. The court found that "at this early stage in the litigation, the correlation analyses, combined with the analysis of the SRAM industry, suffice to establish a plausible methodology for proving an injury" and that "similar correlation analyses and market information have been upheld by numerous courts." *Id.* A correlation analysis was offered in rebuttal to the opinions of plaintiffs' economics expert in *In re Pressure Sensitive Labelstock Antitrust Litigation*, No. 3:03-MDL-1556, 2007 U.S. Dist. LEXIS 85466, at \*60–61 (M.D. Pa. Nov. 19, 2007). Judge Vanaskie stated:

Defendants argue that PSL products do not exhibit a price structure. . . . Defendants argue [plaintiffs' expert economist's] pricing structure analysis was not premised upon a scientifically acceptable method. They note that [the defendants' expert economist] performed a correlation analysis of the transaction data used by [the plaintiffs' expert economist]. [Defendants' expert economist] found that one-third of the resulting correlation coefficients were equal to zero or less than zero, which indicates the lack of similar price movements and that prices actually moved in opposite directions (citations omitted).

*Id.* In *In re Graphics Processing Units Antitrust Litigation*, 253 F.R.D. 478, 493 (N.D. Cal. 2008), the court noted that the plaintiffs' expert economist "heavily relies on correlation analysis." The court stated: "He goes on to present correlation analyses purportedly establishing significant correlations across defendants' graphics products and across all purchasers." *Id.* Finally, an earlier example of plaintiffs' expert doing a correlation analysis in support of a finding of a pricing structure is discussed in *In re Catfish Antitrust Litigation*, 826 F. Supp. 1019, 1041 (N.D. Miss. 1993): "According to [plaintiffs' expert economist], catfish product prices are strongly related to one another over time, and there is a strong correlation between product prices and the prices of live fish . . . ."

<sup>3</sup> See, e.g., *In re Linerboard Antitrust Litig.*, 305 F.3d at 153 ("In reaching its decision, the district court made note of plaintiffs' expert . . . who presented two possible means of assessing impact on a classwide basis—multiple regression analysis, and the benchmark or yardstick approach, which he described as methods of showing 'an antitrust impact by generalized proof.'"); *In re Sulfuric Acid Antitrust Litig.*, No. 03 C 476, 2007 U.S. Dist. LEXIS 20380, at \*24 (N.D. Ill. Mar. 21, 2007) ("[Plaintiffs' expert economist] has proposed a methodology for evaluating the existence and extent of injury through multiple regression analysis and yardstick techniques. These have been found to be acceptable mechanisms on which to base a class action."); *In re OSB Antitrust Litig.*, No. 06-826, 2007 U.S. Dist. LEXIS 56584, at \*17 (E.D. Pa. Aug. 3, 2007) ("[plaintiff's expert's] proposed methods of proving impact on a classwide basis—multiple regression analysis and analysis of the OSB market and Defendants' transactions—are widely accepted in direct purchaser antitrust suits"); *In re Live Concert Antitrust Litig.*, 247 F.R.D. 98, 136 (C.D. Cal. 2007) ("[plaintiffs' expert] testified that he could perform a regression analysis to control for artist quality and other factors in evaluating impact"); *Daniel v. Am. Bd. of Emergency Med.*, 269 F. Supp. 2d 159, 194 (W.D.N.Y. 2003) ("use of multiple variable regression analysis, as proposed by [plaintiffs' expert economist], to prove antitrust impact on a classwide basis has been accepted by the courts as justifying class certification"), *aff'd*, 428 F.3d 408 (2d Cir. 2005); *In re Vitamins Antitrust Litig.*, 209 F.R.D. 251, 267 (D.D.C. 2002) ("[plaintiffs' expert] used multiple regression analyses to evaluate common impact"), *aff'd sub nom. In re Vitamins Antitrust Class Actions*, 327 F.3d 1207 (D.C. Cir. 2003); *In re Flat Glass Antitrust Litig.*, 191 F.R.D. 472, 486 (W.D. Pa. 1999) ("[plaintiffs' expert economist] opines that, by using

ever, the use of both of these techniques can be problematic, leading to flawed inferences about whether impact can be determined on a classwide basis using common evidence.

A correlation analysis depends on the calculation of “correlation coefficients,” each of which is a statistical measurement of the linear relationship between two variables; in the present context, the two variables may be prices of two products over time or prices paid by two customers over time. A calculated correlation coefficient can vary between positive one and negative one.<sup>4</sup> A value of positive one indicates that the two prices have a perfect positive linear relationship.<sup>5</sup> A value of negative one indicates that the two prices have a perfect negative linear relationship.<sup>6</sup> A value of zero indicates that there is no linear relationship between the two price series; the series are, at least in a linear fashion, wholly unrelated to one another. Correlation values between negative one and zero and between positive one and zero indicate that there is some linear relationship between the two price series, but it is not perfect. The closer the correlation coefficient is to positive one, the stronger the linear, positive relationship, and the closer the correlation coefficient is to negative one, the stronger the linear, negative relationship.

There are drawbacks associated with the use of price correlations to draw inferences about common impact. In situations with many products and many customers, relevant individual correlation coefficients can number in the hundreds or thousands. Attempts to aggregate or average price series together in some way, in order to attempt to make a general claim about “average” correlation across all prices paid by all consumers, can lead to incorrect inferences about how prices are or are not correlated. In addition, correlation coefficients may take on a wide range of values between perfect negative correlation (−1.0) and perfect positive correlation (1.0), and without clear rules or

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a regression analysis, plaintiffs can establish the fact of damage for the proposed subclasses”). In *In re Hydrogen Peroxide Antitrust Litigation*, 552 F.3d 305 (3d Cir. 2008), and *In re Plastics Additives Antitrust Litigation*, Nos. 07-2159 & 07-2418, 2009 U.S. App. LEXIS 2177 (3d Cir. Jan. 27, 2009), the Third Circuit recently overturned two decisions that contained similar commentary on the use of regression analysis by plaintiffs’ expert economists. See *In re Hydrogen Peroxide Antitrust Litig.*, 240 F.R.D. 163, 175 (E.D. Pa. 2007) (“[plaintiffs’ expert proposed] to determine impact and damages using a multiple regression analysis”), *rev’d*, 552 F.3d 305; *In re Plastics Additives Antitrust Litig.*, No. 03-2038, 2006 U.S. Dist. LEXIS 69105, at \*33 (E.D. Pa. Sept. 1, 2006) (“Nor is [plaintiffs’ expert economist’s] . . . analysis of the pricing structure within each segment of the plastics additives’ industry speculative: it is based upon multiple regression analysis, a technique frequently utilized to establish class-wide impact. . . .”), *vacated*, 2009 U.S. App. LEXIS 2177 (3d Cir. Jan. 27, 2009).

<sup>4</sup> ORLEY ASHENFELTER, PHILLIP B. LEVINE & DAVID J. ZIMMERMAN, *STATISTICS AND ECONOMETRICS METHODS AND APPLICATIONS* 68–71 (1st ed. 2003).

<sup>5</sup> A perfect positive linear relationship means that if one price is equal to  $P_i$  then the other,  $P_j$ , is equal to  $a + bP_i$  (where  $a$  and  $b$  positive).

<sup>6</sup> A perfect negative linear relationship means that if one price is equal to  $P_i$  then the other,  $P_j$ , is equal to  $a - bP_i$  (where  $a$  and  $b$  positive).

benchmarks as to what level of correlation is sufficient, the correlations offer little if any evidence of whether or to what extent prices are correlated, generally, and therefore provide little meaningful evidence related to any underlying pricing structure.

More importantly, in order for a correlation analysis to be potentially relevant to class certification, there must be an investigation, or analysis, of the reasons for the observed correlation. Correlation may be due to some “economic linkages” among prices or due to a general effect of common factors. An economic linkage may exist between two price series because, for example, the products are substitutes. When the price of one product increases, the demand, and thus the price, of its substitute increases. The two products are said to be economically linked because a price change in one causes a price change in the other. It might be possible, in certain circumstances where such substitution relationships are integral to price determination, to establish impact using common proof because the proof of impact on purchasers of the first product is, via consumer substitution, proof of impact on purchasers of the second product. However, the existence and nature of substitution patterns cannot be measured reliably through simple correlations, but instead require specification and estimation of a demand model.

Alternatively, correlation may be due to a common factor, for example, a cost factor or inflation, that does not bear on the issue of whether the two prices would be commonly affected by some anticompetitive conduct unrelated to that cost factor. Evidence of correlation alone, without evidence of why prices are correlated, is not instructive for class certification purposes. Without such evidence, reliable inferences cannot be drawn from a single price correlation, let alone thousands of individual price correlations, some of which may be positive, while others are negative or zero.

Regression analysis has also been used to attempt to account for the wide price dispersion that exists across a given putative class. The claim is that if such price dispersion can be explained in a single regression model, then the factors relevant to explaining the observed pricing diversity (such as different supply and demand factors) can all be “controlled for” and the effect of the alleged anticompetitive conduct can be measured on a common basis. However, we identify a paradoxical result—the greater the price dispersion due to a diversity of products being included in the class, the better a commonly utilized model appears to fit the data. This “diversity paradox” illustrates the way in which regression analysis can be misused or misinterpreted and the importance of understanding the underlying statistical principles in such models.

The diversity paradox arises when a single regression model is utilized to “explain” transaction prices of different products that are found at different

price points. Pooling such transaction data together creates price dispersion that is not primarily from pricing differences found between different transactions of the same product, but rather from different transactions of wholly different products. Thus, when the common fixed effects approach is used to “control” for the presence of the different products, the regression model appears to fit the data well, and thus explain the price dispersion.<sup>7</sup> Plaintiffs may, for example, point to a high R-squared statistic generated by the regression as evidence of the regression’s explanatory power.<sup>8</sup> However, the explanatory power of the regression is an illusion because the regression is simply explaining the price differences that exist between different products that are found at different price points. For example, a regression that attempts to explain the prices of automobiles by including prices of Mercedes luxury cars and Kia economy cars in one model may fit the data well overall yet explains no more than that Mercedes are higher priced automobiles than Kias. The regression does not explain the reasons why different Mercedes (or Kias) are sold at different prices and may not explain at all how different putative class members were affected by the conduct at issue.<sup>9</sup>

## I. ECONOMIC ANALYSIS IN ANTITRUST CLASS CERTIFICATION

While Federal Rule of Civil Procedure 23 sets out a number of conditions for class certification, the condition that typically generates the most attention from economic experts in antitrust litigation is whether questions of law or fact common to the class predominate over questions affecting only individual

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<sup>7</sup> “Fixed effects” refers to a type of economic model that is estimated using panel data, i.e., data that vary cross-sectionally (such as across different consumers or products) and through time (such as purchases through time). Often, it is believed that there is a relevant unobserved characteristic that differs across the members of the cross section and that this characteristic is time invariant. When this characteristic is modeled as a fixed parameter to be estimated for each member of the cross section, it is called a “fixed effect.” See JEFFREY M. WOOLDRIDGE, *ECONOMETRIC ANALYSIS OF CROSS SECTION AND PANEL DATA* 285–86 (2d ed. 2010). Fixed effects are suggested in this setting because they are a relatively simple way to econometrically account for differences across members of the cross section (such as consumers and/or products). As we discuss below, its simplicity can lead to an incorrect inference related to the effectiveness of the estimated model.

<sup>8</sup> The R-squared statistic, a statistic generated in a regression analysis, indicates how much of the variation in the variable of interest (here, most likely, price) is explained by the variation in the independent, or explanatory, variables. The R-squared statistic varies between 0 and 1, or 0 and 100 percent, where 0 represents that none of the pricing variation is explained, and 1 (or 100 percent) represents that all, or 100 percent, of the pricing variation is explained. A regression with a “high” R-squared statistic may be used to claim that the regression fits the data well and explains the observed price dispersion.

<sup>9</sup> A model that includes variables to control for different models of Mercedes may be able to “explain” price differences between E-class and S-class vehicles, for example, but could not explain why the same E-class automobile is sold at the same point in time by different dealers at different prices.

class members.<sup>10</sup> The critical question is that of antitrust injury, or “impact.”<sup>11</sup> Consider a direct purchaser case involving allegations of price fixing. At the class certification stage, a key question is whether there is common proof of impact to direct purchasers under the assumption that the alleged anticompetitive behavior took place. That is, the question is whether the determination that each putative class member has been overcharged can be made using common proof and a common methodology.<sup>12</sup>

Plaintiffs have often argued, and some courts have accepted, a “presumption” of common impact based on an expert economist’s opinion that the allegedly anticompetitive conduct would have generally impacted the competitive process. Thus, the argument continues, absent the conduct, competition would be greater and prices paid by proposed class members would have been lower. The conclusion that follows is that all proposed class members were in fact injured. This is often referred to as the “*Bogosian* short-cut.”<sup>13</sup> Evidence utilized by the expert economist might include a showing of

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<sup>10</sup> See Robert H. Klonoff, *Antitrust Class Actions: Chaos in the Courts*, 11 STAN. J.L. BUS. & FIN. 1 (2005); William H. Page, *Introduction: Reexamining the Standards for Certification of Antitrust Class Actions*, ANTITRUST, Summer 2007, at 53. Predominance is found in the Federal Rules of Civil Procedure, FED. R. CIV. P. 23(b)(3). Rule 23(a) lists prerequisites to a class action, including: (1) the class is so numerous that joinder of all members is impracticable; (2) there are questions of law or fact common to the class; (3) the claims or defenses of the representative parties are typical of the claims or defenses of the class; and (4) the representative parties will fairly and adequately protect the interests of the class. FED. R. CIV. P. 23(a). Rule 23(b) states an action may be maintained as a class action if the prerequisites of subdivision (a) are satisfied, and in addition, one of the following three conditions are met: (1) the prosecution of separate actions by or against individual members of the class would create a risk of inconsistent or varying adjudications or adjudications with respect to individual members of the class, which would as a practical matter be dispositive of the interests of the other members not parties to the adjudications; or (2) the party opposing the class has acted or refused to act on grounds generally applicable to the class; or (3) the court finds that the questions of law or fact common to the members of the class predominate over any questions affecting only individual members, and that a class action is superior to other available methods for the fair and efficient adjudication of the controversy. FED. R. CIV. P. 23(b). “The party seeking class certification bears the burden of showing that each of the four requirements of Rule 23(a) and at least one of the requirements of Rule 23(b) are met.” *In re Graphics Processing Units Antitrust Litig.*, 253 F.R.D. 478, 483 (N.D. Cal. 2008).

<sup>11</sup> See Ellen Meriwether, *Rigorous Analysis in Certification of Antitrust Class Actions: A Plaintiff’s Perspective*, ANTITRUST, Summer 2007, at 55, 57 (“In an antitrust case, the major contested issue under Rule 23 is almost always ‘predominance,’ and to put even a finer point on it, whether impact (or antitrust injury) can be proven with evidence common to the class.”).

<sup>12</sup> The question of impact, that is, whether or not proposed class members were affected by the alleged conduct, is different from the question of the amount of impact, or damages.

<sup>13</sup> A statement of the *Bogosian* short-cut is found in the eponymous Third Circuit decision:

If, in this case, a nationwide conspiracy is proven, the result of which was to increase prices to a class of plaintiffs beyond the prices which would obtain in a competitive regime, an individual plaintiff could prove fact of damage simply by proving that the free market prices would be lower than the prices paid and that he made some purchases at the higher price. If the price structure in the industry is such that nationwide the conspiratorially affected prices at the wholesale level fluctuated within a

market and product characteristics said to be conducive to common proof of impact, including concentrated markets with barriers to entry, largely controlled by defendants who sell a commodity-like product with few substitutes.<sup>14</sup> However, in most real-world markets such characteristics are largely subjective, and if the alleged conspiracy took place in what is admittedly a more complex economic market (or markets) with diverse buyers, diverse methods of distribution, non-homogeneous products, and price dispersion, then antitrust impact based on a simple description of general market characteristics cannot be presumed. As a result, more rigorous economic and econometric analyses of the marketplace have come to play a role in determining whether antitrust impact can be established for all putative class members using a common methodology and common evidence.<sup>15</sup>

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range which, though different in different regions, was higher in all regions than the range which would have existed in all regions under competitive conditions, it would be clear that all members of the class suffered some damage, notwithstanding that there would be variations among all dealers as to the extent of their damage.

*Bogosian v. Gulf Oil Corp.*, 561 F.2d 434, 455 (3d Cir. 1977). This approach is widely used. See generally Ian Simmons et al., *Without Presumptions: Rigorous Analysis in Class Certification Proceedings*, ANTITRUST, Summer 2007, at 61, 63 (identifying cases where “[c]ourts have applied this ‘Bogosian short-cut’ often to find that the plaintiff can prove impact and injury on a classwide basis”); *In re Alcoholic Beverages Litig.*, 95 F.R.D. 321 (E.D.N.Y. 1982).

<sup>14</sup> An often-cited decision that applies the *Bogosian* short-cut along with a “deferential analysis of the plaintiffs’ expert report,” Page, *supra* note 10, at 53, is the Third Circuit’s opinion in *In re Linerboard Antitrust Litigation*:

There is more to this case than exclusive reliance on the presumed impact theory. The district court used a belt and suspenders rationale to support its conclusion that the putative class had met its burden of showing impact. In addition to relying on the *Bogosian* short cut, it credited the testimony of plaintiffs’ experts, opinions that were supported by charts, studies and articles from leading trade publications. These experts suggested that advanced econometrics models could be effectively prepared to establish class-wide impact.

305 F.3d 145, 153 (3d Cir. 2002).

<sup>15</sup> A review and discussion of courts’ standards in antitrust class certification is found in the Summer 2007 symposium in *Antitrust*. See Blair & Durrance, *supra* note 1; Meriwether, *supra* note 11; Page, *supra* note 10; Simmons et al., *supra* note 13. As reported by Simmons et al.:

In recent years, many courts have exhibited greater willingness to test the viability of methodologies that experts propose to show classwide impact and injury using common proof, and are increasingly skeptical of plaintiffs’ experts who offer only generalized and theoretical opinions that a particular methodology may serve this purpose without also submitting a functioning model that is tailored to market facts in the case at hand.

Simmons et al., *supra* note 13, at 65. Accord Klonoff, *supra* note 10. For an update to their 2007 article, see Ian Simmons & Alexander P. Okuliar, *Rigorous Analysis in Antitrust Class Certification Rulings: Recent Advances on the Front Line*, ANTITRUST, Fall 2008, at 72. Simmons and Okuliar find that recent court opinions support the conclusion that the trend towards more rigorous analysis at the class certification phase continues. *Id.* at 74–75. For example, they note that with respect to *Cordes & Co. Financial Services v. A.G. Edwards & Sons, Inc.*, 502 F.3d 91 (2d Cir. 2007):

The Second Circuit’s message is clear: district judges must roll up their sleeves and dive into the econometric models offered by experts to understand and evaluate the

More rigorous economic analyses seek to determine antitrust impact by analyzing how transaction prices for individual class-member purchasers are determined, and then to establish how that price-setting process would be affected by the alleged conduct. For class certification to be appropriate, this must take place on a classwide basis using a common methodology and common evidence.<sup>16</sup> When there is broad diversity in the prices paid by the putative class, it should not be presumed that an alleged conspiracy, or other anticompetitive conduct, impacted, or injured, each class member. The anticompetitive conduct alleged will not generally be so specific and all-encompassing that, if one assumes it took place, it necessarily follows that *all* putative class members were overcharged. For example, the conduct alleged could be as broad as allegations that competitors met a number of times over a five-year period and discussed prices. When these competitors sell many highly differentiated products, using different distribution methods, at prices that vary not only across products, but across customers, and prices of particular products sold to particular customers are individually negotiated and change over time differently, the classwide impact of such meetings cannot be immediately presumed.<sup>17</sup>

In such situations the statistical analysis and econometric modeling of prices is increasingly utilized in support of class certification.<sup>18</sup> The argument

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prospect of the model's success when applied to the facts in the case. Anything short of this is an abuse of discretion and subject to remand.

Simmons & Okuliar, *supra*, at 75. See also Wendy L. Bloom, *Why Economics Now Matters for Antitrust Class Actions at the Class Certification Stage*, GLOBAL COMPETITION POL'Y, Winter 2009, Vol. 1, No. 1, at 1; Donald Hawthorne & Margaret Sanderson, *Rigorous Analysis of Economic Evidence on Class Certification in Antitrust Cases*, ANTITRUST, Fall 2009, at 55; John D. Majoras, *Opening the Curtain: Why Economics Is Taking Center Stage in Class Certification Battles in Antitrust Cases*, GLOBAL COMPETITION POL'Y, Spring 2008, Vol. 6, No. 2, at 1. Recent decisions in the Third Circuit appear to be continuing this trend. *In re* Plastics Additives Antitrust Litig., Nos. 07-2159 & 07-2418, 2009 U.S. App. LEXIS 2177 (3d Cir. Jan. 27, 2009); *In re* Hydrogen Peroxide Antitrust Litig., 552 F.3d 305 (3d Cir. 2008).

<sup>16</sup> FED. R. CIV. P. 23(b)(3).

<sup>17</sup> A review of the issue is presented in the recent class certification decision in *In re Graphics Processing Units Antitrust Litigation*, 253 F.R.D. 478, 483-84 (N.D. Cal. 2008): "Antitrust decisions have been mixed in determining whether certification is warranted where complex chains of distribution with highly varying purchasers and products are involved." The court finds:

In sum, no uniform approach has emerged. The decisions indicate that evaluating the requirements for class certification in this context involves a particularized analysis of the specific industry and chain of distribution. At times, the complexity of the defendants' distribution chain along with the varying products and purchasers involved have prevented broad certifications. Factors favoring certification have been price lists and commodity products as opposed to individually negotiated deals and customized products.

*Id.* at 489.

<sup>18</sup> In a recent opinion, the district court states:

In industries involving varying products and complex pricing structures, antitrust plaintiffs have in recent years trended toward presenting an econometric formula or

is that if a common econometric analysis, or model, can show that all the relevant prices are determined in a systematic way by common economic forces, then classwide impact from the anticompetitive conduct can be established using common evidence.<sup>19</sup> In other words, if the alleged conduct directly applies only to some subset of products or customers, impact on the rest of the putative class follows from the determination that all of the relevant prices are linked via common economic forces.

Courts differ on whether, at the class certification phase, it is necessary to fully develop and actually perform the econometric analysis.<sup>20</sup> However, what is increasingly necessary to support class certification is an analysis of how prices to individual customers are determined and a conclusion that a common econometric analysis could be feasibly performed to show impact to the class.<sup>21</sup> Two such analyses that have been used by those arguing for certification of proposed classes are a showing that there is a pricing structure, and the development of a regression model of price determination.

#### A. PRICING STRUCTURE

In support of class certification, an approach to dealing with the presence of price dispersion among the putative class is to argue that there is a pricing structure that connects all the different prices together in some systematic way.<sup>22</sup> The existence of a pricing structure implies that although there may be

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other statistical analysis to show class-wide impact. The idea is to account for differences from transaction to transaction by assigning variables to certain conditions relating to the transaction (*e.g.*, product features or type of purchaser) or by establishing some type of correlation between product lines or purchasers. Several courts have accepted proffered econometric analysis at the certification stage.

*Id.* at 491. Accord ABA SECTION OF ANTITRUST LAW, *ECONOMETRICS: LEGAL, PRACTICAL, AND TECHNICAL ISSUES* 195–96 (2005) [hereinafter ABA *ECONOMETRICS*].

<sup>19</sup> It may be that such a model shows some members of the putative class are not affected, or injured, by the anticompetitive conduct. But the determination of which class members are not impacted is made using common proof and thus such a showing would likely not defeat class certification. See ABA *ECONOMETRICS*, *supra* note 18, at 182 n.11.

<sup>20</sup> See Klonoff, *supra* note 10. Compare *In re Cardizem CD Antitrust Litig.*, 200 F.R.D. 297, 321 (E.D. Mich. 2001) (“[T]he Court’s inquiry is limited to whether or not the proposed methods are so insubstantial as to amount to no method at all.” (quoting *In re Potash Antitrust Litig.*, 159 F.R.D. 682, 697 (D. Minn. 1995))), with *In re Graphics Processing Units Antitrust Litig.*, 253 F.R.D. at 497 (the court made a searching inquiry of plaintiffs’ expert’s econometric modeling and concluded that “[d]irect-purchaser plaintiffs’ proffered econometric models are grossly lacking and do not suffice”).

<sup>21</sup> See *supra* note 15; ABA *ECONOMETRICS*, *supra* note 18, at 197–201.

<sup>22</sup> See ABA *ECONOMETRICS*, *supra* note 18, at 217 (“Class members do not have to buy the same homogeneous product at a uniform price to be found to be in the same class. Classes have been certified that include different members that purchased different products or paid substantially different prices for the products or services they purchased. However, this generally requires statistical evidence showing that there is a stable structural relationship between the prices paid by the different class members.”).

complex price dispersion among products and customers, the complexity can be ignored and the pricing structure as a whole can be thought of in the same way as the simple case of one homogeneous price. The claim is that common proof can be used to show classwide impact because, if one price is affected by the alleged conduct, then all prices are affected. Conduct that impacts any one price is conduct that impacts the entire pricing structure and thus is conduct that impacts the entire putative class.<sup>23</sup>

Attempts to show the existence of a pricing structure sometimes are focused on the existence of a “list” or “benchmark” price. Theoretically, such a benchmark price is a price that anchors the pricing structure. All other prices in the structure are somehow determined relative to this price. For example, even in instances when there is individualized negotiation over price, the price that is so determined can be argued to be part of the pricing structure if the negotiations took place in the shadow of a common benchmark price.<sup>24</sup>

The concept of a pricing structure, and the methods used in its determination, have been subject to recent critiques. In particular, commentators have argued that the determination of a pricing structure has often involved nothing more than a “visual inspection” of price series before concluding that prices move together.<sup>25</sup> Some recent judicial opinions, both certifying a class and failing to certify a class, have gone beyond visual inspection and referred to the usage of “correlations” in the context of expert economic testimony on the existence of a pricing structure, and thus common impact. In Part III below, we address this usage of correlations and discuss in depth what a finding of price correlation is, and is not, in the context of class certification. We find that, in general, reliance on correlations to establish whether impact can be

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<sup>23</sup> See, e.g., John H. Johnson & Gregory K. Leonard, *In the Eye of the Beholder: Price Structure as Junk Science in Antitrust Class Certification Proceedings*, ANTITRUST, Summer 2008, at 108, 109 [hereinafter *Price Structure*] (“The general intuition is that even in the face of different prices for a given product, if all of the prices stay in fixed relation to each other, then a conspiracy to fix prices would impact all customers. Under this paradigm, a ‘rising tide floats all boats’; if one customer pays a higher price, all of the customers ultimately pay a higher price. Therefore, common proof could be used to demonstrate fact of injury—showing one customer is harmed is enough to show they all were harmed.”); see also Johnson & Leonard, *Class Certification*, *supra* note 1, at 341, 348.

<sup>24</sup> A recent review of class certification decisions shows that many of the decisions hinge on the finding, or not, of list and benchmark prices and a common pricing structure. *In re Graphics Processing Units Antitrust Litig.*, 253 F.R.D. 478; *In re Carbon Black Antitrust Litig.*, No. 03-10191-DPW, 2005 U.S. Dist. LEXIS 660 (D. Mass. Jan. 18, 2005).

<sup>25</sup> Johnson & Leonard, *Price Structure*, *supra* note 23, at 108–11 (discussing that the concept of a pricing structure is not found in standard, peer-reviewed, economics). Furthermore, Johnson and Leonard argue that attempts to determine the existence of a pricing structure often boil down to a visual inspection of multiple price series and a conclusion that “prices ‘move together.’” *Id.* at 108. Johnson and Leonard argue that this is junk science because it is subjective and the conclusion is not based on “objective economic scientific practice.” *Id.*

determined on a classwide basis is misguided and insufficient to draw such conclusions.

### B. REGRESSION MODELS OF PRICING

Beyond using a simple statistic like correlation (and other evidence, such as price lists) to infer the existence of a pricing structure, economists analyzing class certification have utilized regression analysis to investigate the actual construction of a common economic model of price determination.<sup>26</sup> In complex settings, where there are many differentiated products, prices vary both across products and customers, and there are varied economic circumstances affecting each price, regression analysis has been identified as a way to account for this variation.<sup>27</sup> The claim is that if such price dispersion can be explained in a single regression model, then the effect of the alleged anticompetitive conduct can be measured on a common basis. In Part IV, we show that reliance on regression analysis to determine a common model of pricing where there is substantial price dispersion must be met with careful skepticism.

## II. PRICING STRUCTURE AND CORRELATIONS

A number of recent antitrust class certification judicial opinions, both certifying a class and failing to certify a class, have referred to experts' usage of "correlations" in the determination of a pricing structure and thus common impact. While the use of correlations would appear to be an improvement over "visual inspection,"<sup>28</sup> their use in the context of determining pricing structure raises a number of issues.

<sup>26</sup> See ABA ECONOMETRICS, *supra* note 18, at 220–24.

<sup>27</sup> See Nieberding & Cantor, *supra* note 1. What is critical is whether the dispersion in pricing is systematic and whether there are identifiable and measurable factors that can account for the dispersion. Blair and Durrance point out that such factors need to be both systematic and observable. Blair & Durrance, *supra* note 1, at 71–72. When relevant factors are unobservable, or only observable via individualized inquiry, there is no way to construct a common economic model. *Id.* at 72. A related point is made by Johnson and Leonard, who identify a "common proof paradox." Johnson & Leonard, *Class Certification*, *supra* note 1. The paradox is that to determine whether there are individualized factors relevant to the determination of antitrust impact these factors need to be empirically measured and their importance tested. But measurement implies individualized inquiry, which is what the determination of common proof is meant to avoid. Johnson and Leonard note that reliance on economic theory can get around the common proof paradox, but that any theory must be carefully constructed to rigorously fit the facts of the relevant marketplace. *Id.* at 344. Johnson and Leonard argue that real-world marketplaces, which often display price dispersion, do not lend themselves to a simple theory of price determination that can be modeled using a simple economic model. They discuss how an empirical analysis of "pricing structure" in a marketplace exhibiting price dispersion leads to the common proof paradox. *Id.*

<sup>28</sup> However as reported in the SRAM opinion, both defendants and plaintiffs in that case agreed that "the power of correlation analysis is low." *In re* Static Random Access (SRAM)

## A. CORRELATIONS AND THE AVERAGING OR COMBINING OF PRICE SERIES

Correlation coefficients have certain characteristics that limit their usefulness in class certification contexts. First, a correlation coefficient measures the degree of linear relationship between two variables (such as prices) only. Second, in order to calculate a correlation coefficient between the two variables, the data for each variable must be “matched.” That is, there must be patched pairs of observations for each variable included in the calculation.

The first constraint implies that if the goal of the analysis is to uncover a pricing structure among prices paid for many products and by many consumers, many correlations must be calculated. The second constraint can lead to an inability to calculate correlations in some instances. Generally, it is necessary to calculate the correlation between two time series of prices. Each price is observed at the same point in time, and the calculated correlation indicates whether the two price series move together over time. Thus, there must be observations of each price at the same time: for example, two sets of daily prices (one observation of each price per day) or two sets of monthly prices (one observation of each price per month). But, if the data in a particular case represent periodic purchases of a number of products, it could well be the case that a price series for one consumer’s purchases of one product cannot be matched with the price series of another consumer’s purchases of the same, or different, product. Consider, for example, one consumer who buys office supplies in even months and another consumer who buys office supplies in odd months. There are no coincident purchases. Therefore, without further analysis or assumptions, there is no way to create a matched sample of the two prices, and no way to calculate a price correlation.<sup>29</sup>

Attempts to aggregate or average price series together in some way, in order to attempt to make a general claim about “average” correlation across all prices paid by all consumers, can lead to incorrect inferences about how prices are or are not correlated. In fact, the process of aggregating prices across products and consumers (or over time) runs counter to the stated goal of such analysis: to find an underlying pricing structure that links together all prices paid for all products by all putative class members. Aggregating or averaging obscures the very differences in prices that the analysis should attempt to explain.<sup>30</sup>

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Antitrust Litig., No. C 07-01819 CW, 2008 U.S. Dist. LEXIS 107523, at \*46 (N.D. Cal. Sept. 29, 2008).

<sup>29</sup> Further analysis might support assumptions that prices to the consumers should be thought of as constant over some time period, therefore creating the possibility of deriving a matched sample.

<sup>30</sup> See ABA ECONOMETRICS, *supra* note 18, at 220 (“Sometimes the prices used by economists are averages of a number of different prices charged to different customers or for somewhat different products. Using such averages can lead to serious analytical problems. For example,

The problem inherent with correlations of averaged prices is easily illustrated. Consider a situation in which there are four customers, all buying the same product. Two customers are brokers and two customers are distributors. One broker's prices increase over time and the other broker's prices remain constant over time. Averaging the brokers' prices would result in a price series that increases over time. Further assume that one distributor's prices fall over time and the other distributor's prices rise sharply, so that the average of the two distributors' prices increases somewhat over time. Correlating the average of the brokers' prices with the average of the distributors' prices would result in a positive correlation, given that both the averaged brokers' price and the averaged distributors' price increase over time. However, in actuality, there would be no correlation between the two brokers' prices and negative correlation between one of the broker's prices and one of the distributor's prices. Averaging of the individual customers' prices obscures these true relationships and leads to a false conclusion about how the prices paid by these customers relate to one another. Lack of correlation, or negative correlation, among individual price series may be obscured by the averaging process.<sup>31</sup> Appendix A provides a mathematical explanation and a simple numerical example.

Another approach to deriving an "average" correlation from a multitude of price series leads to similar problems as those that arise in averaging the underlying prices. We call this approach "stacking."<sup>32</sup> This approach attempts to create two price series (thus amenable to the calculation of a correlation coefficient) from many individual price series by stacking price series on top of one another.

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averages can hide substantial variation across individual cases, which may be key to determining whether there is common impact. In addition, average prices may combine the prices of different package sizes of the same product or of somewhat different products. When this happens, the average price paid by a customer can change when the mix of products that the customer buys changes—even if the price of no single product changed.”)

<sup>31</sup> In *In re Graphics Processing Units Antitrust Litigation*, 253 F.R.D. 478 (N.D. Cal. 2008), the court specifically criticized the plaintiffs' expert economist for engaging in such averaging, finding:

In essence, [plaintiffs' expert economist] has evaded the very burden that he was supposed to shoulder—*i.e.*, that there is a common methodology to measure impact across individual products and specific direct purchasers. His report says little about how specific product pricing was correlated across buyers or whether prices paid for multiple products by particular direct purchasers were correlated. If data points are lumped together and averaged before the analysis, the averaging compromises the ability to tease meaningful relationships out of the data.

*Id.* at 493.

<sup>32</sup> This approach was utilized by plaintiffs' expert in *In re Graphics Processing Units Antitrust Litigation*. The court found that “[plaintiffs' expert] ‘stacks’ the *averaged* data to create artificial correlations.” *Id.* at 495.

A simple example illustrates the method and its inherent problem. Consider two different products' prices, where one product is a "low" priced product and the other product is a "high" priced product. Two customers purchase each of the two products. Assume the correlation between the two series (one for each customer) of low product purchase prices is zero and the correlation between the two series of high product prices is zero. Now arrange each of the customer's prices as a stack of the two products' prices, such that a correlation coefficient is calculated using the stacked prices to determine whether the prices paid for the two products by one customer are correlated with the prices paid for the two products by the other customer. Each of the two stacks of prices (one for each customer) contains a set of "low" prices and then a set of "high" prices. The coincident movement in each of the stacks of prices from the low level price to the high level price creates the illusion of correlation when there is none. A finding of correlation between the two price series is due entirely to the stacking. Figure 1 provides a graphical example of the problem described.

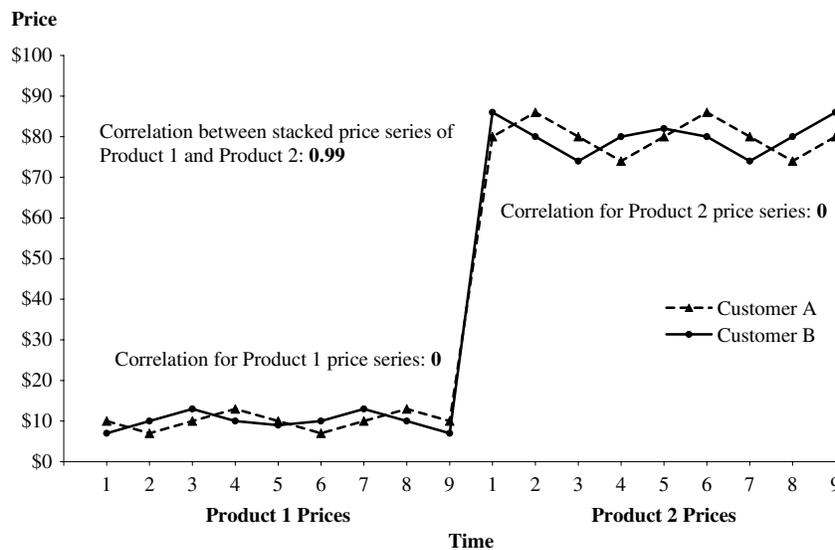


FIGURE 1: CORRELATION IN STACKED PRICE SERIES

Appendix B provides a mathematical explanation and a simple numerical example.

Using price data that has not been averaged and/or stacked will avoid the problems discussed above. However, pursuing a correlation analysis with all the relevant individual price series, rather than averaging or stacking, can imply, in reasonable real-world settings, that there will be many relevant correla-

tions.<sup>33</sup> In a complex putative class with dispersed prices that differ among heterogeneous products and consumers, there could be dozens, if not hundreds or even thousands of relevant price series. And this multiplies the number of relevant correlations. For example, if there are as few as ten relevant price series, there are forty-five correlations when a correlation is calculated for each price series against each other price series.<sup>34</sup>

There are other important issues inherent in the usage of correlations to determine the existence of pricing structure. It is to those that we now turn.

#### B. THE ECONOMIC RELATIONSHIPS THAT GIVE RISE TO A CORRELATION IN PRICES

As discussed above, a correlation coefficient measures the strength of the linear relationship between two series—in this setting, two price series. It is important to recognize that examining price correlations is not an analysis of the economics of how prices are determined. As is well known, “Correlation does not imply causality. Two variables may move together but not be causally related . . . .”<sup>35</sup> The fact that two price series are correlated does not identify what economic forces actually cause changes in each series, and whether, and how, such changes may be connected. To be meaningful the analysis of price correlations must be done with a careful consideration of the correct economic context. An analysis of the economic context must accompany price correlations in order to be at all relevant to “testing” for the presence of a “pricing structure.”<sup>36</sup>

There are two potentially relevant perspectives on the underlying source of correlation found between two price series. The first is that changes in both price series are driven by changes in the same economic factors that directly affect both. For example, in the cases where there are different consumers buying the same product, changes in both consumers’ prices may be driven by changes in the price of an input to the common product, such as changes in

<sup>33</sup> The problem of matching two time series of prices by date must also be overcome.

<sup>34</sup> The formula for determining the number of correlations when there are  $N$  relevant price series is  $N(N-1)/2$ . In *In re Graphics Processing Units Antitrust Litigation*, the defendants’ expert calculated “hundreds of thousands of correlation coefficients.” 253 F.R.D. at 494.

<sup>35</sup> ASHENFELTER ET AL., *supra* note 4, at 71; *see also* ABA ECONOMETRICS, *supra* note 18, at 5.

<sup>36</sup> In a particular case there may be relevant price series that differ along consumer lines and/or along product lines. Two price series being compared, via the calculation of a correlation coefficient, may involve different consumers buying the same product (prices when consumer A buys product X compared to prices when consumer B buys product X), or may involve the same consumers buying different products (prices when consumer A buys product X compared to prices when consumer A buys product Y), or may involve different consumers buying different products (prices when consumer A buys product X compared to prices when consumer B buys product Y).

the price of washing machines to two consumers being driven by changes in the price of steel. Similarly, a change in demand conditions, such as a change in the price of dryers, could affect prices to both consumers.

Correlation between the prices of different products may also be explained by common supply and demand factors. If both products are produced with the same input and that input cost changes, the prices of the two products may also change. For example, changes in the price of both washing machines and bicycles may be affected by changes in the price of steel. These effects can be labeled as “common factor” effects. There is a common factor to both price series, and when the common factor changes, both price series change, giving rise to price correlation.

A second perspective is that changes in both price series are driven by changes in economic factors that do not directly affect both, but rather only directly affect one. Both prices are impacted by a change in one of these independent factors because the direct impact on one price leads to changes in the other price via economic forces that connect the two series. For example, in cases involving different, or differentiated, products, this may occur via substitution by consumers, for example, steel bicycles and aluminum bicycles. A change in the price of steel will directly impact the price of steel bicycles, which may lead consumers to substitute aluminum bicycles, thus impacting the price of aluminum bicycles. We call this type of effect an “economic linkage” effect. The economic linkage between the two price series means that when a factor changes that directly affects only one series the other series is also affected via consumer substitution. It is the linkage that gives rise to price correlation.

These two different economic effects that may explain a measured price correlation—the common factor effect and the economic linkage effect—have quite different meanings for the use of a price correlation to support class certification.<sup>37</sup> When the putative class includes differentiated products

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<sup>37</sup> There is a substantial literature on whether it is appropriate to use price correlations in the definition of antitrust markets. *E.g.*, Jonathan B. Baker, *Market Definition: An Analytical Overview*, 74 ANTITRUST L.J. 129 (2007) [hereinafter *Market Definition*]; Patrick J. Coe & David Krause, *An Analysis of Price-Based Tests of Antitrust Market Delineation*, 4 J. COMPETITION L. & ECON. 983 (2008); George J. Stigler & Robert A. Sherwin, *The Extent of the Market*, 28 J.L. & ECON. 555 (1985); Gregory J. Werden & Luke M. Froeb, *Correlation, Causality, and All that Jazz: The Inherent Shortcomings of Price Tests for Antitrust Market Delineation*, 8 REV. INDUS. ORG. 329 (1993); Jonathan B. Baker, *Why Price Correlations Do Not Define Antitrust Markets: On Econometric Algorithms for Market Definition* (FTC Working Paper No. 149, 1987) [hereinafter *Price Correlations*]. The focus of this literature is on the ability of price correlations to identify two products or geographic regions that should be considered to be in the same market. The basic idea is that consumer substitution between products or geographic regions will tend to make prices, or price changes, correlated with one another. What we are calling the common factor effect is discussed in this literature as “spurious correlation,” or measured price correlation that is not due to customer substitution, but rather due to the two product prices being correlated

sold to various direct purchasers at a wide variety of prices, a measured price correlation may or may not reflect economic forces that are relevant to class certification.

### 1. *The Common Factor Effect*

The common factor effect implies that there is an economic factor common to both prices series, such that changes in this factor lead to common changes in the prices, which implies price correlation. The question is what relevance such correlation has to the issue of class certification, and is the existence of such a common factor relevant to whether or not a common methodology, or common proof, can determine antitrust impact on the consumers who are paying the two prices.

Price correlation due to the common factor effect simply establishes the existence of a factor that has the ability to affect both prices. The correlation does not establish that a different factor, such as any anticompetitive actions on the part of suppliers, will therefore also have a common impact.<sup>38</sup> Drawing this conclusion requires the presumption that the effect of any anticompetitive act is similar to the effect of the common factor. In a vacuum, this is simply an empirically based version of the *Bogosian* short-cut, whereby the researcher has established certain features of the defined marketplace and then draws the conclusion that an anticompetitive act would commonly impact all participants in that marketplace.<sup>39</sup> However, the correlation alone is not proof

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because they have common costs or demand. *See, e.g., Baker, Market Definition, supra; Coe & Krause, supra; Werden & Froeb, supra; Baker, Price Correlations, supra.* This correlation is spurious because “it does not flow from any economic interdependencies between the two products.” *Werden & Froeb, supra*, at 333; *see also ABA ECONOMETRICS, supra* note 18, at 5. Thus, the focus in this literature is on what we are calling the economic linkage effect, or correlation that arises due to economic interdependence, such as consumer substitution that comes about due to a shock to one product’s price (or geographic area) that is not felt directly by the other product (or geographic area). The market definition setting means that this literature does not consider the case, relevant to class certification, of two distinct consumers purchasing the same good.

<sup>38</sup> Johnson & Leonard, *Price Structure, supra* note 23, at 112 n.29 (“it is possible that customer-level prices might have moved together in response to some market conditions (e.g., changes in marginal cost), but not others (e.g., a price-fixing conspiracy)”).

<sup>39</sup> It appears that in some instances courts have been persuaded by pricing structure and correlation analyses that are specifically said to uncover that prices are determined by common factors, thus allowing them to adopt a *Bogosian*-like short-cut. *See In re Pressure Sensitive Labelstock Antitrust Litig.*, No. 3:03-MDL-1556, 2007 U.S. Dist. LEXIS 85466, at \*52–53 (M.D. Pa. Nov. 19, 2007) (“[plaintiffs’ expert economist] found that PSL prices exhibited a pricing structure, suggesting prices are impacted by similar forces.”); *In re Carbon Black Antitrust Litig.*, No. 03-10191-DPW, 2005 U.S. Dist. Lexis 660, at \*65 (D. Mass. Jan. 18, 2005) (quoting the plaintiffs’ expert report: “My empirical analysis of Defendants’ sales transactions indicates a similarity in movement of carbon black prices over time. This demonstrates that carbon black prices are affected by similar forces, such as the alleged coordinate [sic] price behavior of the Defendants.”).

that antitrust impact on one group of consumers will necessarily mean impact on another group of consumers.<sup>40</sup>

Claims of common impact based on correlation, when the correlation exists due to the common factor effect, can lead to nonsensical results. Returning to the example of washing machines and bicycles, price correlation induced by the common factor of steel (an input to both) should not lead to a conclusion that a price-fixing conspiracy among bicycle suppliers to raise the price of bicycles should commonly affect both bicycle and washing machine prices.<sup>41</sup> In any scenario where the cost of an input plays an important role in determining the price of the products at issue, the correlation coefficient between the two products' prices could be found to be relatively high. The example demonstrates the fallacy of concluding common impact because a "high" correlation coefficient appears to imply that common factors "predominate." The correlation in the example is due solely to the prices of the products being affected by a certain common cost and without additional analysis or evidence cannot be used to infer common impact from some anticompetitive conduct. Moreover, the converse is also true. Even if there is no correlation between two products' prices, it is possible that some alleged anticompetitive conduct impacted both prices. The question of common impact is more reliably analyzed by studying the evidence of impact, not evidence solely related to the effects of other common economic factors.

## *2. The Economic Linkage Effect*

A price correlation due to an economic linkage effect is not necessarily subject to the same criticisms as those of a price correlation measured due to a common factor effect. A price correlation based on an economic linkage effect could be interpreted to imply that if one putative class member is affected by anticompetitive conduct, then, because the prices are economically linked, other putative class members will also be affected.

A measured price correlation that is due to the economic linkage effect indicates that a factor that impacts only one of the prices will lead to an impact on the other of the prices because there are economic relationships, such as substitution relationships between the products. For example, if anticompe-

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<sup>40</sup> A relevant case to consider is where a defined putative class contains products that are inputs into other products also in the class, for example, a class that contained both purchasers of bicycle tires and purchasers of bicycles. In such a case it is almost by definition that correlation between tire prices and bicycle prices is due to the common factor being tire prices. Thus, the existence of such correlation could support a conclusion that anticompetitive acts that impacted purchasers of tires would likely impact purchasers of bicycles. However, this conclusion does not work in reverse. Such correlation does not support a conclusion that anticompetitive acts that impacted purchasers of bicycles would likely impact purchasers of tires.

<sup>41</sup> Thus, the use of the word "spurious" in the market definition literature on the use of price correlations.

titive acts directly led to an overcharge on the prices paid for one product, then consumer substitution towards a second product may increase its demand, causing its price to increase. Both products' prices are affected due to the economic linkage. In this hypothetical circumstance, where the price change of one product causes a price change in the second product, proof of impact on consumers of the first product is proof of impact on consumers of the second product. If a measured price correlation captures an economic linkage effect, then the correlation is potentially relevant to the analysis of common impact.<sup>42</sup> However, as described above, the correlation analysis itself cannot be used to reliably establish such economic linkages.

### C. USING CORRELATIONS TO TEST FOR RELEVANT PRICING STRUCTURE

The conclusion from the discussion above is that for a correlation analysis to be relevant to class certification beyond a presumption of common impact, it must test for the existence of economic linkage effects, or for common factor effects directly relevant to the alleged anticompetitive conduct.<sup>43</sup> Simple price correlations are not capable of uncovering important economic linkages. The "linkages" between products' prices may exist because of substitution patterns among the products, which cannot be measured reliably through simple correlations but instead require specification of a demand model. Demand substitution patterns may be highly complex, especially in circumstances where there are numerous products that are similar in certain ways but dissimilar in other ways that may be particularly important to certain customers. The possibility and extent of demand substitution may be customer-specific, depending on such factors as the technology used by a customer or the application for which a customer purchases the product at issue. Price correlations are too simplistic to be used to evaluate or analyze these issues.

In addition, the nature of the linkages may be more complex than what can be shown through simple correlation. For example, there may be a lag between the time a price effect felt by one consumer causes a price effect felt by another consumer. In this case, there may be no contemporaneous price corre-

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<sup>42</sup> This does not necessarily imply that all such purchasers could be included in a class or collect damages. The legal issue of whether a plaintiff has legal standing, for example, is a separate issue that could preclude such purchasers from being included in a class.

<sup>43</sup> In theory, the irrelevant common factor effects should be removed from the price series prior to measuring price correlation. A possible method is to use regression analysis to estimate the impact on each price series of the common factors and then calculate the correlation of the estimated residuals. See e.g., Margaret E. Slade, *Exogeneity Tests of Market Boundaries Applied to Petroleum Products*, 34 J. INDUS. ECON. 291 (1986); Stigler & Sherwin, *supra* note 37.

lation, but some more complex pattern of correlation that must be measured using lagged values of prices or more complex models.<sup>44</sup>

#### D. STATISTICAL AND ECONOMIC SIGNIFICANCE

Another issue that must be considered if relying on price correlations to support a finding of a pricing structure is what, precisely, is the standard for a finding that prices are correlated. A correlation coefficient can range between  $-1$  and  $1$ . Only a correlation of exactly  $1$  implies that two prices are perfectly positively related. However, this is highly unlikely to be achieved in practice. The question then is how much correlation is enough.

The statistical significance of any given correlation coefficient can be determined using standard techniques. However, the standard test of whether the coefficient is statistically different from zero (at a confidence limit like 95 percent) is not necessarily informative. A very low measured correlation could be statistically significant yet not economically significant, in that it shows very little relationship between the prices. The statistical significance simply means that, statistically speaking, the relationship is not zero.

The question of how much correlation is enough has been raised, but not resolved, in the literature on using price correlations in antitrust market definition analysis.<sup>45</sup> Sherwin suggests the possibility of using benchmark correlations for market definition purposes where the benchmark is a calculated correlation between two products that are clearly in the same relevant market.<sup>46</sup> For example, if, through some analysis, it has been established that two products are in the same relevant market and the correlation coefficient between those two products is  $0.85$ , then the benchmark of  $0.85$  is used to determine whether or not other products are in the relevant market. On its own,

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<sup>44</sup> This possibility has led to other price-based tests being proposed in the market definition literature, for example, Granger causality tests, stationarity tests, and cointegration tests. See Coe & Krause, *supra* note 37 (discussing the different approaches).

<sup>45</sup> For example, Stigler and Sherwin write: "What is the level of correspondence between two price series, either directly or in first differences, that determines that they are in the same market? . . . We believe that no unique criterion exists, quite aside from the fact that the degree of correspondence of two price series will vary with the unit and duration of time, the kind of price reported, and other factors." Stigler & Sherwin, *supra* note 37, at 562. Werden and Froeb note that Stigler and Sherwin lack a "fixed rule for determining whether a correlation is high enough to place the two products in the same relevant market." Werden & Froeb, *supra* note 37, at 346.

<sup>46</sup> Robert A. Sherwin, *Comments on Werden and Froeb—Correlation, Causality, and All that Jazz*, 8 REV. INDUS. ORG. 355, 357 (1993) ("Since markets often are not neatly divisible, the absence of exact criteria is a virtue reflecting the continuities that can exist in the degrees of substitution among products of regions. Stigler & Sherwin do provide some guidance, however. Price correlation levels of two products obviously in the relevant market (for example, the products of the two merging firms in many cases) can be used as a benchmark."). This approach is loosely followed by Coe and Krause, *supra* note 37, in their evaluation of price correlations versus other suggested tests.

without further analysis of the reasons for correlation or lack of correlation, this method is too crude to be a reliable indicator of what products or customers would be commonly affected by some alleged conduct.

The problem with determining the appropriate level of correlation in the context of class certification can be easily illustrated.

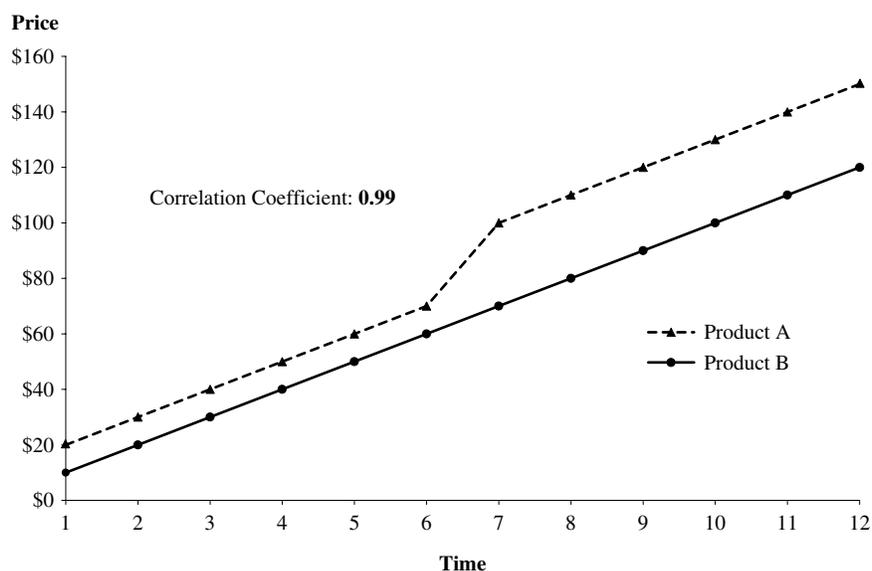


FIGURE 2: CORRELATION IN DIVERGENT PRICE SERIES

Figure 2 shows two price series, product A and product B, in which the two prices move together (perfectly positively correlated) until product A's price level is pushed up to a higher level. Product B stays on the same trajectory. This example could be generated by the two products' prices being driven by a common factor, such as a change in technology, but the anticompetitive conduct affecting only product A. The correct inference in this example is that customers of product B are not affected by the alleged conduct. However, the calculated correlation coefficient, even with the divergence between the two price series, is over 0.99. On the basis of the correlation coefficient one might presume that these prices are correlated to a degree that any overcharge to the customers of product A would be also felt by the customers of product B.<sup>47</sup>

<sup>47</sup> This thought experiment could easily be changed to considering the two price series in a time period when there was no allegation of an anticompetitive overcharge, and the increase in product B's price was due to a cost shift. Again, the incorrect inference based on the calculated correlation coefficient of over 0.99 would be that these prices are highly correlated, thus, any

However, this would be an error. The high correlation is driven by the common factor; the alleged conduct does not have a common impact.

Finally, as pointed out above, in a setting with many types of consumers and many products, there will be a multitude of relevant price correlations. Many of the calculated correlations may be positive and high, but there also may be many that are positive and low, or negative, or statistically insignificant.<sup>48</sup> Plaintiffs' reliance on correlation would imply, in this circumstance, that there is no pricing structure that covers the entire putative class or, stated differently, there are at least segments of the proposed class that require individualized analysis. However, given the problems associated with drawing conclusions based on correlation alone, one could not necessarily conclude that some subset of prices with "high" correlations is evidence of an underlying pricing structure for those products.

#### E. IMPLICATIONS FOR PRICING STRUCTURE AND CORRELATIONS

The issues described above demonstrate the difficulties associated with drawing any inference related to common impact or an underlying pricing structure based on price correlations. Evidence of price correlation without analysis related to why the prices are correlated is not instructive for class certification purposes. Some theory and accompanying evidence about how prices of individual products or to individual consumers are determined should precede the calculation of price correlations. Without such evidence, it is unlikely that any reliable inference can be drawn from a single price correlation, let alone thousands of individual price correlations, some of which may be positive, while others are negative or zero.<sup>49</sup>

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overcharge felt by the customers of product A (in the alleged class period) would also be felt by the customers of product B.

<sup>48</sup> See *In re Graphics Processing Units Antitrust Litig.*, 253 F.R.D. 478, 494–95 (N.D. Cal. 2008) (discussion). The defendant's expert economist disaggregated the data utilized by the plaintiffs' expert economist and, because of the diversity of products and purchasers, calculated "hundreds of thousands" of correlation coefficients. *Id.* Well over 50 percent were found to be negative or not statistically different from zero. *Id.*; see also *In re Pressure Sensitive Labelstock Antitrust Litig.*, No. 3:03-MDL-1556, 2007 U.S. Dist. LEXIS 85466, at \*60–61 (M.D. Pa. Nov. 19, 2007) (discussion).

<sup>49</sup> Our discussion of price correlations has not addressed the common situation in which only limited data are available. Relevant transaction data may not be kept in the course of business, or may not be produced in the litigation. In such a case it is not possible to calculate all of the relevant correlations. If an inference is to be drawn about the existence of a classwide pricing structure, it must rely on more than just the correlations that can be calculated. There must be further evidence that connects the missing price series to those for which data are available.

### III. REGRESSION ANALYSIS AND A COMMON MODEL OF PRICE DETERMINATION

Regression analysis is another empirical tool that has been used to claim antitrust impact can be established using a common methodology.<sup>50</sup> Economic experts for plaintiffs have concluded that if the wide price dispersion that exists across a given putative class can be adequately explained by a common economic model, then, as with a finding of a pricing structure as discussed above, the determination of antitrust impact is predominantly common. The rationale is that the factors relevant to explaining the observed pricing diversity can be “controlled for” in a common model. Thus, for any one member of the putative class the determination of antitrust impact will rely on a model that is common to the class.

Unlike correlation analysis, regression analysis is a statistical tool that is used in the context of a hypothesized economic model that posits a causal relationship among economic variables.<sup>51</sup> Casual relationships between a dependent variable (in this context, prices) and multiple independent variables (such as the economic determinants of prices like relevant demand and supply factors, possibly including a variable that captures the presence of the alleged conspiracy) are hypothesized. Data for the dependent and each of the independent variables are collected and used to estimate the posited relationships and to test their statistical significance. Regression analysis is a way to do this estimation and testing using available data.<sup>52</sup>

<sup>50</sup> See ABA ECONOMETRICS, *supra* note 18, at 220–24; Jonathan B. Baker & Daniel L. Rubinfeld, *Empirical Methods in Antitrust Litigation: Review and Critique*, 1 AM. L. & ECON. REV. 386 (1999); Blair & Durrance, *supra* note 1; Johnson & Leonard, *supra* note 1; Nieberding & Cantor, *supra* note 1; see also *supra* note 3 for discussion in class certification decisions.

<sup>51</sup> For an overview of regression analysis in a litigation context, see Daniel I. Rubinfeld, *Reference Guide on Multiple Regression*, in FED. JUDICIAL CTR., REFERENCE MANUAL ON SCIENTIFIC EVIDENCE 179–227 (2d ed. 2000).

<sup>52</sup> The type of economic model estimated in this setting is generally a “reduced form” model, or a “reduced form price equation.” Baker and Rubinfeld state:

The model is called ‘reduced form’ because the price equation is thought of as derived from other, prior economic relationships—in this case, the interaction of a demand function with a supply relation. In consequence, the parameters of a reduced form equation are typically themselves functions of a number of the structural parameters (the parameters of the underlying economic relationships).

Baker & Rubinfeld, *supra* note 50, at 391; see also ABA ECONOMETRICS, *supra* note 18, at 221–22. This model differs from a “structural” model where, for example, the demand function and the supply relation would be estimated separately. The reduced form versus structural difference is important to consider in the context of what correlation analysis attempts to show. The distinction between a common factor effect and an economic linkage effect cannot be determined in a reduced form model. Thus, a reduced form price equation cannot speak to what kind of economic effect leads to a relationship between two observed price series. As Baker and Rubinfeld write: “Reduced form models are least desirable when the key question for litigation depends on structural parameters, as these are typically very difficult to recover from reduced forms.” Baker & Rubinfeld, *supra* note 50, at 405. A reduced form price equation does not

## A. REGRESSION ANALYSIS AND ANTITRUST IMPACT

By using regression analysis to estimate an economic model, the argument is that if transaction-level data are utilized in the regression, then the economic model being estimated is a model that can uncover the relevant determinants of prices paid by each individual buyer in each of their transactions. Thus, if a variable included in the model captures the presence of the alleged conspiracy, then (again, in theory) the model will not only establish the existence of impact, but also its quantum. Similarly, the claim is made that if the model does not directly estimate the impact of the alleged conspiracy but rather estimates “but for” prices, then the existence and quantum of impact can be established by comparing the estimated “but for” prices with the actual transaction prices paid by each consumer.

In real-world markets, there are issues and complexities that often make this idealized analysis intractable. One important consideration with regard to determination of impact to proposed class members is that regression models are not generally capable of producing coefficient estimates that are specific to particular transactions. That is, while a regression model that contains a variable (or variables) that measured the effect of the alleged conspiracy could be used to establish “average” amounts of damage across groups of transactions (or groups of proposed class members), it cannot be used to address the question of whether or not a particular transaction was impacted, or the amount a particular transaction was impacted. For example, a regression model based on all putative class members’ price, which includes a single variable to measure the effect of an alleged conspiracy on price, returns one estimate of the overcharge for all transactions.<sup>53</sup> Such a regression is not capable of addressing a key issue at the class certification stage, that is, whether or not particular transactions (or particular class members) were or were not affected by the alleged conspiracy.<sup>54</sup> This specification would assume rather than test whether individual class members were affected identically by the alleged conduct. Trying to incorporate more variables in the model that allow for different measures of the effect of an alleged conspiracy on different trans-

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enable the kind of determination of pricing structure that a correlation analysis does if structured properly to uncover economic linkage effects.

<sup>53</sup> ABA ECONOMETRICS, *supra* note 18, at 222 (“The reduced-form pricing equation assumes that a conspiracy has the same effect on every purchaser and focuses on an average effect, which may hide variation across class members. If one is attempting to test whether there is an impact on all members of a proposed class, however, that assumption is not valid, as it assumes the very proposition that is being tested.”). Even with a more complex and sophisticated econometric model, for example a model where the distribution of the coefficients is recoverable, there is no way to associate particular coefficient results with particular transactions or particular proposed class members.

<sup>54</sup> See Richard Schmalensee, *Economic Analysis of Class Certification*, GLOBAL COMPETITION POL’Y, Spring 2008, Vol. 6, No. 2, at 1.

actions (or for different proposed class members) may be infeasible. In many cases, there is not enough data for statistically sound estimates to include a different variable for each class member and this will certainly be the case if the model attempts to measure impact at the individual transaction level.<sup>55</sup> Similarly, it is infeasible for a regression model that does not directly estimate the impact of the alleged conspiracy but rather estimates “but for” prices to have coefficient estimates that are specific to particular transactions, or specific to particular putative class members.

There are additional issues that should be considered when using regression analysis in the context of class certification. The necessary data are often unavailable.<sup>56</sup> Unavailable data could include data on the determinants of price, such as the factors that may affect negotiated prices between buyers and sellers. A different, but equally serious problem exists if the necessary data may be obtained only through individualized inquiry, for example, by analyzing the negotiations between individual buyers and the defendant.<sup>57</sup> In some cases, individual-level price data itself may not be available.<sup>58</sup>

However, it is a reality of economic modeling that data are never perfect and rarely complete. This raises the question, at the class certification stage, of how complete an empirically estimated economic model must be to establish that antitrust impact can be established using a common methodology.<sup>59</sup> In some cases, courts appear to have been persuaded by an economic expert’s conclusion that such modeling could be done—even though it has not been done.<sup>60</sup> In other instances, even where some preliminary modeling has been undertaken, courts have rejected it as woefully incomplete.<sup>61</sup>

#### B. REGRESSION ANALYSIS AND THE DIVERSITY PARADOX

It is often argued that circumstances in the marketplace are too complex for common modeling to be undertaken. Defendants may point out that a proposed class includes diverse products and customers and that the supply and demand variables that affect the relevant prices are different and cannot be

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<sup>55</sup> See ABA ECONOMETRICS, *supra* note 18, at 223 (“Estimating a model that allows each purchaser to experience a different price increase requires an enormous amount of data and will not often be feasible.”).

<sup>56</sup> See discussion *supra* note 27.

<sup>57</sup> This circumstance is explored in some detail in Johnson & Leonard, *Price Structure*, *supra* note 23.

<sup>58</sup> When individual customer transactions data is not available, but some kind of average, or aggregate, price data is, then similar issues to those discussed above in Part II.A arise. Using average, or aggregate, price data in a regression model can, at best, uncover antitrust impact at the average, or aggregate level. This does not establish impact at the individual level.

<sup>59</sup> See discussion *supra* note 15.

<sup>60</sup> See discussion *supra* note 20.

<sup>61</sup> See *id.*

included in a common model. In the face of this type of complexity, plaintiffs may claim that such differences can be explained using, for example, a fixed effects regression model.<sup>62</sup> A common fixed effects model, however, may or may not be able to address the important and relevant reasons for observed price dispersion. Any inference from such a model must consider carefully what price dispersion the model is, and is not, explaining.

Price dispersion can exist for a variety of reasons, including, for example, cross-sectional dispersion due to differences in products, product characteristics, and proposed class members, as well as price dispersion across time. In some cases, it may be possible to explain price dispersion across products but not possible to explain price dispersion across customers for a given product; fixed effects models may be capable of controlling for time-invariant product characteristics but, without individualized analysis, may not be able to explain time-invariant customer characteristics. Moreover, even if the time-invariant customer characteristics are investigated and can be included in a model, evidence of their importance is likely to suggest the existence of time-varying customer specific factors that affect price. This complicates the analysis further and increases the likelihood that individualized analysis of each proposed class member over time is necessary in order to determine impact.

Additionally, there is no clear standard for determining whether a particular regression model has sufficiently explained prices.<sup>63</sup> Traditional statistical measures, such as the R-squared statistic from the regression, have been relied upon.<sup>64</sup> As with the correlation coefficient, discussed above, however, there is no standard for how high an R-squared statistic needs to be before the model

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<sup>62</sup> See *supra* note 7; *In re Live Concert Antitrust Litig.*, 247 F.R.D. 98, 136 (C.D. Cal. 2007) (“[plaintiffs’ expert] stated that he could account for differences in the artist or concert through the use of a ‘fixed effects approach’ through the insertion of dummy variables into the regression analysis”). In the context of a regression proposed for damages, see *In re Pressure Sensitive Labelstock Antitrust Litig.*, No. 3:03-MDL-1556, 2007 U.S. Dist. LEXIS 85466, at \*65 (M.D. Pa. Nov. 19, 2007) (“In response to [defendants’ expert economist] criticism that the damages formula would not account for unique product and customer characteristics of the transactions, [plaintiffs’ expert economist] suggested that a fixed effects model could be employed to account for product- and customer-specific variables.”).

<sup>63</sup> This question is never reached in the instances where the description of the model is enough for plaintiffs to meet their burden, rather than the actual estimation of a model. If a description of a model is found to be sufficient at the class certification stage, the analysis could be rejected at a later point in the litigation, which may or may not raise issues for the certification of the class.

<sup>64</sup> See *supra* note 8. A higher R-squared is used to claim a “better” model. Though the R-squared statistic does capture, statistically, the model’s ability to explain price dispersion, it is still subject to the old adage that “garbage in means garbage out.” Simply adding more and more explanatory variables will increase the R-squared statistic even if the variables have no plausible economic significance. A variation of the R-squared statistic, called the Adjusted R-squared, attempts to correct for this by “penalizing” the statistic when irrelevant independent variables are added to the model. See PETER KENNEDY, A GUIDE TO ECONOMETRICS 79–80 (6th ed. 2008).

is “good enough.”<sup>65</sup> Furthermore, reliance on the R-squared statistic leads to the paradoxical result that the more diverse the proposed class is, the more likely it is that a fixed effects model will return a high R-squared statistic and the false conclusion that the individual prices have been explained with a common model.

Price dispersion may exist simply because the products have different characteristics and those characteristics have different values.<sup>66</sup> However, prices of a given product may also vary across different proposed class members. Such differences in price may reflect customers’ relative bargaining power, negotiating talents, relationships with the buyer, added services that are sold with the product to certain customers, or a host of other characteristics that may or may not be identifiable without analyzing the particular circumstances surrounding the buyer, seller, and prices.

It is the process of controlling for variation in price, using fixed effects, that leads to the diversity paradox: the more diverse the products and consumers in a putative class, the higher the R-squared in the regression model that attempts to “control” for this diversity. In a general fixed effects model, the individual prices paid by a particular customer for a particular product are modeled to depend upon a set of supply, demand, and structural factors, as well as fixed effect variables which do not vary over time but are different for each customer and product.<sup>67</sup> The fixed effect variables “control” for the cross-sectional price variation by shifting the model up and down for each customer-product combination. It is these “controls” in a setting of wide cross-sectional price dispersion which lead to a high R-squared.

Figure 3 illustrates this effect with hypothetical pricing data for a product with a price that varies around \$10. There is a corresponding (potentially) explanatory factor, labeled here simply as “ $X_t$ ” also shown. These data have been constructed so that the variable  $X_t$  has no explanatory power. Thus an estimated model, where the price of the product depends on  $X_t$ , yields an R-

<sup>65</sup> See ABA ECONOMETRICS, *supra* note 18, at 409; KENNEDY, *supra* note 64, at 26.

<sup>66</sup> Modeling differences in prices across products with fixed effect models can be difficult in certain circumstances. Specifying the model so that each product has an associated fixed effects coefficient may not be possible if the number of products is large relative to the number of price observations—for example, if each product identifier is considered a different product. In addition, it may or may not be appropriate to consider each product identifier as a separate product. What constitutes a separate product may require highly technical and product-specific information and analysis. An alternative specification, where fixed effect coefficients are associated with product characteristics, may also pose problems. For example, there may be numerous different product characteristics or product characteristics that are difficult to observe.

<sup>67</sup> Consider the following general model:

$$P_{ijt} = X_{ijt}\beta + c_{ij} + \mu_{ijt},$$

where  $P_{ijt}$  is the price paid by consumer  $i$  for product  $j$  at time  $t$ ;  $X_{ijt}$  is a set of supply, demand, or structural factors which are multiplied by coefficients,  $\beta$ ;  $c_{ij}$  is the fixed effect (it does not vary by time) for consumer  $i$  and product  $j$ ; and  $\mu_{ijt}$  is the error term.

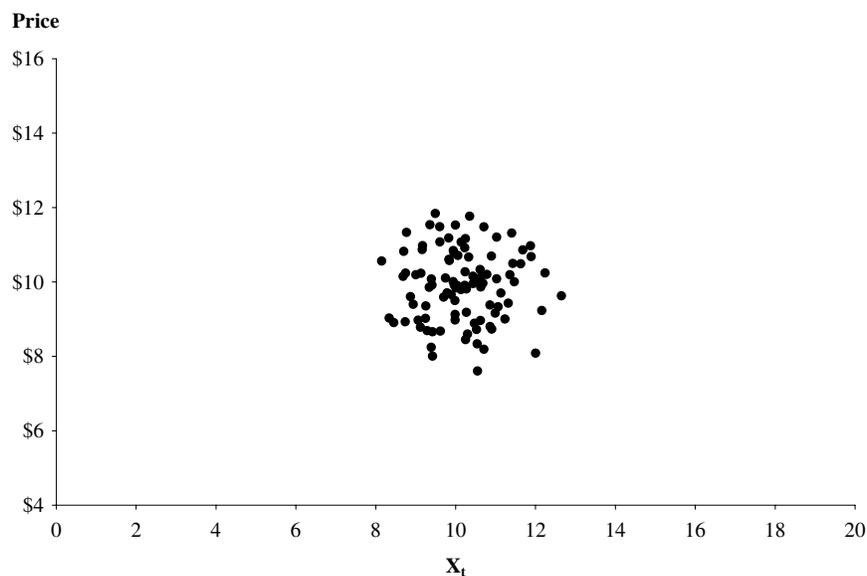


FIGURE 3: PRICES OF PRODUCT 1

squared of 0. Figure 4 adds another set of hypothetical pricing data for a similar but somewhat different product that varies around \$20. For example, the two hypothetical products here could differ in terms of size or capacity, with the first product being sold at prices lower than the first because the first product has less capacity. Again, the data has been constructed so that the variable  $X_t$  has no explanatory power. Thus an estimated model, where the second product's price depends on  $X_t$ , also yields an R-squared of 0.

Now consider estimating the model where both products' prices are included together in one model, the same explanatory variable,  $X_t$ , is included in the model, and fixed effect variables that differentiate the two products are included. Estimation of this model, including both products' prices together in one model, yields an R-squared of 0.92, or 92 percent.<sup>68</sup> Because, as shown, the factor  $X_t$ , the only explanatory variable considered, has no explanatory power for either of the two products' prices, the high R-squared from the combined model is due entirely to the explanatory power of the fixed effect

<sup>68</sup> More formally, the model estimated using the data in Figure 3,  $P_{1t} = X_t\beta + c_1 + \mu_{1t}$ , yields an R-squared of 0. (For ease of exposition we drop the customer index on P, and drop both the customer and product indices on X. The "t" indices attached to both the price and the explanatory variable, X, indicate that the data used in the estimation vary over time.) The model estimated using the data in Figure 4,  $P_{2t} = X_t\beta + c_2 + \mu_{2t}$ , yields an R-squared of 0. The model for the two products combined and including the fixed effect variables,  $P_{it} = X_t\beta + c_i + \mu_{it}$ , where  $i=1,2$ , yields an R-squared of 0.92, or 92 percent.

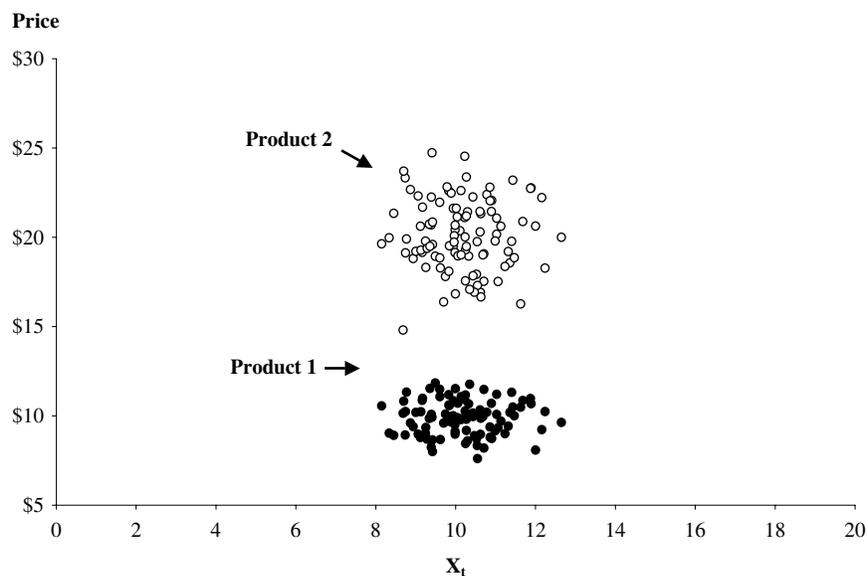


FIGURE 4: PRICES OF PRODUCT 1 AND PRODUCT 2

variables that do nothing more than distinguish the two products. The “high” explanatory power of the model, or the fact that the model’s independent variables largely “explain” the observed price dispersion, is entirely due to the pooling together in the putative class of two products (or consumers) that engage in transactions at different average price points. The independent variable in the model is not capturing the underlying economic determinant of prices. Rather, the model is simply reporting the fact that there are two different price series, at two different general levels, that have been pooled together. If a fixed effects model (as above) were used in a model where flat screen television prices (at an average of \$1,000) were pooled with milk prices (at an average of \$2) the resulting model’s R-squared would be high because the model would accurately capture the variation due to the difference in the average price of flat screen televisions and milk. It would clearly be an error in inference if it was concluded (on the basis of this alone) that a valid common model for these two very different sets of products could be determined and, thus, the determination of antitrust impact is predominantly common.

The greater the cross-sectional variation (or diversity) in the putative class, the higher the R-squared of the resulting estimated model. That is, the greater

the price dispersion, the better the “common” model appears to fit the data.<sup>69</sup> This is the result we refer to as the diversity paradox.

### C. IMPLICATIONS FOR REGRESSION ANALYSIS

The diversity paradox implies that use of fixed effects models to satisfactorily control for and explain observed price dispersion in a complex putative class must be met with careful skepticism. The use of fixed effects models, as described above, will likely lead to an estimated model that appears to “explain” a great deal of the pricing variation but may or may not explain any of the relevant differences in prices across putative class members.<sup>70</sup>

If differentiated products and consumers are pooled together and the resulting price dispersion is said to be explained in a regression model by common economic forces, then those economic forces should have some standalone explanatory power on each of the different categories of products and consumers. There are methods that can be used to address whether price variation for given products is explained by a common economic model. For example, if the proposed regression were estimated for product and consumer categories individually (thus, no fixed effect variables), then it may be possible to measure the remaining pricing variation explained by the explanatory variables in the model. Alternatively, it is possible to calculate the R-squared for a transformed fixed effects model, the “fixed-effects,” or “within” transformation.<sup>71</sup> This transformation removes the fixed effects coefficients by using differences from the mean as independent variables. For example, if it is posited that income affects price, then income as a variable in the model would enter as the difference between income at a particular point in time and the mean income over the sample period. Because the fixed effects variables do not change over time, this differencing drops all of the fixed effects variables from the estimation.<sup>72</sup> The “prices” now being modeled are in terms of differences from their category means, so the data no longer contain the price-level differences between transactions involving different products or consumers. The R-Square from the estimated model will not be influenced by the effect

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<sup>69</sup> In the example above, if the pricing data for the second product, which varied around \$20, varied around higher and higher values, the resulting R-squared from the model described above would also be higher and higher. Thus, the greater the diversity, the greater the R-squared.

<sup>70</sup> The model described above is equivalent to one in which there is an indicator, or dummy, variable for each consumer/product combination. The misleading inference that is drawn from the interpretation of the R-squared statistic in a model with a variety of dummy variables is identified in KENNEDY, *supra* note 64, at 237 (“Care must be taken in evaluating models containing dummy variables designed to capture structural shifts or seasonal factors, since these dummies could play a major role in generating a high  $R^2$ , hiding the fact that the independent variables have little explanatory power.”).

<sup>71</sup> See WOOLDRIDGE, *supra* note 7, at 302–04 (providing details).

<sup>72</sup> The model being estimated becomes  $(P_{ijt} - \bar{P}_{ij}) = (X_{ijt} - \bar{X}_{ij})\beta + \mu_{ijt}$ .

discussed above, where the fixed effects explained the variation among different price levels because the price levels have been taken out of the data. The R-squared is solely influenced by the explanatory power of the supply, demand, and structural variables found in the model.<sup>73</sup>

However, if separate models were estimated for different product and customer groups, or if the fixed effects model is transformed, the issue discussed above relating to the lack of a benchmark R-squared, or a standard for how much price dispersion is explained by any regression, remains. Moreover, in practice it is likely that the value of the R-squared statistics will vary across models estimated for particular products and customers, further complicating the issue of whether or not the proposed model is sufficient to conclude there is evidence of common impact or an underlying pricing structure. Only a truly robust model will withstand the scrutiny suggested by these issues.

The diversity paradox means that in a setting with wide cross-sectional price dispersion across the putative class, the empirical establishment of a “good enough” common-to-the-class economic model of price determination must not rest on simply “explaining” or “controlling” the variation using fixed effects. Other explanatory variables that capture the true underlying economics must also have real explanatory power. Only then would it be valid to use such a model to help draw a conclusion that the determination of antitrust impact is predominantly common.

#### IV. CONCLUSION

The trend towards more rigorous economic analysis at the class certification phase in antitrust matters has resulted in greater reliance by economic experts on statistical and econometric tools, such as correlations and regressions, to inform their opinions on whether impact on the class can be shown with a common methodology using common evidence. Defendants’ experts’ reliance on price dispersion across both products and customers has been met by plaintiffs’ experts’ claims that “high” correlations and regressions with “high” R-squared statistics are indicative of an underlying common price structure and evidence that impact can be determined with a common model.

In this article, we review the use of these empirical tools for drawing inferences about the existence of a “price structure.” In certain cases both can be usefully used to uncover relevant economic relationships that can inform a decisionmaker about the merits of class certification. However, as we show, they are also prone to misuse and misinterpretation. The interpretation of a

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<sup>73</sup> For example, utilizing this transformation (modeling prices as differences from their category means) on the model described above, using the data in Figure 4, yields an estimated R-squared of 0.0002. This is as expected because, by construction, the sole explanatory variable  $X_i$  has no explanatory power.

correlation analysis or a regression analysis proffered in the context of class certification must be done carefully and with an understanding of what such analyses do and do not say about the economics of transactions by a given putative class. As the trend towards more rigorous economic analysis at the class certification phase continues, decisionmakers are faced with the challenges of becoming more sophisticated in their understanding of the use and misuse of such statistical and econometric tools as correlations and regressions.

APPENDIX A: CORRELATION BIASES IN AVERAGE PRICE SERIES

Consider two price series, X and Y, each representing T individual price observations (labeled  $x_i$  and  $y_i$ , for  $i = 1$  to T).<sup>74</sup> We designate the sample correlation between two price series, X and Y, as  $\rho(X, Y)$ .<sup>75</sup> We designate the sample variance of an individual price series as  $\text{Var}(X)$ .<sup>76</sup> Now assume that each element of X and Y is constructed by averaging a number of price series (X is an average of  $X^1, \dots, X^N$ , and Y is an average of  $Y^1, \dots, Y^M$ );<sup>77</sup> so that an element of X is an average of elements of  $X^1, \dots, X^N$ , or

$$x_i = \frac{\sum_{j=1}^N x_i^j}{N} \tag{3}$$

Similarly, the elements of Y are also averages. The individual price series could represent prices paid by different consumers for the same product, or prices paid by different consumers for different products, or prices paid by the same consumer for different products.

It can be shown that

$$\rho(X, Y) = \sum_{j=1}^N \sum_{k=1}^M \omega_{jk} \rho(X^j, Y^k) \tag{4}$$

where

$$\omega_{jk} = \frac{\sqrt{\text{Var}(X^j)\text{Var}(Y^k)}}{\sqrt{\text{Var}(\sum_{j=1}^N X^j)\text{Var}(\sum_{k=1}^M Y^k)}} \tag{5}$$

That is, the correlation between the two average price series is a weighted sum of the correlations between the individual series that make up the two

<sup>74</sup> This example assumes that there are relevant matched prices in the two price series, X and Y.

<sup>75</sup> The sample correlation is defined as

$$\frac{\sum_{i=1}^T (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^T (x_i - \bar{x})^2 \sum_{i=1}^T (y_i - \bar{y})^2}} \tag{1}$$

<sup>76</sup> The sample variance is defined as

$$\frac{\sum_{i=1}^T (x_i - \bar{x})^2}{T - 1} \tag{2}$$

<sup>77</sup> For ease of exposition we will assume that each of these series,  $X^j$  and  $Y^k$ , also has T price observations.

average series. This implies that negative, or zero, correlations between individual price series may become obscured by the averaging process. The correlation between X and Y could be found to be high and positive when a number of correlations between individual series  $X^j$  and  $Y^k$  are zero or negative.

Consider the simple example shown in Table 1, which shows four monthly price series.<sup>78</sup> One scenario (call this the one-product scenario) is that these series represent prices paid for a single product purchased by four different consumers (A, B, C, and D). Another scenario (call this the two-product scenario) is that these represent prices paid for two different products by four different consumers (the first product, X, is bought by consumers A and B, and the second product, Y, is bought by consumers C and D).

TABLE 1: EXAMPLE OF CORRELATION BIAS IN  
AVERAGE PRICE SERIES

	January	February	Change
Price Series A	\$10	\$21	Up \$11
Price Series B	\$10	\$9	Down \$1
Average A and B	\$10	\$15	Up \$5
Price Series C	\$10	\$7	Down \$3
Price Series D	\$10	\$23	Up \$13
Average C and D	\$10	\$15	Up \$5

In the example in Table 1, we see the following. In January, all purchasers are paying \$10 and so the average price for both A and B and for C and D purchasers is \$10. In February, the prices paid by A and B are \$21 and \$9, respectively. An average calculation would indicate that the average price for A and B is \$15 for February. Also in February, the prices paid by consumers C and D are \$7 and \$23, respectively, and so the average price for consumers C and D is also \$15 for February.

Therefore, comparing the monthly average prices shows that the average prices for consumers A and B moves from \$10 to \$15 between January and February and the average price for consumers C and D also moves from \$10 to \$15 between January and February. The conclusion drawn would be that the two average prices move in the same direction and by the same amount (e.g., the averages would be perfectly and positively correlated).

<sup>78</sup> This example assumes that there are relevant matched prices at the monthly level.

However, this methodology of averaging prices paid (in either the one-product or two-product interpretation) obscures important discrepancies in the individual purchasers' price movements. Evaluating the relationship between prices paid by individual consumers shows that among the A and B purchasers the prices move in different directions. Similarly, among the C and D purchasers the prices also move in different directions. Further, the prices of A and C purchasers move in different directions, and prices of B and D purchasers move in different directions.

## APPENDIX B: CORRELATION BIASES IN STACKED PRICE SERIES

Consider four price series,  $X^1$ ,  $X^2$ ,  $Y^1$ , and  $Y^2$ . Each series has  $T$  elements. Now assume that there is no correlation between price series  $X^1$  and  $Y^1$ , and there is no correlation between price series  $X^2$  and  $Y^2$ , that is,  $\rho(X^1, Y^1) = 0$  and  $\rho(X^2, Y^2) = 0$ .

Now consider two “stacked” series,  $X$  and  $Y$  that are each constructed with  $2T$  elements, such that for  $X$ ,  $x_i$  is equal to  $x_i^1$  for  $i = 1$  to  $T$ , and equal to  $x_i^2$  for  $i = T+1$  to  $2T$ , with  $Y$  constructed similarly. It can be shown that

$$\rho(X, Y) = \frac{\frac{T}{2}(\bar{X}^1 - \bar{X}^2)(\bar{Y}^1 - \bar{Y}^2)}{\sqrt{\sum_{i=1}^{2T} (x_i - \bar{X})^2 \sum_{i=1}^{2T} (y_i - \bar{Y})^2}} \quad (6)$$

If the means of the two price series,  $X^1$  and  $X^2$ , and the means of the two price series,  $Y^1$  and  $Y^2$ , are different, then even though there is zero correlation between  $X^1$  and  $Y^1$ , and there is zero correlation between  $X^2$  and  $Y^2$ , there is a correlation between the stacked price series. That is, under certain circumstances using stacked price series manufactures correlation where none exists in the underlying price series. Note also that even though  $\rho(X^1, Y^1) = \rho(X^2, Y^2)$ , it is the case that  $\rho(X^1, Y^1) = \rho(X^2, Y^2) \neq \rho(X, Y)$  except under particular conditions. Stacking does not generate some kind of “average” correlation.

This phenomenon can be understood by considering the case of two price series of different levels that are stacked on top of one another. For example, consider  $X^1$  and  $Y^1$  to represent low price products and  $X^2$  and  $Y^2$  to represent high price products (thus  $\bar{X}^1 < \bar{X}^2$  and  $\bar{Y}^1 < \bar{Y}^2$ ). In this case we can see, via the equation above, that  $\rho(X, Y) > 0$ . This is because a price series at a high level of prices is being stacked on top of a price series at a low level of prices (in both  $X$  and  $Y$ ).

Consider Tables 2 and 3. In each there are four months of prices paid by two types of customers. In Table 2 there are prices for product  $X$  paid by customer type A and customer type B. In Table 3 there are prices for product  $Y$  paid by customer type A and customer type B. The Table 2 data is constructed such that the correlation coefficient between product  $X$  prices paid by customers A and product  $X$  prices paid by customers B is zero. Similarly, Table 3 shows product  $Y$  prices where these prices are constructed such that the correlation coefficient between customer A prices and customer B prices is zero. Importantly, these prices have been constructed such that the product  $Y$  prices are substantially higher (by ten times) than the product  $X$  prices. These different price levels are important in understanding why stacking leads to incorrect results.

TABLE 2: EXAMPLE OF ZERO CORRELATION PRICE SERIES

	Product	Customer A Price	Customer B Price
January	X	0.5	0.5
February	X	1.5	1.5
March	X	0.5	1.5
April	X	1.5	0.5
Correlation Coefficient Between Prices Paid by Customer A and Customer B is <b>ZERO</b>			

TABLE 3: EXAMPLE OF ZERO CORRELATION PRICE SERIES

	Product	Customer A Price	Customer B Price
January	Y	5	5
February	Y	15	15
March	Y	5	15
April	Y	15	5
Correlation Coefficient Between Prices Paid by Customer A and Customer B is <b>ZERO</b>			

Now consider Table 4. This is where the data is “stacked” so that the four price series become two. The product Y data for each customer is stacked on top of the product X for that customer. This creates two eight-month series out of two sets of four-month series.

The distortion due to stacking is shown with the calculated correlation coefficient based on the stacked prices. That coefficient is 0.62, even though the underlying prices at the individual product level have zero correlation. Because the product X and product Y prices in the example are so different in price level, stacking them introduces correlation when, in fact, there is none.<sup>79</sup>

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<sup>79</sup> This problem due to stacking was discussed by the court in the class certification decision in *In re Graphics Processing Units Antitrust Litigation*, 253 F.R.D. 478, 495 (N.D. Cal. 2008). The court found that the use of stacking by the plaintiffs’ economics expert artificially inflated his measured correlation coefficients.

TABLE 4: EXAMPLE OF CORRELATION BIAS IN  
STACKED PRICE SERIES

	Product	Customer A Price	Customer B Price
January	X	0.5	0.5
February	X	1.5	1.5
March	X	0.5	1.5
April	X	1.5	0.5
January	Y	5	5
February	Y	15	15
March	Y	5	15
April	Y	15	5
Correlation Coefficient Between Prices Paid by Customer A and Customer B is <b>0.62</b>			