

Centralization vs. Delegation by a Firm that Supplies to Rivals

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Abstract

Consider a vertically integrated input monopolist, M , that supplies to its downstream unit, division 1, and a downstream rival, firm 2. Downstream competition may be differentiated-products Bertrand or Cournot. We compare two organizational structures for the integrated firm. Under *Centralization*, M sets an input price only to firm 2, and division 1's choice (price or quantity) maximizes integrated profit, including the upstream profit from total input sales. With *Delegation*, M sets observable input prices, w_1 and w_2 , to both downstream competitors, division 1 maximizes only its own (downstream) profit, and M internalizes division 1's profit when setting input prices. We show that these structures are not equivalent: the integrated firm generically earns higher profit with Delegation. This result holds whatever the nature of downstream competition.

Charging an observable input price w_1 is valuable solely because—by signaling a change in division 1's downstream choice—it provides the integrated firm with an additional instrument for altering the choice of firm 2, beyond relying solely on the input price w_2 . Indeed, we show that Delegation yields the same profit as the integrated firm could earn if it remained centralized but could act as a Stackelberg leader downstream and, hence, could influence firm 2's choice directly through its own downstream choice (instead of indirectly via w_1 under Delegation).

The desired change in firm 2's choice is *a priori* more complex than in standard two-stage models of competition because here firm 2 is both a downstream rival and an input customer. Seemingly, there is a tradeoff: e.g. with Bertrand competition, an increase in firm 2's price benefits division 1 but reduces the profitable input sales to firm 2. Despite the seeming ambiguity, we establish a sharp result: starting from the level of w_2 under Centralization, with Delegation the integrated firm wants to move firm 2's choice in the opposite direction to the prediction of standard two-stage games with no supply relationship: it wants to induce a *reduction in firm 2's price* under Bertrand competition or an increase in firm 2's quantity under Cournot competition. These results do not hinge on whether, in each case, the downstream variables are strategic substitutes or strategic complements (though the direction of change in w_1 will vary).

If the integrated firm charges firm 2 a two-part tariff for the input instead of a linear price, the first two results extend—Delegation still dominates Centralization and achieves the same outcome as Stackelberg leadership—and we show how the incentives to shift firm 2's choice are altered.

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1. Introduction

In the familiar successive monopoly setting with linear input pricing, vertical integration eliminates double marginalization and yields equal profit under two alternative organizational structures for the integrated firm: the downstream variable (price or quantity) is chosen to maximize overall profit, and there is no economically meaningful input price; or the downstream division maximizes its own profit and buys the input from the upstream division at marginal cost. Instead of downstream monopoly, consider strategic competition—duopoly for simplicity—and partial integration: the input monopolist, M , is vertically integrated with a downstream unit, division 1, and continues supplying firm 2. Does the choice of organizational structure now matter?

We compare two structures. Under *Centralization*, M sets an input price only to firm 2, and then downstream variables are chosen simultaneously, with division 1 acting to maximize integrated profit, including the upstream profit from total input sales. Under *Delegation*, M sets observable input prices to both downstream competitors, then downstream choices are made simultaneously, with division 1 acting to maximize only its own profit (as explained later, this assumption can be relaxed substantially); here, M internalizes division 1's profit when setting input prices. We show that the integrated firm generically earns higher profit with Delegation — whatever the downstream competition (in prices or quantities, and strategic substitutes or strategic complements) and whether the integrated firm uses linear pricing for the input to the rival or a two-part tariff.

The logic is that commitment to an observable input price w_1 for the downstream division provides the integrated firm with an additional instrument for affecting the choice of firm 2 beyond relying solely on its input price w_2 . Under Centralization, the integrated firm's downstream choice is based on a shadow marginal cost equal to the input resource cost plus the opportunity cost arising if division 1's output reduces input sales to firm 2. In equilibrium, firm 2 correctly anticipates division 1's shadow marginal cost and, hence, its downstream choice. Under Delegation, the upstream supplier can set an observable and economically relevant w_1 different from the (ex post) shadow marginal cost. The value of this additional instrument w_1 derives entirely from using it to signal a change in division 1's downstream choice in order to alter firm 2's choice. Indeed, we show that Delegation

yields the same profit as the integrated firm could earn if it remained centralized but could act as a Stackelberg leader downstream and, hence, could influence firm 2's choice directly through its own choice (instead of indirectly via w_1).

More surprisingly, the change in firm 2's choice desired by the integrated firm runs opposite to that in standard two-stage models of competition with no supply relationship (e.g. Fudenberg and Tirole, 1984; Bulow, Geanakoplos, and Klemperer, 1985), where the goal is to make the rival's behavior "softer." Because firm 2 here is both a downstream competitor and an input customer, its choices have opposing effects on the integrated firm's profits. For example, when firm 2 and division 1 engage in (differentiated) Bertrand competition, a rise in firm 2's price would increase division 1's profit but lower the upstream division's profit from input sales to firm 2. Despite these opposing effects, we establish a strong result: starting from the level of w_2 under Centralization (with linear input pricing), under Delegation the integrated firm wants to induce (via w_1) a decrease in firm 2's price if the downstream competition is Bertrand, or an increase in firm 2's output if the competition is Cournot. These results hold whether the downstream variables (prices or quantities) are strategic substitutes or strategic complements (though the required level of w_1 will differ). We also discuss how the incentives change if the integrated firm charges firm 2 a two-part tariff.

The closest work to ours is by Arya, Mittendorf, and Yoon (2008). Our analysis is broader in several important respects, e.g. allowing general non-linear demands, Bertrand competition (not only Cournot), and two-part tariffs; but they consider a richer set of potential organizational arrangements, as we discuss in the concluding section. There, we also contrast our work to that of Bonanno and Vickers (1988) on the strategic advantage of vertical separation by rival suppliers, and to the literature on strategic advantages of establishing competing divisions (e.g. Schwartz and Thompson, 1986; Baye, Crocker, and Ju, 1996). We also discuss briefly the plausibility of Delegation. Empirically, vertically integrated firms have broad latitude in how they structure their inter-division dealings, so the strategic advantages of the Delegation structure may have practical implications.

In the rest of the paper, Section 2 describes the model and Section 3 presents the results, first for linear pricing of the input and then for two-part tariffs. Section 4 concludes.

2. The Setting

Consider an input monopolist, M , that supplies to its downstream unit, division 1, and to an independent downstream rival, firm 2. The downstream choice variables x_1 and x_2 are either per-unit prices (p_1 and p_2) or quantities (q_1 and q_2), thereby allowing either Bertrand or Cournot competition downstream.¹ We consider two games, representing alternative organizational structures of the partially integrated firm.

Centralization: First, M publicly sets a per-unit input price w_2 to firm 2. (Two-part tariffs are discussed in Section 3.2.) Then, division 1 and firm 2 simultaneously set downstream variables $\mathbf{x} = (x_1, x_2)$, consumers purchase, and firm 2 pays for M 's input. Firm 2 chooses x_2 to maximize its profit $\Pi_2(\mathbf{x}; w_2)$. There is no explicit input price to division 1, and x_1 is chosen to maximize integrated profit, i.e. division 1's profit plus the profit from input sales. M sets w_2 to maximize integrated profit. This is a standard representation of behavior by an integrated firm.²

Delegation: First, M publicly commits to a pair of input prices $\mathbf{w} = (w_1, w_2)$ where w_1 denotes the price to division 1. Given these prices, division 1 and firm 2 choose downstream variables \mathbf{x} simultaneously, consumers purchase, and M receives input payments. Division 1 chooses x_1 to maximize only its profit $\Pi_1(\mathbf{x}; w_1)$ as would an independent firm, but supplier M now sets both w_1 and w_2 to maximize integrated profit. (As explained later, the assumption that division 1 maximizes only its profit can be relaxed substantially.)

For any pair of publicly observed input prices \mathbf{w} under Delegation, assume there exists a unique downstream equilibrium in pure strategies, with the choice of downstream rival k ($k = 1, 2$) denoted $X_k^*(\mathbf{w})$ and its corresponding output level denoted $Q_k^*(\mathbf{w})$. As a

¹ The integrated input monopolist prefers to maintain firm 2 as an active buyer of the input rather than foreclose firm 2 (under both Bertrand and Cournot downstream competition) if firm 2 is a sufficiently efficient competitor. See Arya, Mittendorf, and Sappington (2008).

² Under this representation, Chen (2001) compares partial integration to no integration. Arya, Mittendorf, and Sappington (2008) compare the outcomes under partial integration when downstream competition is Bertrand or Cournot. Neither article considers the Delegation structure.

function of the downstream variables \mathbf{x} , a firm's output level is denoted $Q_k(\mathbf{x})$.³ To simplify, we assume for each firm fixed proportions, one-to-one, between the output and input quantities, so the function $Q_k^*(\mathbf{w})$ is also k 's input demand (and $Q_k(\mathbf{x})$ is the input demand conditional on the downstream variables). We also assume that the marginal cost of producing the input is constant at c , and that all the relevant functions are differentiable.⁴

The integrated firm's total profits are given by:

$$V(\mathbf{x}; w_2) = \Pi_1(\mathbf{x}; w_1) + (w_1 - c)Q_1(\mathbf{x}) + (w_2 - c)Q_2(\mathbf{x}). \quad (1)$$

Note that this integrated profit function depends on the downstream variables \mathbf{x} and the input price w_2 , but not directly on w_1 since input payments from division 1 are a pure transfer (though under Delegation w_1 will affect V indirectly, by changing the equilibrium values of \mathbf{x}).

Under Delegation, supplier M sets w_1 and w_2 to maximize

$$V^D(\mathbf{w}) \equiv V(X_1^*(\mathbf{w}), X_2^*(\mathbf{w}); w_2). \quad (2)$$

Under Centralization, denote the downstream equilibrium choices by the functions $X_1^C(w_2)$ and $X_2^C(w_2)$. Supplier M now sets w_2 to maximize

$$V^C(w_2) \equiv V(X_1^C(w_2), X_2^C(w_2); w_2). \quad (3)$$

Let w_2^C denote the profit-maximizing choice; x_1^C and x_2^C denote the resulting equilibrium downstream choices; and V^C the corresponding maximized profit of the integrated firm.

³ We thus have $Q_k^*(\mathbf{w}) \equiv Q_k(X_1^*(\mathbf{w}), X_2^*(\mathbf{w}))$. Similarly, we define $\Pi_k^*(\mathbf{w}) \equiv \Pi_k(X_1^*(\mathbf{w}), X_2^*(\mathbf{w}); w_k)$. Under Cournot competition, $Q_k(\mathbf{x}) = x_k$ and $Q_k^*(\mathbf{w}) = X_k^*(\mathbf{w})$.

⁴ We thus are implicitly assuming there are no binding capacity constraints and the downstream products are differentiated (i.e. imperfect substitutes). We also assume the downstream firms' reaction functions are either strictly increasing (strategic complements) or strictly decreasing (strategic substitutes), and have slopes smaller than unity in absolute value. In addition, we assume $\partial \Pi_k / \partial x_k$ is either strictly increasing or strictly decreasing in w_k , so that X_k^* is either strictly increasing or strictly decreasing in w_k (for $k=1,2$). All these conditions are assumed to hold over the relevant range of the variables.

We now offer two useful observations. First, the best response function of firm 2 is the same under both regimes:

$$R_2(x_1; w_2) \equiv \arg \max_{x_2} \Pi_2(\mathbf{x}; w_2). \quad (4)$$

Second, although there is no explicit input price w_1 under Centralization, the integrated firm faces a shadow marginal cost (C_1 below) of supplying the input to its downstream division. This shadow marginal cost is equal to the resource cost c plus any opportunity cost due to reduced input sales to firm 2 resulting from 1's expansion:

$$C_1(w_2) \equiv c + (w_2 - c)D_{21}(w_2), \quad (5)$$

where $D_{21} \equiv -\frac{\partial Q_2/\partial x_1}{\partial Q_1/\partial x_1}$ is the input diversion ratio—i.e. decreased input sales to firm 2 per extra unit of input to division 1. (The input diversion ratio D_{21} is evaluated at $x_1 = X_1^C(w_2)$ and $x_2 = X_2^C(w_2)$, hence is a function of w_2 .) Note that $D_{21} > 0$ if downstream competition is Bertrand, since division 1's output expansion (induced by cutting price) displaces some sales of firm 2; and $D_{21} = 0$ if competition is Cournot, since division 1 then takes firm 2's output, and thus also firm 2's input purchases, as given.⁵

3. Results

We now compare the two regimes, starting with the case where the integrated firm uses linear pricing for the input to firm 2, and then considering also two-part tariffs.

3.1 Linear Pricing of the Input

Under Delegation, let $W_1^D(w_2)$ denote the integrated firm's optimal choice for w_1 given w_2 , i.e. $W_1^D(w_2) \equiv \arg \max_{w_1} V^D(\mathbf{w})$. We establish the following result.

⁵ Arya, Mittendorf, and Sappington (2008) invoke this distinction to show that, unlike in standard duopoly, Bertrand competition downstream can yield higher prices than Cournot when a partially integrated input monopolist sells also to a downstream rival, because the monopolist internalizes a higher opportunity cost under Bertrand than under Cournot.

Proposition 1. (i) Under Delegation, the integrated firm can achieve the same profit as under Centralization by setting $w_2 = w_2^C$ and $w_1 = C_1(w_2^C)$, i.e. $V^C = V^D(C_1(w_2^C), w_2^C)$.

(ii) Therefore, the integrated firm's equilibrium profit is (weakly) higher with Delegation, i.e. $V^C = V^D(C_1(w_2^C), w_2^C) \leq V^D(W_1^D(w_2^C), w_2^C) \leq V^D(w_1^D, w_2^D) = V^D$.

Proof. (i) Under Delegation, suppose the integrated firm were to set $w_2 = w_2^C$ and $w_1 = C_1(w_2^C)$, i.e. the same input price to firm 2 as with Centralization, and an input price to division 1 equal to the shadow marginal cost prevailing with Centralization. This would induce the same downstream equilibrium choices as with Centralization:

$$X_1^*(C_1(w_2^C), w_2^C) = x_1^C \quad \text{and} \quad X_2^*(C_1(w_2^C), w_2^C) = x_2^C. \quad (6)$$

To see this, observe first that under Delegation, division 1's best response function maximizes $\Pi_1(\mathbf{x}; w_1)$ and, hence, is implicitly determined by:

$$\frac{\partial \Pi_1}{\partial x_1} = 0, \quad (7)$$

whereas with Centralization, division 1's best response function maximizes $V(\mathbf{x}; w_2)$ and, hence, using (1), is implicitly determined by

$$\frac{\partial \Pi_1}{\partial x_1} + (w_1 - c) \frac{\partial Q_1}{\partial x_1} + (w_2 - c) \frac{\partial Q_2}{\partial x_1} = 0. \quad (8)$$

Setting $w_1 = C_1(w_2)$ from (5) makes (7) identical to (8), so division 1's best response will be the same function of w_2 and x_2 in both cases. Since firm 2's best response to w_2 and the expected x_1 is also the same function in both cases, setting $w_2 = w_2^C$ and $w_1 = C_1(w_2^C)$ under Delegation would replicate the Centralization outcome.

(ii) Under Delegation, the integrated firm's optimal choice for w_1 conditional on $w_2 = w_2^C$ is typically not $C_1(w_2^C)$. Profit could be (weakly) increased by setting w_1 at the optimal level $W_1^D(w_2^C)$. (The direction of desired deviation in w_1 will be discussed later in Corollary 1.) The actual optimum will generally involve changing both w_1 and w_2 , thereby further increasing profit compared to Centralization.

It is useful to clarify why the ability to charge division 1 an observable input price $w_1 \neq C_1(w_2)$ under Delegation benefits the integrated firm. The direct effect of changing w_1 (holding \mathbf{x} constant) on integrated profit is zero because it is a pure transfer; the change in x_1 induced by a small change in w_1 also has no effect on integrated profit because, given w_2 and $x_2 = X_2^C(w_2)$, setting $w_1 = C_1(w_2)$ already induces the x_1 that maximizes integrated profit. Therefore, a small change in w_1 affects integrated profit solely by altering firm 2's choice, x_2 , in response to the expected change in x_1 . The same outcome could be achieved if, under Centralization, the integrated firm could act as a Stackelberg leader in the downstream competition, as in the following game.⁶

Leadership: As with Centralization, M publicly commits only to w_2 . But now downstream choices occur sequentially, with first the integrated firm choosing its downstream variable x_1 to maximize integrated profit (not division 1's profit) and firm 2 then choosing its best response, $x_2 = R_2(x_1; w_2)$.

Proposition 2. The integrated firm's profit is the same under Delegation or Leadership.

Proof. Under Leadership, the integrated firm sets w_2 and x_1 to maximize:

$$V(x_1, R_2(x_1; w_2); w_2). \quad (9)$$

Under Delegation, let $w_1 = W_1(x_1; w_2)$ be the inverse function of $x_1 = X_1^*(w_1, w_2)$. Setting w_1 and w_2 to maximize $V(X_1^*(\mathbf{w}), X_2^*(\mathbf{w}); w_2)$ is equivalent to setting w_2 and x_1 to maximize:

$$V(x_1, X_2^*(W_1(x_1; w_2), w_2); w_2). \quad (10)$$

Since $X_2^*(W_1(x_1; w_2), w_2) \equiv R_2(x_1; w_2)$, the two maximization problems are identical.

⁶ Lu, Moresi, and Salop (2007) show the same result as the ensuing Proposition 2 assuming Bertrand competition. On the general connection between strategic delegation and Stackelberg leadership in the competition game, see Vickers (1985, sections I and II). Adapted to our setting, his agent appointment game—that precedes downstream competition—corresponds to whether the integrated firm initially adopts the Centralization structure and sets only an input price w_2 or the Delegation structure and sets an additional input price w_1 . The agent appointment game maximizes the integrated firm's profit if and only if it implements the same downstream outcome as Stackelberg leadership by the integrated firm.

Our Delegation structure in which division 1 maximizes solely its downstream profit offers one way to implement the downstream Stackelberg leader outcome for the integrated firm. The same outcome could be implemented under alternative objective functions for division 1, as long as division 1's downstream choice is affected by the input price w_1 (and not entirely independent of it as occurs when division 1 treats w_1 as a pure transfer within the integrated firm). We will expand on this point in the Conclusion.

Delegation and public commitment to an input price w_1 , therefore, are a means of shifting division 1's best response function in the ensuing downstream game so as to change firm 2's behavior. The general strategic incentives for commitment through vertical delegation are well known (e.g. Vickers, 1985; Bonnano and Vickers, 1988; Caillaud and Rey, 1994). However, the analysis of incentives is seemingly more complex here, because here firm 2 is both a downstream rival and an input customer. Nevertheless, we can establish a fairly general result. Recalling that $x_k = p_k$ under Bertrand competition and $x_k = q_k$ under Cournot ($k = 1,2$), we have:

Proposition 3. At $(x_1, x_2, w_2) = (x_1^C, x_2^C, w_2^C)$, V is decreasing in x_2 if downstream competition is Bertrand and increasing in x_2 if downstream competition is Cournot.

Proof. From (2) and (1), we have:

$$\frac{\partial V^C}{\partial w_2} = \frac{\partial V}{\partial x_1} \frac{\partial X_1^C}{\partial w_2} + \frac{\partial V}{\partial x_2} \frac{\partial X_2^C}{\partial w_2} + Q_2^C = \frac{\partial V}{\partial x_2} \frac{\partial X_2^C}{\partial w_2} + Q_2^C = 0, \quad (11)$$

where the second equality follows from $\partial V/\partial x_1 = 0$ at $(x_1, x_2, w_2) = (x_1^C, x_2^C, w_2^C)$. It then follows from (12) and $Q_2^C > 0$ that $(\partial V/\partial x_2)(\partial X_2^C/\partial w_2) < 0$. If downstream competition is Bertrand, we have $\partial X_2^C/\partial w_2 > 0$, implying $\partial V/\partial x_2 < 0$; if it is Cournot, $\partial X_2^C/\partial w_2 < 0$, implying $\partial V/\partial x_2 > 0$.

Observe that Proposition 3 identifies opposite incentives to those in a standard duopoly environment where firm 1 does not supply firm 2.⁷ There, starting from the

⁷ See, for example, Fudenberg and Tirole (1984), Bulow, Geanakoplos, and Klemperer (1985), Fershtman and Judd (1987), or the survey by Shapiro (1989).

equilibrium with simultaneous downstream choices and holding x_1 constant, firm 1 would gain from a rise in 2's price under Bertrand competition or a fall in 2's output under Cournot. This is the usual "softening downstream competition" effect. It is present also in our context, but is necessarily dominated by the opposing input supply effect. Given that the integrated firm under Centralization sets the input price to firm 2 at the profit-maximizing level, w_2^C , a small reduction in w_2 would have the following equal but opposite effects: profit from inframarginal input sales would fall, implying that profit must rise (equally) from increased input sales—even after incorporating the loss in division 1's downstream profit caused by firm 2's expansion. Thus, holding w_2 constant at w_2^C , the integrated firm would gain on balance, despite the loss to division 1, if firm 2 were exogenously to increase its input purchases and downstream sales.

Building on the above observation, the next result identifies the integrated firm's incentive to change w_1 under Delegation, relative to the shadow marginal cost under Centralization, $C_1(w_2^C)$.

Corollary 1. Holding w_2 constant at w_2^C , under Delegation:

- (i) With Bertrand competition, the integrated firm wants to induce a fall in 2's price, implying $W_1^D(w_2^C) < (>) C_1(w_2^C)$ if prices are strategic complements (substitutes).
- (ii) With Cournot competition, the integrated firm wants to induce a rise in 2's quantity, implying $W_1^D(w_2^C) > (<) C_1(w_2^C)$ if quantities are strategic substitutes (complements).

Proof. Starting at $w_2 = w_2^C$ and $w_1 = C_1(w_2^C)$, under Delegation the integrated firm's desired change in w_1 is determined by the sign of (from (3)):

$$\frac{\partial V^D}{\partial w_1} = \frac{\partial V}{\partial x_1} \frac{\partial X_1^*}{\partial w_1} + \frac{\partial V}{\partial x_2} \frac{\partial X_2^*}{\partial w_1} = \frac{\partial V}{\partial x_2} \frac{\partial X_2^*}{\partial w_1}. \quad (12)$$

Observe that $\partial X_2^*/\partial w_1 = (\partial R_2/\partial x_1)(\partial X_1^*/\partial w_1)$, and we assumed $\partial X_1^*/\partial w_1 > (<) 0$ if downstream competition is Bertrand (Cournot), while by definition $\partial R_2/\partial x_1 > (<) 0$ if downstream variables are strategic complements (substitutes).

As noted, fixing w_1 in our setting is akin to an "investment" that affects marginal cost in a standard two-stage game and thereby shifts division 1's best-response function in the

downstream competition. The change in w_1 needed to induce the desired change in x_2 will therefore depend on familiar issues — whether downstream competition is in prices or quantities and whether these choice variables are strategic complements or substitutes.⁸ Since the basic logic is the same for parts (i) and (ii) of Corollary 1, we will explain only (i).

For Bertrand competition, we know from Proposition 3 that the integrated firm’s overall profit would increase if firm 2 were to *reduce* its price p_2 (holding p_1 and w_2 constant at their Centralization equilibrium values). Under Delegation, the integrated firm can induce firm 2 to cut its price by lowering w_1 to signal a reduction in division 1’s price p_1 in the “normal” case where prices are strategic complements (or via an increase in w_1 if prices are strategic substitutes). A small change in p_1 from the initial equilibrium has a zero first-order effect on the integrated firm’s profit and, hence, the change in w_1 is profitable. Essentially, lowering p_1 is an imperfect way to shrink firm 2’s margin and *reduce double marginalization* in input sales to firm 2.⁹

Finally, we note that Proposition 3 identifies the incentives regarding w_1 under Delegation while holding w_2 constant at the Centralization solution w_2^C . In equilibrium, the integrated firm would typically adjust also w_2 . However, since the adjustment in w_2 is only in response to the change in x_2 (induced by w_1), one would expect the equilibrium change in x_2 to track the direction indicated by Proposition 3 and Corollary 1. This expectation is confirmed for the linear example reported in the Appendix.

3.2 Two-Part Tariff for the Input

Suppose that instead of linear pricing of the input, the integrated firm uses a two-part tariff: a pair (w_2, f_2) , where f_2 is a fixed upfront fee. Proposition 1 continues to hold—the integrated firm still earns (weakly) higher profit under Delegation. The logic is as follows. Denote the downstream choices that maximize overall industry profit (downstream plus

⁸ See Fudenberg and Tirole (1984), Bulow, Geanakoplos, and Klemperer (1985), or Shapiro (1989, pp. 385-386).

⁹ However, because the decrease in p_2 is induced by lowering p_1 , firm 2’s output is likely to decline on balance relative to Centralization (still holding $w_2 = w_2^C$). The linear demand example in the Appendix supports this intuition.

upstream) as the “monopoly solution,” x_1^m and x_2^m . Under Delegation, the integrated firm can attain this monopoly profit, by setting the marginal input prices at the levels w_1^m and w_2^m that will induce division 1 and firm 2 to choose (unilaterally) x_1^m and x_2^m , and setting f_2 to extract firm 2’s profits.¹⁰

With Centralization, the integrated firm generally cannot implement the monopoly solution even with two-part tariffs. The ability to charge firm 2 an upfront fixed fee will typically lead the integrated firm to reduce w_2 below the optimal level for linear pricing. However, the fixed fee does not affect the shadow marginal cost of supplying to division 1 as a function of w_2 , which remains $C_1(w_2)$ from (5). Generally, this will *not* be the integrated firm’s optimal w_1 conditional on w_2 under Delegation. For example, suppose that maximizing industry profit requires a symmetric outcome downstream, $x_1 = x_2 = x^m$, and equal input marginal prices under Delegation, $w_1 = w_2 = w^m$. Under broad conditions, we have $C_1(w_2) < w_2$.¹¹ Thus, Centralization cannot achieve the downstream monopoly outcome: if the integrated firm sets $w_2 = w^m$, its shadow marginal cost will be $C_1(w^m) < w^m$, yielding asymmetric downstream choices skewed to favor division 1.

Since Delegation attains the monopoly outcome, Proposition 2 also continues to hold. Under Leadership, the integrated firm can achieve the monopoly outcome by setting $w_2 = w_2^m$ and $x_1 = x_1^m$, which will induce firm 2 to set $x_2 = x_2^m$. And by definition, it can do no better than the monopoly outcome. Thus, Delegation and Leadership yield the same payoff.

Unlike Propositions 1 and 2 that extend to two-part tariffs for the input, the direction in which the integrated firm wants to shift firm 2’s choice when moving from Centralization to Delegation can differ with two-part tariffs from the pattern shown in Proposition 3 with linear pricing of the input. Under Centralization, if firm 2 accepts the

¹⁰ When division 1 and firm 2 are competitors, to implement x_1^m and x_2^m the integrated firm typically must set the marginal input prices above the marginal cost c . The qualitative results do not change if firm 2 has an alternative source of supply and, as a result, the integrated firm cannot extract all the profits of firm 2.

¹¹ From (5), $C_1 = c$ with Cournot competition (since $D_{21} = 0$) and, typically, $C_1 < w_2$ with differentiated Bertrand competition (since typically we have $w_2 > c$ and $D_{21} < 1$ when both firms use the input in the same fixed proportion to output).

two-part tariff offer (w_2, f_2) (as it will in equilibrium), the downstream outcome is given by the functions $X_1^C(w_2)$ and $X_2^C(w_2)$ as with linear pricing. Denote firm 2's profit gross of the fixed fee as $\Pi_2^C(w_2) = \Pi_2(X_1^C(w_2), X_2^C(w_2); w_2)$. In equilibrium, the integrated firm will extract firm 2's profit by setting $f_2 = \Pi_2^C(w_2)$. Therefore, its profit function is: $V^{C,T}(w_2) = V^C(w_2) + \Pi_2^C(w_2)$, where $V^C(w_2)$ is the same as with linear pricing (see (3)). The integrated firm chooses $w_2^{C,T} \equiv \arg \max_{w_2} V^{C,T}(w_2)$, and induces the equilibrium downstream values $x_1^{C,T}$ and $x_2^{C,T}$.

Proposition 4. At $(x_1, x_2, w_2) = (x_1^{C,T}, x_2^{C,T}, w_2^{C,T})$:

- (i) V is decreasing in x_2 if downstream competition is Bertrand and prices are strategic complements (and increasing or decreasing in x_2 if prices are strategic substitutes).
- (ii) V is decreasing in x_2 if downstream competition is Cournot and quantities are strategic substitutes (and increasing in x_2 if quantities are strategic complements).

Proof. Let $R_1^C(x_2, w_2)$ denote division 1's reaction function under Centralization. The analogue to (12) is now:

$$\frac{\partial V^{C,T}}{\partial w_2} = \frac{\partial V^C}{\partial w_2} + \frac{\partial \Pi_2^C}{\partial w_2} = \frac{\partial V}{\partial x_2} \frac{\partial X_2^C}{\partial w_2} + Q_2^C - Q_2^C + \frac{\partial \Pi_2}{\partial x_1} \frac{\partial X_1^C}{\partial w_2} = 0, \text{ or} \quad (13a)$$

$$\frac{\partial V^{C,T}}{\partial w_2} = \frac{\partial V}{\partial x_2} \frac{\partial X_2^C}{\partial w_2} + \frac{\partial \Pi_2}{\partial x_1} \left(\frac{\partial R_1^C}{\partial x_2} \frac{\partial X_2^C}{\partial w_2} + \frac{\partial R_1^C}{\partial w_2} \right) = 0. \quad (13b)$$

(i) For Bertrand competition: $\partial X_2^C / \partial w_2 > 0$, $\partial \Pi_2 / \partial x_1 > 0$, and $\partial R_1^C / \partial w_2 > 0$ (since the shadow marginal cost $C_1(w_2)$ in (5) increases with w_2); thus, (13b) implies $\partial V / \partial x_2 < 0$ if prices are strategic complements ($\partial R_1^C / \partial x_2 > 0$). (If $\partial R_1^C / \partial x_2 < 0$, the term in parentheses in (13b) has an ambiguous sign, hence so does $\partial V / \partial x_2$.)

(ii) For Cournot competition: $\partial X_2^C / \partial w_2 < 0$, $\partial \Pi_2 / \partial x_1 < 0$, and $\partial R_1^C / \partial w_2 = 0$ (there is no input diversion in Cournot, so the shadow marginal cost $C_1(w_2)$ is always equal to c); thus, (13b) implies $\partial V / \partial x_2 < 0$ if quantities are strategic substitutes ($\partial R_1^C / \partial x_2 < 0$), and $\partial V / \partial x_2 > 0$ if instead $\partial R_1^C / \partial x_2 > 0$. (The sign is now unambiguous in both cases because the shadow marginal cost does not depend on w_2 , unlike for Bertrand competition.)

Thus, for Bertrand competition and the “normal” case of strategic complements, we have the same pattern as shown in Proposition 3 with no fixed fee (i.e. $f_2 = 0$): starting at the Centralization solution, the integrated firm wants to induce a reduction in p_2 . But for Cournot competition and the “normal” case of strategic substitutes, the pattern is now reversed since here the integrated firm wants to induce a *decrease* in q_2 .

These patterns can be understood by comparing the optimal input pricing conditions (13a) or (13b) to condition (11) for linear pricing. Rearranging (11) gives $(\partial V/\partial x_2)(\partial X_2^C/\partial w_2) = -Q_2^C$.¹² With two-part tariffs, rearranging (13a) gives $(\partial V/\partial x_2)(\partial X_2^C/\partial w_2) = -(\partial \Pi_2/\partial x_1)(\partial X_1^C/\partial w_2)$. Thus, term Q_2^C is absent: changing w_2 no longer affects the integrated firm’s net profit from inframarginal input sales to firm 2, due to the required adjustment in the fixed fee which fully extracts firm 2’s profit. Instead, the novel effect under a two-part tariff is $(\partial \Pi_2/\partial x_1)(\partial X_1^C/\partial w_2)$: changing w_2 alters the integrated firm’s equilibrium downstream choice, X_1^C , which impacts firm 2’s expected profit—an effect now internalized by the integrated firm via its fixed fee to firm 2. Since $Q_2^C > 0$, if $(\partial \Pi_2/\partial x_1)(\partial X_1^C/\partial w_2) > 0$ then $\partial V/\partial x_2$ will have the same sign as under linear pricing; while if $(\partial \Pi_2/\partial x_1)(\partial X_1^C/\partial w_2) < 0$, the sign of $\partial V/\partial x_2$ will switch.

For Bertrand competition, $\partial \Pi_2/\partial x_1 > 0$. Thus, $\partial V/\partial x_2$ will have the same sign as under linear pricing if $\partial X_1^C/\partial w_2 = [(\partial R_1^C/\partial x_2)(\partial X_2^C/\partial w_2) + \partial R_1^C/\partial w_2] > 0$, which holds if $\partial R_1^C/\partial x_2 > 0$ (since $\partial X_2^C/\partial w_2 > 0$ and $\partial R_1^C/\partial w_2 > 0$ for price competition). In words: a small cut in w_2 and, hence, in firm 2’s equilibrium downstream price will—in the “normal” case where prices are strategic complements ($\partial R_1^C/\partial x_2 > 0$)—induce a reduction in firm 1’s downstream price, which lowers firm 2’s profit and the attainable fixed fee f_2 .¹³ At the optimal w_2 , this loss in f_2 due to a small cut in w_2 is just offset by a *gain* in variable profits from the induced reduction in p_2 . Thus, starting from the two-part tariff equilibrium and assuming prices are strategic complements, the integrated firm would gain from an exogenous reduction in p_2 , as under linear pricing of the input.

¹² With Bertrand competition $\partial X_2^C/\partial w_2 > 0$, implying $\partial V/\partial x_2 < 0$, whereas both inequalities are reversed under Cournot competition, as stated in Corollary 1.

¹³ Under linear pricing, a reduction in w_2 decreases profits, instead, from inframarginal input sales to firm 2.

For Cournot, $\partial \Pi_2 / \partial x_1 < 0$, so $\partial V / \partial x_2$ will *switch* sign compared to linear pricing if $\partial X_1^C / \partial w_2 = (\partial R_1^C / \partial x_2)(\partial X_2^C / \partial w_2) > 0$.¹⁴ This holds if $\partial R_1^C / \partial x_2 < 0$ (since $\partial X_2^C / \partial w_2 < 0$ for quantity competition). An increase in w_2 causing a decrease in firm 2's equilibrium downstream quantity q_2 , will—in the “normal” case where quantities are strategic substitutes—lead to an increase in firm 1's quantity q_1 , which *lowers* firm 2's profit and, hence, the attainable fixed fee f_2 . At the optimal w_2 , this loss term caused by a small rise in w_2 is just offset by a gain in variable profits from the induced decrease in q_2 . Thus, the integrated firm would gain from an exogenous *reduction* in firm 2's quantity, unlike with linear pricing of the input.

4. Discussion and Conclusions

We conclude with a brief discussion of related work, and of the plausibility of Delegation by a vertically integrated firm. Bonanno and Vickers (1988) noted the strategic advantage of commitment to observable input prices. They consider differentiated Bertrand competition between two suppliers where prices are strategic complements. If both suppliers are vertically integrated, each provides the input to its downstream retail unit at marginal cost. If both are vertically separated, each sells through a different single retailer and captures the latter's profit with a two-part tariff. For any input price set by one supplier (including marginal cost if integrated), the other supplier prefers to vertically separate and raise the input price somewhat above marginal cost, so as to coax an increase in the rival's downstream price. We showed that the incentive to influence a downstream rival through an observable input price to one's own downstream unit extends to a vertically integrated firm that supplies to the rival. However, because the rival is also an input customer, the incentive is to induce a *reduction* in its downstream price.¹⁵

¹⁴ Recall that $\partial R_1^C / \partial w_2$ is zero under Cournot, since the integrated firm's shadow marginal cost of supplying division 1 under Centralization is unaffected by w_2 because there is no input diversion.

¹⁵ Also, because our input supplier is a monopolist, it benefits from Delegation whatever the downstream competition. Bonanno and Vickers observe that if downstream variables are strategic substitutes, vertical separation will lead both firms to cut input prices, which intensifies competition and harms both firms (though separation remains unilaterally profitable).

Our assumption that under the Delegation structure the downstream division maximizes its own profit is reminiscent of the literature on the strategic advantage of creating autonomous *competing divisions* (e.g. Schwartz and Thompson, 1986; Baye, Crocker, and Ju, 1996). There, divisional autonomy is strategically beneficial because it makes the firm a tougher competitor, in order to deter entry or induce output contraction by oligopoly rivals when choice variables are strategic substitutes. Here, autonomy involves a *vertical* division and the strategic gain does not hinge on presenting a tough posture, e.g. under Cournot competition, the integrated firm uses Delegation to make itself a softer competitor in the downstream market.

Observe that our organizational Delegation framework embodies two assumptions: the downstream division acts as a profit center to maximize solely its own profit; and the integrated firm can credibly commit to charge the downstream division an input price observable by downstream rivals. The role of the profit center assumption is to prevent the downstream division from “undoing” the effects of an observable input price charged to it by fully internalizing how its choice affects the upstream division’s profit. As explained earlier, all our results will extend to any objective function of the downstream division for which the division’s downstream choice is affected by its input price. For example, if the division maximizes a weighted average of its own profit and the integrated firm’s profit, the integrated firm can implement its preferred downstream outcome by suitably raising the input price to the division compared to the case where the division only maximizes its own profit (Arya, Mittendorf, and Yoon, 2008, Proposition 3).¹⁶ In practice, integrated firms may find it feasible and beneficial to establish divisions as profit centers for incentive reasons; our analysis suggests the benefits can also be strategic in partial integration settings, provided a commitment can be made to charge one’s division an observable input price.¹⁷

¹⁶ These authors further show that decentralization (our Delegation) increases the integrated firm’s profit if the input price to the downstream division is determined by bargaining, for a range of assumptions about the relative bargaining power.

¹⁷ For a general discussion on firms’ ability to make commitments, *see* Shapiro (1989, p. 382).

The latter assumption is more questionable for unregulated firms.¹⁸ If the downstream division is an independent profit center but under common ownership with the upstream supplier, can the integrated firm commit to an arm's length input price? One consideration that may aid such commitment is the supplier's interest in maintaining a reputation for not acting opportunistically to disadvantage independents by secretly favoring its division (e.g. McAfee and Schwartz, 1994). To preserve such a reputation, it may adopt a policy of public and transparent pricing.

¹⁸ Regulated firms may be subject to rules governing input pricing to subsidiaries as well as the subsidiaries' behavior through imputation requirements (e.g. Laffont and Tirole, 2000).

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Appendix: Example

Assume that the marginal cost of the input is constant, $c = 1$, and that there are no other downstream costs. Consumer demand for the products of firms 1 and 2 is given by $q_i = 500 - 200p_i + 100p_j$, where q_i denotes the output of firm i , and p_i and p_j denote the prices of firms i and j , respectively ($i, j \in \{1, 2\}, i \neq j$).¹⁹

We begin with the case of Bertrand competition. Table 1 below shows the equilibrium profits, outputs and prices under Centralization and Delegation.

Table 1: Bertrand Competition Downstream

	Centralization	Delegation
Profit of Integrated Firm	696.97	700.00
Profit of Downstream Rival	59.50	50.00
Output of Integrated Firm, q_1	227.27	250.00
Output of Downstream Rival, q_2	109.09	100.00
Output Price of Integrated Firm, p_1	3.12	3.00
Output Price of Downstream Rival, p_2	3.52	3.50
Input Price Charged to Downstream Rival, w_2	2.97	3.00
Shadow Cost of Supplying Input to Division 1, $C_1(w_2^C)$	1.98	NA
Input Price Charged to Division 1, w_1	NA	1.75
$W_1^D(w_2^C)$	NA	1.74

Table 1 illustrates Proposition 1: Delegation allows the integrated firm to increase its profit relative to Centralization. It also illustrates Proposition 3 and Corollary 1(i): starting from the Centralization solution, the integrated firm wants to induce a *reduction* in firm 2's

¹⁹ In this example where firms 1 and 2 are symmetric, vertical integration of firm 1 and the input monopolist does not lead to foreclosure of firm 2 under Centralization. See Arya, Mittendorf, and Sappington (2008). Our results imply that this also is true under Delegation.

downstream price p_2 ; and since linear demands imply that prices are strategic complements, inducing a reduction in p_2 requires signaling a reduction in p_1 by lowering w_1 : $W_1^D(w_2^C) < C_1(w_2^C)$. That is, at the input price charged to firm 2 under Centralization (2.97), the profit-maximizing input price to division 1 under Delegation is lower than the shadow marginal cost under Centralization, $1.74 < 1.98$. In the actual Delegation equilibrium, the integrated firm raises w_2 slightly, from 2.97 to 3.00, and reduces w_1 to 1.75 (instead of 1.74) from the shadow marginal cost of 1.98. The reduction in w_1 and increase in w_2 lead to a (substantial) decrease in the profit of the downstream rival.

We now turn to the case of Cournot competition.

Table 2: Cournot Competition Downstream

	Centralization	Delegation
Profit of Integrated Firm	682.76	685.71
Profit of Downstream Rival	45.66	48.98
Output of Integrated Firm, q_1	279.31	257.14
Output of Downstream Rival, q_2	82.76	85.71
Output Price of Integrated Firm, p_1	2.86	3.00
Output Price of Downstream Rival, p_2	3.52	3.57
Input Price Charged to Downstream Rival, w_2	2.97	3.00
Shadow Cost of Supplying Input to Division 1, C_1	1.00	NM
Input Price Charged to Division 1, w_1	NM	1.29
$W_1^D(w_2^C)$	NM	1.28

Table 2 also illustrates Proposition 1—the integrated firm’s profit again is higher under Delegation. It also illustrates Proposition 3 and Corollary 1(ii): starting from the Centralization solution, the integrated firm wants to induce an *increase* in firm 2’s quantity q_2 ; and since linear demands imply that quantities are strategic substitutes, this requires signaling a reduction in q_1 by raising w_1 . Thus, moving to Delegation the integrated firm wants to raise the input price to division 1 above the shadow marginal cost under Centralization (which, under Cournot competition, simply equals the resource marginal

cost, $C_1 = c = 1$, independent of w_2): $W_1^D(w_2^C) = 1.28 > 1.00 = C_1$. In the actual Delegation equilibrium, w_1 rises substantially, to 1.29 (from $C_1 = 1$), while w_2 rises slightly, from 2.97 to 3.00, causing the integrated firm's output q_1 to fall and the rival's output q_2 to rise, consistent with the incentives described above. This time, the rival's profit increases, unlike in the Bertrand case.